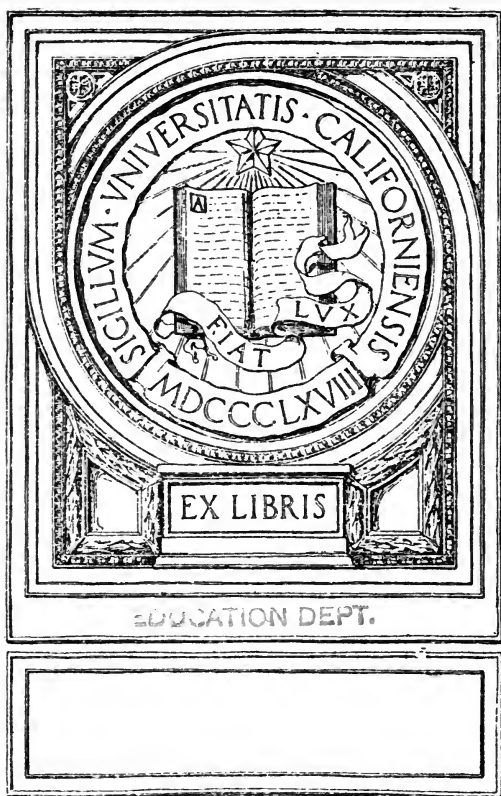


Commercial Arithmetic





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WILLIAMS & ROGERS SERIES

COMMERCIAL ARITHMETIC

BY

OSCAR F. WILLIAMS, A. M.

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EDUCATION DEPT.
W. P. 12

PREFACE.

NO school text-book is used now-a-days in consequence of prefatory compliments paid it by its authors or publishers. In this day of general enlightenment, teachers understand the necessities of their classes, and, as a rule, need no advice as to what or how to teach.

Since the wants of American schools and the ideas of American teachers are various, a variety of text-books upon every topic of school instruction is required, and with the hope and belief that the contents of this volume will more nearly meet the necessities of some schools and the ideas of some teachers, than any of the several good books upon the subject of Arithmetic now in print, the work is respectfully submitted by

THE AUTHORS.

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COMMERCIAL ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the Science of Numbers and the Art of Computation.
2. A **Unit** is a single thing.
3. A **Number** is a unit or a collection of units.
4. The **Unit** of a number is *one* of the collection of units forming the number; thus, the unit of 5 is 1; of 17 dollars, 1 dollar; of 30 pupils, 1 pupil.
5. An **Integer** is a whole or entire number.
6. An **Even Number** is one that can be exactly divided by 2; as, 6, 8, 44.
7. An **Odd Number** is one that cannot be exactly divided by 2; as, 5, 9, 23.
8. A **Composite Number** is one that can be resolved or separated into factors; as, $4 = 2 \times 2$; $12 = 3 \times 2 \times 2$.
9. A **Prime Number** is one that cannot be resolved or separated into factors, being divisible only by itself and unity; as, 1, 2, 3, 5, 7, 19, 83.
10. An **Abstract Number** is one used without reference to any particular thing or quantity; as, 3, 11, 24.
11. A **Concrete Number** is one used with reference to some particular thing or quantity; as, 3 dollars, 11 men, 24 cords of wood.
12. A **Compound Denominate**, or **Compound Number**, is a concrete number expressed by two or more orders of units; as, 3 dollars and 11 cents; 5 pounds, 2 ounces and 15 pennyweights.
13. **Like Numbers** are such as have the same unit value; as, 5, 14, 37; or, 5 men, 14 men, 37 men; or, if denominate, the same kind of quantity; as, 5 hours 14 minutes 37 seconds.
14. **Unlike Numbers** are such as have different unit values; as, 11, 16 days, 265 dollars, 5 pounds, 4 yards.
15. **Ratio** is the comparison of magnitudes. It is of two kinds; *arithmetical* and *geometrical*.
16. **Arithmetical Ratio** expresses a difference.
17. **Geometrical Ratio** expresses a quotient.
18. A **Problem** in Arithmetic is a question to be solved; its *analysis*, the logical statement of its conditions and of the steps required for its solution.

19. The **Conclusion** of the analysis is called the *answer*, or *result*.

20. A **Rule** is an outline of the steps to be taken in a solution.

SIGNS.

21. A **Sign** is a character used to express a relation of terms or to indicate an operation to be performed.

The following are the principal and most useful arithmetical signs:

22. The **Sign of Addition** is a perpendicular cross, $+$. It is called *Plus*, and indicates that the numbers between which it is placed are to be added; thus, $5 + 4$ indicates that 4 is to be added to 5.

23. The **Sign of Subtraction** is a short horizontal line, $-$. It is called *Minus*, and indicates, when placed between two numbers, that the value of the number on its right is to be taken from the value of the number on its left; thus, $8 - 3$, indicates that 3 is to be subtracted from 8.

24. The **Sign of Multiplication** is an oblique cross, \times . It indicates that the numbers between which it is placed are to be multiplied together; thus, 7×9 , indicates that the value of 7 is to be taken 9 times.

25. The **Common Sign of Division** is a short horizontal line with a point above and one below, \div . It indicates a comparison of numbers to determine a quotient, it being understood that the number at the left of the sign is to be divided by the one at its right; thus, $20 \div 5$, indicates that 20 is to be divided by 5.

26. The **Sign of Ratio** is the colon, $:$; it also indicates division.

27. The **Sign of Equality** is two short horizontal lines, $=$. It is read *equals*, or, is *equal to*, and indicates that the numbers, or expressions, between which it is placed are equal to each other; thus, $2 + 2 = 4$.

28. The **Signs of Aggregation** are the parenthesis, $()$, brackets, $[]$, brace, $\{ \}$, and vinculum, $—$. They indicate that the quantities included within, or connected by them, are to be taken together and subjected to the same operation.

29. The **Index**, or **Power Sign**, is a small figure placed at the right of and above another figure. It indicates that the number over which it is placed is to be taken as a factor a number of times equal to the numerical value of the index. Thus 4^2 indicates that 4 is to be taken twice as a factor, or multiplied by itself once; 4^3 indicates that 4 is to be used three times as a factor. 4^2 is read 4 squared; 4^3 is read 4 cubed; also, the second power of 4; the third power of 4.

30. The **Root**, or **Radical Sign**, is the character, $\sqrt{}$; it is the opposite of the index, or power sign. When there is no figure in the opening, it indicates that the quantity over which the sign is placed is to be separated into *two* equal factors, or its *square root* taken. A figure placed in the opening indicates the number of equal factors required, or the root to be extracted; as, $\sqrt[3]{27}$, $\sqrt[4]{16}$.

31. The **Dollar Sign** is the character, \$.

32. The **Cent Sign** is the character, ¢.

33. The **Decimal Point** is the period, . ; when employed to separate dollars from cents it is called a *Separatrix*.

Fractional parts of a dollar are expressed only as hundredths; thus \$14.53 is read 14 dollars and 53 cents, or 14 and 53 hundredths dollars.

ABBREVIATIONS AND CONTRACTIONS.

34. The following are some of the principal abbreviations and contractions in common use:

Bbl. or **Bar.** for barrel or barrels.

Bu. for bushel or bushels.

Cd. for cord or cords.

Ct. for cent or cents.

Cwt. for hundred weight or hundred weights.

Cent. for cental or centals.

Da. for day or days.

Doz. for dozen or dozens.

Ft. for foot or feet.

Gal. for gallon or gallons.

Hhd. for hogshead or hogsheads.

In. for inch or inches.

Lb. for pound or pounds.

Mo. for month or months.

Oz. for ounce or ounces.

Pk. for peck or pecks.

Pt. for pint or pints.

Qt. for quart or quarts.

Yd. for yard or yards.

NOTE. — Other and more complete lists of abbreviations and contractions, together with illustrations of their uses, will be given in the advanced part of this work.

NOTATION AND NUMERATION.

35. **Notation** is the method of *expressing* numbers.

There are three ways of expressing numbers.

I. By **Words**; as one, two, three.

II. By **Figures**, called the *Arabic*, or, more properly, the *Indian Notation*; this notation employs the nine digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, and the naught, 0, which is also called zero. and cipher.

By this method a number is written and read with direct reference to its successive periods, commencing with the highest.

III. By **Letters**, called the *Roman Notation*; this notation employs the seven capital letters; I for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred, and M for one thousand.

By this method a number is written and read with direct reference to its successive orders, and multiplication by one thousand is indicated by over-scoring the letter whose value is to be so increased; as, V for five, \overline{V} for five thousand; M for one thousand, \overline{M} for a thousand thousand, or a million.

36. **Numeration** is the method of *reading* numbers expressed by words, figures, or letters.

ARABIC METHOD OF NOTATION.

37. By the **Arabic Method** the value of numbers increases from right to left, and decreases from left to right in a ten-fold ratio; the successive figures from right to left or from left to right are called orders of units, the value of one of any order being ten times the value of one of the order next to its right, and only one-tenth the value of one of the order next to its left; for example, in the number one hundred and eleven, expressed 111, the second 1 is equal in value to ten times the first 1, but to only one-tenth the value of the third 1.

The succession of the orders of units in writing numbers by this method, establishes a decimal system in which the numbers are divided for convenience into periods of three figures, or places, each. Numbers so written are read or enumerated from right to left to *ascertain* their value, and from left to right to *announce* their value. The naught, or cipher, is always read as of the order of the place it occupies.

For example, in reading to ascertain the value of the expression 265017, we begin at the right and name the successive orders of units: units, tens, hundreds, thousands, tens of thousands, hundreds of thousands. Having now determined the names of the given units, we read from the left, and announce the number as two hundred sixty-five thousand seventeen.

FRENCH SYSTEM OF NUMERATION.

38. The separating of written numbers into uniform periods of three figures, or places, as explained above, is known by its origin and use as the French system of numeration. This is the system invariably used in the United States.

39. The **Periods** take their names from the Latin numerals, with certain established variations, and numbers are divided into orders of units and into periods, and are read as shown by the following

French Numeration Table.

Billions Period.			Millions Period.			Thousands Period.			Units Period.		
3, Trillions.	1, Hundreds of Billions.	30 Tens of Billions.	3, Hundreds of Millions.	4 Tens of Millions.	3, Millions.	3, Hundreds of Thousands.	30 Tens of Thousands.	3, Thousands.	1, Hundreds.	30 Tens.	3, Units.

The other successive periods are called Quadrillion, Quintillion, Sextillion, Septillion, Octillion, etc. Commencing with the right figure, which is called units of the first order, or *simple units*, the orders of figures, or units, to the left, are called units of the second order, units of the third order, fourth, fifth, sixth, etc.

ENGLISH SYSTEM OF NUMERATION.

40. There is in use a system known as the English numeration, which gives to each period after thousands, six places, or figures, instead of three as given by the French numeration. Numbers are divided into periods, and enumerated and read by the English numeration as shown by the following

English Numeration Table.

2 Billions.	5 Hundreds of Thousands of Millions.	7 Tens of Thousands of Millions.	3 Thousands of Millions.	1 Hundreds of Millions.	8 Tens of Millions.	1 Millions.					
						9 Hundreds of Thousands.	6 Tens of Thousands.	4 Thousands.	3 Hundreds.	7 Tens.	2 Units.

Each period of the higher orders has also six places.

REMARK.—The English system of Numeration being of no practical value to pupils in the schools of the United States, it will not be hereafter referred to.

41. The Arabic method of notation is based upon the following

General Principles.—1. *The removal of any figure one place toward the left multiplies its value by ten; two places, by one hundred; three places, by one thousand, etc.*

2. *The removal of any figure one place toward the right divides its value by ten; two places, by one hundred, etc.*

3. *A cipher placed after a significant figure multiplies it by ten; two ciphers so placed, multiplies it by one hundred, etc.*

42. Write and read :

1. Nine units of the first order.
2. Five units of the first order and two units of the second order.
3. Eight units of the first order, three of the second, and one of the first.
4. Four units of the fourth order, nine of the third, and two of the second.
5. Two units of the fifth order, nine of the third, and seven of the first.
6. One unit of the sixth order, nine of the fourth, six of the second, and eight of the first.
7. Seven units of the seventh order and seven of the first.
8. Six units of the eighth order, four of the sixth, seven of the fourth, and one of the second.
9. One unit of the ninth order, two of the eighth, three of the seventh, four of the sixth, five of the fifth, six of the fourth, seven of the third, eight of the second, and nine of the first.

43. Express by figures the following numbers:

1. Sixty-four.
2. One hundred forty-eight.
3. One thousand four hundred six.
4. Twenty thousand twenty-one.
5. Three hundred sixty-five thousand.
6. Eighty million forty-two.
7. Ninety thousand nine hundred.
8. Fifty million fifty-one
9. Eighty-seven billion seven thousand twelve.
10. Ninety-seven million ninety-seven thousand ninety-seven.
11. Twenty-one million twenty-five.
12. Sixteen billion sixteen million sixteen.
13. Six hundred eighty-nine thousand nine hundred seven.
14. Nineteen billion five hundred forty-one million eleven thousand eleven.
15. Twenty-seven quintillion eighty-one quadrillion two trillion seven hundred sixty billion one million two.

44. Point off into periods, numerate, and read the following numbers:

- | | | |
|-------------|-------------------|------------------------|
| 1. 386. | 8. 77010016. | 15. 987000460000. |
| 2. 1341. | 9. 200020. | 16. 27510304050. |
| 3. 12406. | 10. 1400246780. | 17. 11002200330044. |
| 4. 79001. | 11. 2100211. | 18. 2234567890. |
| 5. 872463. | 12. 5860092. | 19. 46122555003. |
| 6. 901008. | 13. 34307061. | 20. 621438001246709. |
| 7. 4681005. | 14. 100010001000. | 21. 12345325596001503. |

45. Write in words and read the following numbers:

- | | | |
|-----------|------------|----------------|
| 1. 920. | 5. 50415. | 9. 1406250. |
| 2. 1146. | 6. 100000. | 10. 54790207. |
| 3. 3070. | 7. 521469. | 11. 1021714. |
| 4. 61036. | 8. 201012. | 12. 579607359. |

ROMAN NOTATION AND NUMERATION.

46. By combining, according to certain principles, the letters used in this method of writing numbers, any number can be expressed.

Principles.—1. *Repeating a letter repeats its value.*

Thus, I = one, II = two, X = ten, XX = twenty.

2. *If a letter of any value is annexed to one of greater value, the sum of the two values is indicated; if a letter of any value is prefixed to one of greater value, the difference of their values is indicated.*

Thus, XI denotes $X + I =$ eleven, IX denotes $X - I =$ nine.

3. *A dash — placed over a letter multiplies its value by one thousand.*

Thus, V = five, \overline{V} = five thousand, CD = four hundred, \overline{CD} = four hundred thousand, LXVII = sixty-seven, \overline{LXVII} = sixty-seven thousand.

Table of Roman Numerals with Arabic Equivalents.

I,	1.	XII,	12.	L,	50.	DCC,	700.
II,	2.	XIII,	13.	LX,	60.	DCCC,	800.
III,	3.	XIV,	14.	LXX,	70.	CM,	900.
IV,	4.	XV,	15.	LXXX,	80.	M,	1000.
V,	5.	XVI,	16.	XC,	90.	MM,	2000.
VI,	6.	XVII,	17.	C,	100.	\overline{X} ,	10000.
VII,	7.	XVIII,	18.	CC,	200.	\overline{L} ,	50000.
VIII,	8.	XIX,	19.	CCC,	300.	\overline{C} ,	100000.
IX,	9.	XX,	20.	CD,	400.	\overline{D} ,	500000.
X,	10.	XXX,	30.	D,	500.	\overline{M} ,	1000000.
XI,	11.	XL,	40.	DC,	600.		

47. Read the following expressions:

XCII.	CCXVII.	CMXIX.	\overline{DLXX} .
XXVII.	DCV.	MCLXXIX.	\overline{DCCXLV} .
XXIX.	DCCX.	MCDXCH.	\overline{MDC} .
CLX.	CMXXV.	MDCCLVI.	$\overline{MDCCCLXXXVIII}$.

48. Express by the Roman system the following numbers:

42.	7454.	1629.	45450.	6059.
111.	8709.	1889.	19015.	21021.
666.	62550.	460012.	1111.	4888.
1125.	1620.	3658.	6057.	90909.
7000.	399.	175400.	3113.	5168.
11451.	25406.	1761.	90055.	1890.
997.	48250.	1887.	805000.	1775.
56104.	3700.	1000000.	365.	1893.
3001.	2865.	20000.	1515.	1900.

ADDITION.

49. Addition is the process of combining several numbers into one equivalent number.

50. The Sum or Amount is the result obtained by the addition of two or more numbers.

51. The Sign of Addition is $+$, and is called *Plus*, which signifies *more*. When placed between two numbers or combinations of numbers, it indicates their addition; as, $5 + 2$ is read 5 plus 2, and shows that 5 and 2 are to be added.

52. The Sign of Equality is $=$. When placed between two numbers or combinations of numbers, it indicates that there is no difference in their value; thus, $5 + 2 = 7$, is read 5 plus 2 equals 7, and indicates that the value of 7 equals the value of the sum of the numbers at the left of the sign of equality.

53. Carrying the Tens is the process of reserving the tens and adding them with the next column.

54. Principles.—1. *Only like numbers and like unit orders can be added one to another.*

2. *The sum or amount contains as many units as all the numbers added.*

3. *The sum or amount is the same in whatever order the numbers be added.*

55. Addition is the Reverse of Subtraction and may be proved by it; as, $5 + 2 = 7$. Now if 7 be diminished by 5, the result will be 2, while if 7 be diminished by 2, the result will be 5.

56. Numbers are written for addition either in *vertical* or *horizontal* order.

57. General Rules.—1. *If the sum of two numbers and one of the numbers be given, the unknown number may be found by taking the given number from the sum.*

2. *If the sum of several numbers and all of the numbers but one be given, the unknown number may be found by subtracting the sum of those given from the sum of all the numbers.*

NOTES TO TEACHER.—1. Classes should have frequent and extended drill in rapid mental addition.

2. The following table is given simply to facilitate class drill, preparatory to work in rapid addition.

Addition Table for Class Drill.

6 + 6 = 12	8 + 12 = 20	10 + 22 = 32	13 + 22 = 35	17 + 22 = 39
6 + 7 = 13	8 + 13 = 21	10 + 23 = 33	13 + 23 = 36	17 + 23 = 40
6 + 8 = 14	8 + 14 = 22	10 + 24 = 34	13 + 24 = 37	17 + 24 = 41
6 + 9 = 15	8 + 15 = 23	10 + 25 = 35	13 + 25 = 38	17 + 25 = 42
6 + 10 = 16	8 + 16 = 24			
6 + 11 = 17	8 + 17 = 25	11 + 11 = 22	14 + 14 = 28	18 + 18 = 36
6 + 12 = 18	8 + 18 = 26	11 + 12 = 23	14 + 15 = 29	18 + 19 = 37
6 + 13 = 19	8 + 19 = 27	11 + 13 = 24	14 + 16 = 30	18 + 20 = 38
6 + 14 = 20	8 + 20 = 28	11 + 14 = 25	14 + 17 = 31	18 + 21 = 39
6 + 15 = 21	8 + 21 = 29	11 + 15 = 26	14 + 18 = 32	18 + 22 = 40
6 + 16 = 22	8 + 22 = 30	11 + 16 = 27	14 + 19 = 33	18 + 23 = 41
6 + 17 = 23	8 + 23 = 31	11 + 17 = 28	14 + 20 = 34	18 + 24 = 42
6 + 18 = 24	8 + 24 = 32	11 + 18 = 29	14 + 21 = 35	18 + 25 = 43
6 + 19 = 25	8 + 25 = 33	11 + 19 = 30	14 + 22 = 36	
6 + 20 = 26		11 + 20 = 31	14 + 23 = 37	19 + 19 = 38
6 + 21 = 27	9 + 9 = 18	11 + 21 = 32	14 + 24 = 38	19 + 20 = 39
6 + 22 = 28	9 + 10 = 19	11 + 22 = 33	14 + 25 = 39	19 + 21 = 40
6 + 23 = 29	9 + 11 = 20	11 + 23 = 34		19 + 22 = 41
6 + 24 = 30	9 + 12 = 21	11 + 24 = 35	15 + 15 = 30	19 + 23 = 42
6 + 25 = 31	9 + 13 = 22	11 + 25 = 36	15 + 16 = 31	19 + 24 = 43
	9 + 14 = 23		15 + 17 = 32	19 + 25 = 44
7 + 7 = 14	9 + 15 = 24	12 + 12 = 24	15 + 18 = 33	
7 + 8 = 15	9 + 16 = 25	12 + 13 = 25	15 + 19 = 34	20 + 20 = 40
7 + 9 = 16	9 + 17 = 26	12 + 14 = 26	15 + 20 = 35	20 + 21 = 41
7 + 10 = 17	9 + 18 = 27	12 + 15 = 27	15 + 21 = 36	20 + 22 = 42
7 + 11 = 18	9 + 19 = 28	12 + 16 = 28	15 + 22 = 37	20 + 23 = 43
7 + 12 = 19	9 + 20 = 29	12 + 17 = 29	15 + 23 = 38	20 + 24 = 44
7 + 13 = 20	9 + 21 = 30	12 + 18 = 30	15 + 24 = 39	20 + 25 = 45
7 + 14 = 21	9 + 22 = 31	12 + 19 = 31	15 + 25 = 40	
7 + 15 = 22	9 + 23 = 32	12 + 20 = 32		21 + 21 = 42
7 + 16 = 23	9 + 24 = 33	12 + 21 = 33	16 + 16 = 32	21 + 22 = 43
7 + 17 = 24	9 + 25 = 34	12 + 22 = 34	16 + 17 = 33	21 + 23 = 44
7 + 18 = 25		12 + 23 = 35	16 + 18 = 34	21 + 24 = 45
7 + 19 = 26	10 + 10 = 20	12 + 24 = 36	16 + 19 = 35	21 + 25 = 46
7 + 20 = 27	10 + 11 = 21	12 + 25 = 37	16 + 20 = 36	
7 + 21 = 28	10 + 12 = 22		16 + 21 = 37	22 + 22 = 44
7 + 22 = 29	10 + 13 = 23	13 + 13 = 26	16 + 22 = 38	22 + 23 = 45
7 + 23 = 30	10 + 14 = 24	13 + 14 = 27	16 + 23 = 39	22 + 24 = 46
7 + 24 = 31	10 + 15 = 25	13 + 15 = 28	16 + 24 = 40	22 + 25 = 47
7 + 25 = 32	10 + 16 = 26	13 + 16 = 29	16 + 25 = 41	
	10 + 17 = 27	13 + 17 = 30	17 + 17 = 34	23 + 23 = 46
8 + 8 = 16	10 + 18 = 28	13 + 18 = 31	17 + 18 = 35	23 + 24 = 47
8 + 9 = 17	10 + 19 = 29	13 + 19 = 32	17 + 19 = 36	23 + 25 = 48
8 + 10 = 18	10 + 20 = 30	13 + 20 = 33	17 + 20 = 37	
8 + 11 = 19	10 + 21 = 31	13 + 21 = 34	17 + 21 = 38	24 + 24 = 48
				24 + 25 = 49
				25 + 25 = 50

MENTAL EXERCISES.

58. Add

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
1	5	1	2	2	11	12	25
2	6	3	4	5	14	14	50
3	7	5	6	4	13	18	15
4	8	7	8	8	15	15	25
5	9	9	10	6	12	16	10
6	10	11	12	12	18	20	25
7	11	13	14	10	17	14	15
8	12	15	16	14	20	16	10

59. There are three methods of addition in common use, viz.; the *Elementary method*, the *Result method*, and the *Group method*.

REMARKS.—1. These methods of addition are recommended to be taught in their order to pupils in elementary work; the first, as soon as mastered, should be abandoned for the second, and the second in its turn, when mastered, abandoned for the third.

2. Daily drill in the third method is urgently advised with all pupils during the entire period of their study of Arithmetic. Too much importance can scarcely be attached to this suggestion.

60. The Elementary Method of Addition.

EXAMPLE.—Add 32, 71, 25, 48, 90, 12, and 63.

OPERATION.	EXPLANATION.
32	Having arranged the numbers so that units of like orders stand directly under each other, begin with the last figure in the right-hand,
71	or units' column, and add upward as follows: 3 and 2 are 5, 5 and 8 are 13,
25	13 and 5 are 18, 18 and 1 are 19, 19 and 2 are 21. Having thus obtained the sum,
48	place the 1 beneath the line, in units' column, and treat the 2 as a part of the
90	second, or tens' column, which add upward as before; thus, 2 and 6 are 8,
12	8 and 1 are 9, 9 and 9 are 18, 18 and 4 are 22, 22 and 2 are 24, 24 and 7 are 31,
63	31 and 3 are 34. Having obtained the sum, write it in full at the left of the
341	figure 1 before written, and the result is 341, the numerical expression of the sum of the numbers added.
	TO PROVE.—Add the columns downward; if the two results agree, the work is presumed to be correct.

61. The Result Method of Addition.

EXAMPLE.—Add 32, 71, 25, 48, 90, 12, and 63.

OPERATION.	EXPLANATION.
32	Beginning as before, with the lower figure in units' column,
71	name the <i>result</i> only of each successive addition, thus: 3, 5, 13, 18, 19, 21; then,
25	as before, write the 1 beneath the line in units' column and carrying the 2 to
48	tens' column as a part of it, add upward, thus: 2, 8, 9, 18, 22, 24, 31, 34; as
90	before, write 34 at the left, and the result is 341, the same as before.
12	
63	TO PROVE.—Add the columns downward.
341	

62. The Group Method of Addition.

EXAMPLE.—1. Add 32, 71, 25, 48, 90, 12, and 63.

OPERATION.	EXPLANATION.
$\begin{array}{r} 32 \\ 71 \\ 25 \\ 48 \\ 90 \\ 12 \\ 63 \\ \hline 341 \end{array}$	<p>Treat the same numbers thus: add upward; 3, 13, 21; grouping 2 and 8 for 10 to add to 3, making 13, and 5, 1, and 2 for 8, to add to 13, making 21. Having written the 1 beneath the line, in units' place, carry the 2 or 2 tens, to its column, and again add; 2, 8, 18, 24, 34; grouping 2 and 6 for 8, 9 and 1 for 10, 4 and 2 for 6, and 7 and 3 for 10; then write the result in full as before.</p> <p>To PROVE.—Review the first column by adding downward; 8, 18, 21; grouping 2, 1, and 5 for 8, 8 and 2 for 10, to add for 18, and to this add the remaining figure 3, for 21, the same result as before. Then review the second column by adding downward; 10, 16, 26, 34; grouping 3 and 7 for 10, 2 and 4 for 6, 9 and 1 for 10, and 6 and 2, for 8, with the same result.</p>

EXAMPLE.—2. Add 3417, 2140, 439, 7164, 1538, 5046, 6116, 8735, 971, 4880, 1263, 9270, 192, and 634.

OPERATION.	EXPLANATION.
$\begin{array}{r} 3417 \\ 2140 \\ 439 \\ 7164 \\ 1538 \\ 5046 \\ 6116 \\ 8735 \\ 971 \\ 4880 \\ 1263 \\ 9270 \\ 192 \\ 634 \\ \hline 51805 \end{array}$	<p>Beginning with the lower unit figure add upward; 10, 15, 35, 55, grouping 4, 2, 3, and 1 for 10, which added to 5 gives 15; grouping 6, 6, and 8 for 20 to add to 15 obtaining 35; and grouping 4, 9, and 7 for 20 to add to 35 for 55 the result. Write the units' figure 5 in its place, and carrying the tens' figure 5 to its column proceed thus: 8, 24, 28, 48, 56, 66, 70, grouping the 5 carried and 3 for 8; 9 and 7 for 16 to add to 8 for 24; 6 and 8 for 14 to add to 24 to make 38; 7 and 3 for 10, making 48; 1, 4, and 3 for 8, making 56; 6, 3, and 1 for 10, making 66, to which we add the 4 for 70, the result. Write the cipher of the 70 at the left of the unit figure already written beneath the line and carrying the 7 to the third, or hundreds' column group as before; 16, 26, 36, 48, 58, grouping upward thus: 7, 6, 1, 2 = 16; 2, 8 = 10; 9, 1 = 10; 7, 5 = 12; 1, 4, 1, 4 = 10. Write the 8 in hundreds' column, and carrying the 5 to thousands' column, group 15, 27, 39, 49, 51. 5, 9, 1 = 15; 4, 8 = 12; 6, 5, 1 = 12; 7, 3 = 10, and adding 2 write the result, 51, at the left of the figures before written, thus obtaining 51805 the numerical expression of the sum of the numbers added.</p>

Prove by adding downward, grouping as illustrated above.

REMARK.—Practice in grouping will lead to great proficiency, and after the pupil becomes somewhat skilled, he should be encouraged to skip about somewhat along the column, in order to select those numbers which can be most conveniently grouped. Ordinarily thorough drill in the addition table will greatly assist in grouping, and multiples of the nine digits can be added with ease. Except with very bright pupils, groups greater than 25 are not to be recommended.

HORIZONTAL ADDITION.

63. Numbers when written in horizontal order, as in invoices and other business forms, may be added without being re-written in vertical columns.

REMARKS.—1. In adding numbers written horizontally, more care is requisite that the units added shall be of like order, and greater certainty of correctness can be had by adding first from left to right, and then from right to left.

2. The group method may be employed with equal advantage where numbers are written horizontally.

MENTAL EXERCISES.

64. Add from left to right, and review from right to left.

- | | |
|-----------------------------------|--------------------------------------|
| 1. 5, 3, 6, 1, 8, 2, 7, 9, 4. | 6. 15, 23, 36, 18, 25, 53, 92. |
| 2. 21, 56, 12, 93, 47, 60, 17. | 7. 11, 85, 315, 125, 111, 206. |
| 3. 66, 29, 5, 14, 71, 19, 2, 11. | 8. 8, 42, 87, 20, 112, 108, 94, 128. |
| 4. 149, 865, 73, 40, 5, 13, 502. | 9. 61, 400, 1, 126, 25, 440. |
| 5. 365, 10, 88, 46, 200, 175, 95. | 10. 25, 50, 511, 3, 209, 8, 804. |

WRITTEN EXERCISES.

65. Copy, and add from left to right; review from right to left, preserving results.

1. 510, 297, 69, 841, 638, 203, 40, 7, 700, 28, 9.
2. 1260, 2700, 408, 9206, 51, 7240, 27, 1620.
3. 8809, 1492, 1000, 20, 1, 504, 6620, 7596, 10.
4. 50000, 20000, 8900, 21050, 47800, 14090.
5. 76030, 20500, 38037, 69000, 81, 107, 2, 19975.
6. 346211, 218040, 173508, 973200, 701001, 555555.
7. 604000, 181523, 51, 19406, 200, 309, 5, 2, 8000.
8. 2463911, 7054133, 4444044, 1371005, 6090400.
9. 8500500, 1035660, 5000000, 2987400, 7020319.
10. 416, 49, 2, 7967400, 81, 307, 21021, 190200, 40, 3.

REMARK.—Horizontal addition is rarely practiced with numbers containing more than four or five figures. It may sometimes be employed to advantage in adding dollars and cents; in such cases it is best to omit the dollar sign; as, for \$5.25 write 5.25.

66. Copy and add horizontally; review and preserve results.

1. 5.25, 8.17, 11.40, 1.82, 16.02, 90.70.
2. 146.24, 9.11, 210.10, 46.98, 5.50, 108.12, 4.75.
3. 26.53, 92, 5.71, 108.97, 29.33, 150, 46.07, 19, 76.
4. 231.45, 50, 75, 19.78, 40, 50, 63, 100.
5. 63, 51, 87, 25, 75, 18.09, 95, 1.25, 6.
6. 278.19, 105.29, 80.50, 19.93, 52, 1.
7. 29.30, 403, 51, 73, 1.14, 90, 300, 1.25.
8. 1.13, 9.25, 14, 27.16, 5.01, 8, 25, 1.75.
9. 87.50, 125, 36.21, 9.90, 14.75, 16, 25.25.
10. 117.82, 7.71, 19.03, 15, 49.55, 87.08.
11. 5.40, 88, 35, 90, 112.50, 45.95, 111.50.
12. 100, 79.22, 50.08, 2.25, 7.75, 10, 3, 8.24.
13. 216.24, 92, 15.06, 138.50, 2.38, 9.25.

REMARK.—The teacher may give other examples of the same kind; he will find extensive drill in such work of great value to all grades of pupils, in developing accuracy and rapidity.

EXAMPLES FOR PRACTICE.

67. 1. A grocer's sales were, for Monday, \$241; Tuesday, \$306; Wednesday, \$523; Thursday, \$438; Friday, \$497; on Saturday his sales amounted to \$27 more than the sales of the first three days of the week. What were his total sales during the week?

2. A planter shipped eleven bales of cotton, weighing respectively 492, 504, 523, 487, 490, 500, 516, 499, 512, 511, and 496 pounds. What was the aggregate weight of the shipment?

3. A portable saw-mill cut lumber for the six working days of a week, as follows: On Monday, 5116 feet; Tuesday, 4900 feet; Wednesday, 5750 feet; Thursday, 6100 feet; Friday, 4580 feet; and on Saturday, 6754 feet. What amount of lumber did the mill cut during the week?

4. Find the sum of four units of the second order and five of the first; eight of the fifth, three of the third, and nine of the second; seven of the sixth, one of the fifth, and two of the third; one of the eighth, nine of the third, seven of the second, and six of the first; four of the fifth, three of the fourth, and nine of the third; five of the tenth, one of the ninth, four of the seventh, eight of the third, two of the second, and one of the first.

5. Find the sum of sixty-nine thousand five hundred seven, one thousand six hundred twenty-two, one hundred fifty-six thousand seventy-six, ninety-nine thousand nineteen, forty-one million eighty-seven thousand five, three hundred twenty-five million sixteen thousand eight hundred eighty-eight, six billion ninety-one million four thousand two hundred fifty-six.

6. The British House of Lords was, in 1884, comprised of 4 princes, 23 dukes, 19 marquises, 139 earls, 32 viscounts, 26 bishops, and 272 barons. How many members in all?

7. In 1883, there arrived and settled in the United States, immigrants: Germans, 192,000; English, 100,200; Canadians, 65,100; Irish, 64,400; Scandinavians, 52,200; Italians, 32,500; miscellaneous, 92,700. What was the total number of immigrants?

8. The British national debt in March, 1883, was: Consols, £699,053,100; Bank debts, £13,645,900; Annuities, £27,570,900; Exchequer Bills, £8,754,400; Treasury Bills, £5,431,000; Savings Banks, £1,804,400; and the local debt, £163,501,000. What was the total debt in pounds sterling?

9. In 1866, the U. S. collected as revenue from Customs, \$179,046,651.58; from Internal Revenue, \$309,226,813.42; from Direct Taxes, \$1,974,754.12; from the Public Lands, \$665,031.03; and from other sources, \$29,036,314.23. What was the total government revenue collected that year?

10. The British government collected as revenue in 1882: From Customs, £19,300,000; from Excise, £27,230,000; from Stamps, £11,145,000; from Land Tax, £2,775,000; from Income Tax, £11,662,000; from Post Office, £7,150,000; from Telegraphs, £1,650,000; from Crown Lands, £380,000; from Interest, £1,180,000; from miscellaneous sources, £4,725,000. What was the total revenue of the British government for that year?

11. The dwarf, Borowlaski, was only 39 inches in height; Tom Thumb, 31; Mrs. Tom Thumb, 32; Che-Mah, of China, 25; Lucia Zarate, of Mexico, 20; and Gen. Mite, 21. What was the combined height of the six?

12. The firm of Davis & Drake own land valued at \$39,750; lumber, \$68,125; notes, \$21,700; book accounts, \$17,291; machinery, \$13,250; cash in bank, \$14,238; cash on hand, \$4,232. What is the property value of the firm?

13. In 1880, there were women workers in the United States as follows: artists, 2,061; authors, 320; barbers, 2,902; dressmakers, 281,928; journalists, 288; lawyers, 75; musicians, 13,181; physicians, 2,432; preachers, 165; printers, 3,456; tailors, 52,098; teachers, 154,375. How many women workers in all?

REMARK.—The three following problems can be properly used by the teacher for drill in group adding.

14. The population of the United States, by the census of 1880, was as follows:

Ala.,	1,262,505	Kans.,	996,096	N. Y.,	5,082,871
Alaska,	30,000	Ky.,	1,684,690	N. C.,	1,399,750
Ariz.,	40,440	La.,	939,946	Ohio,	3,138,062
Ark.,	802,525	Me.,	648,936	Oregon,	174,768
Cal.,	864,694	Md.,	934,943	Pa.,	4,282,891
Colo.,	194,327	Mass.,	1,783,085	R. I.,	276,531
Conn.,	622,700	Mich.,	1,636,937	S. C.,	995,577
Dak.,	135,177	Minn.,	780,773	Tenn.,	1,542,359
Del.,	146,608	Miss.,	1,131,597	Tex.,	1,591,749
D. C.,	177,624	Mo.,	2,168,380	Utah,	143,963
Fla.,	269,493	Mont. T.,	39,159	Vt.,	332,286
Ga.,	1,542,180	Nebr.,	452,402	Va.,	1,512,565
Idaho,	32,610	Nev.,	62,266	Wash. T.,	75,116
Ill.,	3,077,871	N. H.,	346,991	W. Va.,	618,457
Ind.,	1,978,301	N. J.,	1,131,116	Wis.,	1,315,497
Ind. T.,	70,000	N. Mex.,	119,565	Wyo. T.,	20,789
Iowa,	1,624,615				

What was the total population?

15. The area of the United States, in square miles, is as follows:

Ala.,	51,540	Kans.,	81,700	N. Y.,	47,620
Alaska,	531,409	Ky.,	40,000	N. C.,	48,580
Ariz.,	112,920	La.,	45,420	Ohio,	40,760
Ark.,	53,045	Me.,	29,895	Oregon,	94,560
Cal.,	155,980	Md.,	9,860	Pa.,	44,985
Colo.,	103,645	Mass.,	8,040	R. I.,	1,085
Conn.,	4,845	Mich.,	57,430	S. C.,	30,170
Dak.,	147,700	Minn.,	79,205	Tenn.,	41,750
Del.,	1,960	Miss.,	46,340	Tex.,	262,290
D. C.,	60	Mo.,	68,735	Utah,	82,190
Fla.,	54,240	Mont. T.,	145,310	Vt.,	9,135
Ga.,	58,980	Nebr.,	76,185	Va.,	40,125
Idaho,	84,290	Nev.,	109,740	Wash. T.,	66,880
Ill.,	56,000	N. H.,	9,005	W. Va.,	24,645
Ind.,	35,910	N. J.,	7,455	Wis.,	54,450
Ind. T.,	69,830	N. Mex.,	122,460	Wyo. T.,	97,575
Iowa,	55,475				

What is the total area?

16. For State tax of 1888, the several Counties of the State of New York were assessed as follows :

Albany,	\$86,606,307	Oneida,	\$58,146,279
Alleghany,	14,395,123	Onondaga,	63,265,536
Broome,	21,383,568	Ontario,	29,389,870
Cattaraugus,	16,050,985	Orange,	42,953,974
Cayuga,	30,631,548	Orleans,	14,816,445
Chautauqua,	25,649,740	Oswego,	23,655,679
Chemung,	18,718,275	Otsego,	22,544,650
Chenango,	17,982,340	Putnam,	7,483,530
Clinton,	9,766,255	Queens,	44,464,675
Columbia,	29,984,129	Rensselaer,	60,545,955
Cortland,	11,108,469	Rockland,	13,394,485
Delaware,	13,921,534	Richmond,	12,271,105
Dutchess,	44,532,280	Saratoga,	23,189,435
Erie,	127,763,104	Schenectady,	12,772,451
Essex,	10,515,260	Schoharie,	10,297,219
Fulton,	8,383,735	Schuyler,	7,248,620
Franklin,	8,026,235	Seneca,	15,347,372
Genesee,	21,384,810	St. Lawrence,	24,476,678
Greene,	13,760,299	Steuben,	22,776,074
Hamilton,	1,157,600	Suffolk,	17,262,646
Herkimer,	23,739,092	Sullivan,	5,427,300
Jefferson,	23,638,204	Tioga,	12,084,525
Kings,	342,116,976	Tompkins,	15,450,670
Lewis,	9,039,285	Ulster,	25,443,000
Livingston,	25,395,180	Warren,	6,555,175
Madison,	19,797,535	Washington,	22,501,173
Monroe,	85,964,190	Wayne,	25,404,569
Montgomery,	23,877,638	Westchester,	82,375,217
New York,	1,500,550,825	Wyoming,	14,922,986
Niagara,	26,097,826	Yates,	12,721,716

What was the total assessed value of the State that year ?

SUBTRACTION.

68. Subtraction is the process of finding the difference between two numbers.

69. The Subtrahend is the number to be subtracted.

70. The Minuend is the number from which the Subtrahend is to be subtracted.

71. The Difference or Remainder is the result obtained by subtracting one number from another.

72. The Sign of Subtraction is $-$. It is called *Minus* and signifies *less*.

When the sign of subtraction is placed between two numbers it indicates that the number placed *after* it is to be taken from the one *before* it.

73. The Complement of a Number is the difference between it and a unit of the next higher order.

Thus the complement of 7 is 3, because 1 ten, the unit of the next higher order, diminished by 7 = 3. Again, the complement of 36 is 64, because the unit of the next higher order, 1 hundred, or 100, diminished by 36 = 64.

74. Principles.—1. *Only like numbers and units of the same order can be subtracted, one from the other.*

2. *The sum of the subtrahend and the remainder must be equal to the minuend.*

75. General Relation of Terms in Subtraction.

I. The Minuend $-$ the Subtrahend = the *Remainder*.

II. The Minuend $-$ the Remainder = the *Subtrahend*.

III. The Subtrahend $+$ the Remainder = the *Minuend*.

76. General Rules.—1. *If the minuend and subtrahend be given, the remainder may be found by subtracting the subtrahend from the minuend.*

2. *If the minuend and remainder be given, the subtrahend may be found by subtracting the remainder from the minuend.*

3. *If the remainder and subtrahend be given, the minuend may be found by adding the remainder to the subtrahend.*

77. To Prove Subtraction.—*Add the remainder to the subtrahend; if the sum equals the minuend, the work is correct.*

Subtraction Table.

78. Find the difference, mentally, between

12 and 6	16 and 12	19 and 10	21 and 16	24 and 5
12 " 7	16 " 13	19 " 11	21 " 17	24 " 6
12 " 8		19 " 12	21 " 18	24 " 7
12 " 9	17 and 3	19 " 13	22 and 3	24 " 8
	17 " 4	19 " 14	22 " 4	24 " 9
13 and 4	17 " 5	19 " 15	22 " 5	24 " 10
13 " 5	17 " 6	19 " 16	22 " 6	24 " 11
13 " 6	17 " 7		22 " 7	24 " 12
13 " 7	17 " 8	20 and 3	22 " 8	24 " 13
13 " 8	17 " 9	20 " 4	22 " 9	24 " 14
13 " 9	17 " 10	20 " 5	22 " 10	24 " 15
	17 " 11	20 " 6	22 " 11	24 " 16
14 and 5	17 " 12	20 " 7	22 " 12	24 " 17
14 " 6	17 " 13	20 " 8	22 " 13	24 " 18
14 " 7	17 " 14	20 " 9	22 " 14	24 " 19
14 " 8		20 " 10	22 " 15	24 " 20
14 " 9	18 and 3	20 " 11	22 " 16	24 " 21
14 " 10	18 " 4	20 " 12	22 " 17	25 and 3
14 " 11	18 " 5	20 " 13	22 " 18	25 " 4
	18 " 6	20 " 14	22 " 19	25 " 5
15 and 4	18 " 7	20 " 15	23 and 4	25 " 6
15 " 5	18 " 8	20 " 16	23 " 5	25 " 7
15 " 6	18 " 9	20 " 17	23 " 6	25 " 8
15 " 7	18 " 10		23 " 7	25 " 9
15 " 8	18 " 11	21 and 3	23 " 8	25 " 10
15 " 9	18 " 12	21 " 4	23 " 9	25 " 11
15 " 10	18 " 13	21 " 5	23 " 10	25 " 12
15 " 11	18 " 14	21 " 6	23 " 11	25 " 13
15 " 12	18 " 15	21 " 7	23 " 12	25 " 14
		21 " 8	23 " 13	25 " 15
16 and 4	19 and 3	21 " 9	23 " 14	25 " 16
16 " 5	19 " 4	21 " 10	23 " 15	25 " 17
16 " 6	19 " 5	21 " 11	23 " 16	25 " 18
16 " 7	19 " 6	21 " 12	23 " 17	25 " 19
16 " 8	19 " 7	21 " 13	23 " 18	25 " 20
16 " 9	19 " 8	21 " 14	23 " 19	25 " 21
16 " 10	19 " 9	21 " 15		25 " 22
16 " 11				

REMARKS.—1. Frequent and thorough use of the Subtraction Table will result in great facility in all operations in this subject, and will also aid in additions and rapid work in arithmetical computations in general.

2. The above table, like the one in addition, is given for the teacher's reference, to save time and labor in rapid mental exercises.

79. When any Figure in the Minuend is Less than the Corresponding Figure in the Subtrahend.

EXAMPLE.—From 435 take 176.

OPERATION.	EXPLANATION.—
435 Minuend.	It is readily observed that the units figure 6 of the subtrahend cannot be taken from the corresponding figure of the minuend; therefore analyze the minuend, and transform it into 4 hundreds,
176 Subtrahend.	2 tens, 15 units; then from the 15 units take the 6 units of the subtrahend, obtaining 9 units as a remainder, which write as the units of the result;
259 Remainder.	having reduced one of the tens of the minuend to units, we have only 2 tens remaining in the tens' column of the minuend, and since this is numerically less than the tens' figure in the subtrahend, transform as before, and read the 4 hundreds and 2 tens as 3 hundreds and 12 tens; then taking the 7 tens of the subtrahend from the 12 tens thus produced, write the remaining 5 tens for the second or tens' figure in the result; having taken 1 from the hundreds' column, we have 3 remaining in that column, from which take the 1 hundred of the subtrahend 7, obtaining 2 as the third or hundreds' figure of the result. Thus we conclude that 176 subtracted from 435, leaves a remainder of 259.

REMARKS.—1. This process is called “borrowing tens,” as each left-hand order is tenfold greater than the order at its right.

2. Having mastered the theory, the ordinary and most convenient method for practice is to leave the minuend figure in its original form and, when borrowing is necessary, add 1 to the succeeding subtrahend figure.

Again, apply this method to the example;

OPERATION.	EXPLANATION.—
435	Subtract 6 from 15 leaving 9, which write in units' column;
176	(adding 1 to 7) subtract 8 from 13 leaving 5, which write in its column; (adding
259	1 to 1) subtract 2 from 4 leaving 2, which write in its column.

Rule.—I. *So write the numbers to be subtracted, that units of the same order stand in the same vertical line.*

II. *Begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend. When it is necessary, transform, or borrow ten, and mentally add one to the next subtrahend figure.*

III. *Write results in their proper order.*

EXAMPLES FOR PRACTICE.

- | | |
|--|--|
| <p>80. 1. From 1524 take 911.</p> <p>2. From 3128 take 1519.</p> <p>3. From 4055 take 2033.</p> <p>4. From 27410 take 13520.</p> <p>5. From 80500 take 30500.</p> <p>6. From 123706 take 59341.</p> | <p>7. From 520200 take 368977.</p> <p>8. From 80090 take 23084.</p> <p>9. From 3406268 take 1998765.</p> <p>10. From 303005 take 89700.</p> <p>11. From 2046 take 1597.</p> <p>12. From 40509300 take 9619475.</p> |
|--|--|
13. In Germany there are 2,436,000 land owners, and in France 3,226,000. How many more in France than in Germany?
14. A dealer bought 1,732 sheep and sold to A 51, B 147, C 34, D 1000, and to E the remainder. How many did E purchase?

15. A farmer raised 1,130 bushels of wheat, 958 of barley, 1,275 of oats, and 1,762 of corn. If he keep for seed and feed, 116 bushels of wheat, 84 of barley, 600 of oats, and 1,150 of corn, how many bushels of grain will he have left to sell?

16. The equatorial diameter of the earth is 41847194 feet, and the polar diameter 41707308 feet. How many feet greater is the equatorial than the polar diameter?

17. If the sailing distance from New York to Queenstown be 2890 miles, how far from the latter port will a steamer be after running 1296 miles from the port of New York?

18. Texas contains 274356 square miles and New York 47156 square miles. How many times may the area of New York be taken from the area of Texas and what number of square miles will remain?

19. The area of Brazil is 3956000 square miles and of the United States 3026504 square miles. How many square miles greater is Brazil than the United States?

20. A man bought a farm for 3250 dollars. He built a house on it at a cost of 1850 dollars, fences costing 416 dollars and then sold it for 7500 dollars. What was his gain?

21. I bought 23,240 acres of Dakota land, and sold at times 1000, 320, 520, 640, 3200, 2520, 160, and 1920 acres. How many acres had I remaining?

22. During a five years' partnership a firm gained \$123,475. If the gain the first year was \$11,425; the second, \$9,500; the third as much as the first and second, less \$1,120; the fourth equal to the second and third; how much must have been gained the fifth year?

23. The cost of my lot was \$1,750. I paid for mason work on my house, \$1,210; for carpenter work, \$5,145; for plumbing, \$985; for decorating, \$1,650; for painting, \$625; for grading, sodding, and fencing grounds, \$590. The interest on outlays to date of sale was \$315. I then sold the property at a loss of \$20, receiving cash \$6,000, and a note for the remainder. What was the face of the note?

24. My book-keeper's salary is \$1,450 per year. If he requires for his rent, \$365; for personal expenses, \$170; and for support of his family, \$775; what amount will he have left at the end of the year?

25. A Boston bicyclist journeying to San Francisco, distant 3,432 miles, ran the first week of six days, an average of 77 miles per day; the second week, 92 miles; the third, 84 miles; the fourth, 106 miles; the fifth, 95 miles, and reached his destination at the end of the sixth week. How many miles did he run the last week?

MULTIPLICATION.

81. Multiplication is the process of taking one of two numoers as many times as there are units in the other.

82. One of the numbers is called the *Multiplicand* and the other the *Multiplier*. The numbers are also called *Factors* of the product.

83. The **Multiplicand** is the factor multiplied.

84. The **Multiplier** is the factor by which the multiplicand is multiplied.

85. The **Product** is the result obtained by multiplying one number by another.

86. The **Factors** of a number are such numbers as will, when multiplied together, produce the given number.

87. A **Continued Product** is the result obtained by multiplying several factors together.

88. The **Sign of Multiplication** is an oblique cross, \times . It is read "*times*," or "*multiplied by*," and indicates that the numbers between which it is placed are to be multiplied together, or their product obtained. Thus, 5×2 is read 5 times 2, or, 5 multiplied by 2.

REMARKS.—1. In practice, the *multiplier* is regarded as an *abstract* number and the *multiplicand* as a *concrete* number; but as the resulting product is the same whichever factor is used as a multiplier, the above relation is recognized only in explanations of work done.

2. Where the multiplicand is concrete, the product will be concrete and of the same denomination as the multiplicand.

89. Multiplication is a short method of performing addition, and like addition may be proved by subtraction. Thus, $2 \times 2 = 4$; that is, two taken twice as a factor $= 4$, or, 2 added to 2 $= 4$. We prove this by subtraction, 2 from 4 leaves 2.

Again, $6 \times 7 = 42$; that is, 6 taken seven times as a factor $= 42$, or, seven 6's added $= 42$; this may be proved by subtracting seven 6's in succession from 42, when nothing remains.

90. General Rules.—1. *If the multiplicand and multiplier be given, the product may be found by multiplying those factors together.*

2. *If the product and multiplier be given, the multiplicand may be found by dividing the product by the multiplier.*

3. *If the product and multiplicand be given, the multiplier may be found by dividing the product by the multiplicand.*

4. *If the product of two numbers and one of the numbers be given, the other may be found by dividing the product by the number given.*

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	2
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	3
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	4
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	5
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150	6
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175	7
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200	8
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225	9
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	10
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	11
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	12
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325	13
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350	14
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375	15
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	16
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	17
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450	18
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475	19
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500	20
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525	21
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550	22
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575	23
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600	24
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625	25
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25		

NOTE.—It will be of great advantage to the student to fully master the above table. Any delay caused by following this suggestion will be offset by time gained in subsequent work; such mastery will so increase the rapidity of work in business applications as to greatly lessen the labor of accounting.

91. Multiply

EXAMPLES FOR ORAL DRILL.

1. 42 by 3.	11. 102 by 4.	21. 144 by 10.	31. 595 by 13.
2. 31 by 4.	12. 511 by 8.	22. 52 by 11.	32. 70 by 22.
3. 27 by 2.	13. 125 by 6.	23. 45 by 13.	33. 90 by 25.
4. 60 by 5.	14. 340 by 2.	24. 201 by 15.	34. 150 by 23.
5. 51 by 4.	15. 416 by 3.	25. 65 by 20.	35. 118 by 11.
6. 75 by 6.	16. 99 by 7.	26. 411 by 17.	36. 906 by 15.
7. 91 by 2.	17. 133 by 9.	27. 932 by 12.	37. 450 by 19.
8. 29 by 3.	18. 208 by 4.	28. 43 by 19.	38. 375 by 14.
9. 57 by 2.	19. 666 by 5.	29. 111 by 23.	39. 250 by 18.
10. 95 by 5.	20. 89 by 8.	30. 207 by 22.	40. 789 by 11.

92. When either Factor is a Number within one's thorough knowledge of the Multiplication Table.

EXAMPLE.—1. Multiply 417 by 5.

OPERATION.	EXPLANATION.—
$\begin{array}{r} 417 \\ 5 \\ \hline 2085 \end{array}$	Write the multiplier 5 below the unit figure of the multiplicand, and multiply each figure of the multiplicand by the multiplier, thus; 5 times 7 = 35, or, 3 tens + 5 units; write the 5 units in units' place and reserve the 3 tens to add to the next product; next, 5 times 1 ten are 5 tens, and adding the 3 tens reserved gives 8 tens, which write in tens' place; next, 5 times 4 hundreds are 20 hundreds, or 2 thousands; write a naught, or cipher, in the hundreds' place and the 2 in the thousands' place, thus completing the multiplication and obtaining 2085 as the product of 417 multiplied by 5.

EXAMPLE.—2. Multiply 123 by 12.

OPERATION.	EXPLANATION.—
$\begin{array}{r} 123 \\ 12 \\ \hline 1476 \end{array}$	Multiply each figure of the multiplicand by the multiplier, 12; 12 times 3 = 36, or, 3 tens + 6 units; write the 6 in units' place and reserve the 3 tens to add to the next product; next, 12 times 2 tens are 24 tens, and adding the 3 tens reserved gives 27 tens, or 2 hundreds + 7 tens; write the 7 in tens' place and reserve the 2 hundreds to add to the next product; next, 12 times 1 hundred are 12 hundreds, and adding the 2 hundreds reserved gives 14 hundreds, or, 1 thousand 4 hundreds, which write in hundreds' and thousands' places, thus completing the multiplication and obtaining 1476 as the product of 123 multiplied by 12.

Rule.—Write the factors one below the other, arranged so that figures of like orders will stand in the same vertical line. Multiply each figure of the upper factor, beginning at the right, by the lower factor, placing in order the last figure of the product so obtained, and carrying to the next product all figures except the last; continue so doing until the last product is found, which write in full.

93. Multiply

EXAMPLES FOR MENTAL PRACTICE.

1. 201 by 8.	7. 1325 by 2.	13. 811 by 16.	19. 641 by 13.
2. 507 by 5.	8. 2108 by 11.	14. 1603 by 9.	20. 7122 by 5.
3. 1001 by 12.	9. 511 by 15.	15. 3006 by 14.	21. 984 by 8.
4. 311 by 6.	10. 293 by 12.	16. 249 by 7.	22. 2260 by 12.
5. 805 by 9.	11. 1801 by 13.	17. 519 by 8.	23. 461 by 14.
6. 1203 by 8.	12. 684 by 14.	18. 1122 by 11.	24. 3542 by 15.

EXAMPLES FOR WRITTEN PRACTICE.

94. Multiply

- | | | |
|-----------------|-----------------------|-------------------------|
| 1. 2168 by 9. | 6. 356142 by 18. | 11. 99084160024 by 15. |
| 2. 31046 by 16. | 7. 2147603 by 8. | 12. 294640205580 by 9. |
| 3. 599 by 12. | 8. 15286097 by 15. | 13. 66205380777 by 7. |
| 4. 2170 by 13. | 9. 508093240 by 13. | 14. 897352468004 by 12. |
| 5. 50890 by 11. | 10. 6381201432 by 14. | 15. 21430206041 by 15. |

95. When the Multiplier consists of two or more figures.

EXAMPLE.—Multiply 417 by 352.

OPERATION.	
	Hundreds. Tens. Units.
Multiplicand.	417
Multiplier.	352
<hr/>	
Units.	834
Tens.	2085
Hundreds.	1251
<hr/>	
	146784

EXPLANATION.—Write the numbers one below the other in the same unit order from the right. Then, beginning with the unit figure of the lower factor multiply; 2 times 7 units are 14 units = 1 ten + 4 units; write the 4 units in units' column and add the 1 ten to the next product; next, 2 times 1 ten are 2 tens and the 1 ten added makes 3 tens, which write in tens' place; next, 2 times 4 hundreds are 8 hundreds, which write in hundreds' place, giving 834 as the first partial product, or the product of the upper factor multiplied by the unit figure of the lower factor. Next take the tens' figure of the lower number as a multiplier; 7 taken 5 tens or 50 times = 35 tens, or 350; write the 5 of the number 35 in tens' column, or below the 3 tens of the first partial product, and carry the 3 of the 35 to the next product; next, 5 times 1 are 5 and the 3 to carry added make 8, which write under the 8 of the first partial product; then, 5 times 4 are 20, which

write still to the left, making the second partial product 2085 tens. Next, take the third figure, or hundreds, of the lower factor, as a multiplier; 3 times 7 hundreds are 21 hundreds; write the 1 in the hundreds' place and reserve the 2 for the next product; then, 3 times 1 are 3 and 2 to carry makes 5, which write in its order; then, 3 times 4 are 12, which write still to the left, having as a product 1251. Since, however, the several figures of the factor taken as a multiplier were of successive orders of units,

The first partial product 834 =	834 simple units.
The second partial product 2085 = 2085 tens =	20850 " "
The third partial product 1251 = 1251 hundreds =	125100 " "

And the sum = 146784

Therefore, 146784 is found to be the product of the numbers 417 and 352.

Rule.—I. *Place the multiplier below the multiplicand, the unit figures in the same vertical line.*

II. *Beginning with the unit figure, multiply all the figures of the multiplicand by each successive figure of the multiplier, writing the first figure obtained in each partial product directly below the figure by which it was multiplied. Add the partial products.*

REMARK.—The object of writing each succeeding partial product below and one place to the left of its predecessor, is that units of the same grade, or order, may, for convenience in adding, be found in the same vertical line; this arrangement precludes the necessity of filling the vacant orders with ciphers.

As before shown, the arrangement of factors will not vary the result; as, $4 \times 5 = 20$, also $5 \times 4 = 20$; therefore, in business or school practice, arrange factors in such order as to save time and space; by so doing, problems otherwise long and difficult, may be solved by mental processes.

EXAMPLE.—Multiply 120000 by 7256.

EXPLANATION.—Consider the factors as reversed in order; thus, 7256×120000 . Then multiply the 7256 mentally by 12, and to the product, 87072, annex four ciphers, because the 12 was not 12 simple units but 12 units of the fifth order, or tens of thousands.

96. When one Factor is 10, 100, 1000, 10000, or 1 with any number of ciphers annexed.

EXAMPLE.—Multiply 324 by 1000.

EXPLANATION.—Since there are three ciphers in the multiplier, annex three to the multiplicand, 324, thus obtaining the product, 324000.

Rule.—*To the one factor annex as many ciphers as there are ciphers in the other factor.*

97. A Composite Number is a number that may be resolved or separated into integral factors; or, it is a number that may be formed by multiplying together two or more numbers; thus, $12 = 4 \times 3$; or $12 = 2 \times 2 \times 3$; or $4 \times 3 = 12$; or $2 \times 2 \times 3 = 12$.

98. When the Multiplier is a Composite Number.

Whenever it is required to find the product of numbers one or more of which is composite, the result may be obtained by using as multipliers the factors of such composite number or numbers; thus, $6 \times 4 = 24$, or $6 \times (2 \times 2) = 24$.

Rule.—*Separate the multiplier into its factors. Multiply the multiplicand by one of these factors, that product by another factor, and so on, using in succession all the factors; the last product will be the result required.*

REMARK.—Since the order in which factors are used will not vary the product, the student is recommended to seek the simplest number—the one most easily factored—as a multiplier.

EXAMPLES FOR PRACTICE.

- 99.** 1. Multiply 41 by 15, using as factors 3 and 5.
2. Multiply 17 by 21, using as factors 7 and 3.
3. Multiply 111 by 24, using as factors 3, 2, and 4.
4. Multiply 1157 by 36, using as factors 6 and 6.
5. Multiply 2019 by 45, using as factors 5, 3, and 3.
6. Multiply 87002 by 96, using as factors 6, 4, and 4.
7. Multiply 54235 by 144, using as factors 12 and 12.
8. Multiply 54235 by 144, using as factors 9 and 16.
9. Multiply 54235 by 144, using as factors 9, 4, and 4.
10. Multiply 54235 by 144, using as factors 3, 3, 2, and 8.
11. Multiply 54235 by 144, using as factors 3, 3, 2, 2, and 4.
12. Multiply 54235 by 144, using as factors 3, 3, 2, 2, 2, and 2.
13. Multiply 81 by 64, using as factors 8 and 8.
14. Multiply 64 by 81, using as factors 9 and 9.
15. Multiply 81 by 64, using as factors of 81, 9 and 9, and as factors of 64, 8 and 8.

EXAMPLES COMBINING ELEMENTARY PRINCIPLES PREVIOUSLY EXPLAINED.

100. Multiply

- | | |
|---|---|
| 1. 1431 by 7000. | 6. 3500 by 72 = 6×12 . |
| 2. 900 by 21046. | 7. 1921 by 450 = $9 \times 5 \times 10$. |
| 3. 1969 by 54 = 9×6 . | 8. 321058 by 144 = 12×12 . |
| 4. 171548 by 1500 = 15×100 . | 9. 504 by 288 = $9 \times 8 \times 4$. |
| 5. 1653 by 25000 = $5 \times 5 \times 1000$. | 10. 1043 by 105 = $7 \times 3 \times 5$. |

11. A clerk sold 9 shirts at 80 cents each, 2 neck-ties at 35 cents each, 10 collars at 25 cents each, a pair of gloves for 75 cents, and two suits of underwear at 95 cents per suit. What was the price of all?

12. I bought 15 cows at 32 dollars per head, a pair of horses for 245 dollars, a harness for 22 dollars, and 81 sheep at 3 dollars per head; what was the total cost of my purchases?

13. The cost of furnishing a house was, for parlor and library furniture 762 dollars, halls 150 dollars, dining room and kitchen 295 dollars, chambers 648 dollars, stoves and furnace 350 dollars, carpets and curtains 825 dollars, what was the total cost?

14. 14250 dollars was paid for four houses, the first costing 2750 dollars, the second 400 dollars more than the first, the third 250 dollars less than the first and second together, and the fourth the remainder. Find the cost of the fourth house?

15. Find the difference between the continued products of $91 \times 4 \times 3 \times 11 \times 9$ and $5 \times 5 \times 12 \times 4 \times 6 \times 7$.

16. Find the difference between seven units of the sixth order and the continued product of $15 \times 6 \times 5 \times 12 \times 4 \times 7 \times 11 \times 8 \times 2 \times 9$.

17. A merchant having 17462 dollars to his credit in a bank, gave checks as follows: for dry goods 5416 dollars, groceries 5995 dollars, boots and shoes 1416 dollars, hardware 1850 dollars, and drew out 500 dollars for family expenses; what amount was left in the bank?

18. Exchanged a city block valued at 35000 dollars, for a farm of 175 acres valued at 95 dollars per acre, eight horses at 110 dollars each, 14 cows at 28 dollars each, 225 sheep at 4 dollars each, farm machinery valued at 825 dollars, and received the balance in cash. How much cash was received?

19. A drover bought 135 horses at an average price of 115 dollars for 100 of them, and 125 dollars per head for the remainder; he sold 25 at 100 dollars per head, twice that number at twice the price per head, and the remainder at 67 dollars per head. How much was gained or lost?

20. A ranchman sold to a trader, 46 ponies at 60 dollars per pair, 116 calves at 9 dollars per head, 41 cows at 35 dollars per head, and a pair of mules for 375 dollars. He received in part payment, 15 barrels of flour at 9 dollars per barrel, 11 hundred weight of bacon at 12 dollars per hundred weight, 4 suits of clothes at 22 dollars per suit, 2 saddles at 13 dollars each, a wagon at 75 dollars, a set of furniture for 58 dollars, and the remainder in cash. What amount of cash did the trader pay?

MISCELLANEOUS EXAMPLES.

101. 1. The United States export 105,000 sewing machines yearly. If each machine does the work of 12 women, what is the value of the labor thus contributed by the United States to other nations each year of 306 working days, if labor be estimated at \$1 per day?

2. The Union Pacific Railway is 1777 miles in length, and was built at an average cost of \$106,775 per mile; what was the total cost of construction?

3. The bills issued by the U. S. Treasury for National Bank circulation, are in denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, and \$1000. How much money has one possessing 73 bills of each denomination?

4. The gold coins of the U. S. are in denominations of \$1, \$2.50, \$3, \$5, \$10, and \$20. How much money in a bag containing 365 of each of these coins?

5. The U. S. notes — *greenbacks* — are of the following denominations, viz.: \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, \$1000, \$5000, and \$10000. How large a debt could be paid with 7 of each of the above-named greenbacks?

6. How many feet of wire will be required to fence a field 1116 ft. square, with six wires on each of the four sides?

7. What is the amount of the following bill?

28 lb. Lard @ 11¢ per lb.

110 lb. Beef @ 14¢ per lb.

46 bu. Salt @ 15¢ per bu.

50 “ Butter @ 32¢ per lb.

17 “ Apples @ 45¢ per bu.

4 pk. Onions @ 35¢ per pk

61 lb. Pork @ 9¢ per lb.

15 bu. Potatoes @ 75¢ per bu.

8. Find the total cost:

4 cd. Hard Wood @ \$6 per cord.

13 tons Furnace Coal @ \$5 per ton.

11 “ Soft Wood @ \$3 per cord.

7 “ Stove Coal @ \$6 per ton.

9 loads Kindling @ \$2 per load.

2 “ Cannel Coal @ \$9 per ton.

9. Find the cost of

7 lb. Tea @ 65¢ per lb.

9 lb. Java Coffee @ 31¢ per lb.

50 “ A Sugar @ 7¢ per lb.

52 “ Br. Sugar @ 5¢ per lb.

15 “ Cheese @ 13¢ per lb.

25 “ C Sugar @ 6¢ per lb.

10. What must be paid for the following goods?

7 yd. Prints @ 7¢ per yd.

11 yd. Jeans @ 19¢ per yd.

61 “ Sheetting @ 13¢ per yd.

29 “ Calico @ 9¢ per yd.

77 “ Ticking @ 15¢ per yd.

14 “ Delaine @ 23¢ per yd.

17 “ Drilling @ 16¢ per yd.

25 “ Gingham @ 12¢ per yd.

11. Find the total cost:

67 yd. Moquette Carpet @ \$3 per yd.

32 yd. Border No. 1 @ \$3 per yd.

131 “ Brussels “ @ \$2 per yd.

70 “ “ “ 2 @ \$2 per yd.

100 “ Ingrain “ @ \$1 per yd.

45 “ “ “ 3 @ \$1 per yd.

12. The Boston “boot-maker” will enable a workman to make 300 pairs of boots daily. How many pairs can he make in a year having 309 working days?

13. My grain sales for the year 1888 were as follows :

516 bu. White Wheat @ 85¢ per bu.	250 bu. Peas @ 95¢ per bu.
723 " Red " @ 95¢ per bu.	287 " Rye @ 92¢ per bu.
941 " Barley @ 73¢ per bu.	635 " Beans @ 75¢ per bu.
1625 " Oats @ 32¢ per bu.	321 " Buckwheat @ 85¢ per bu.

How much was received for all ?

14. In New York State a bushel of barley weighs 48 lb., of clover seed 60 lb., of flax seed 55 lb., of beans 62 lb., of buckwheat 48 lb., of rye 56 lb., of corn 58 lb., of oats 32 lb., of potatoes 60 lb., of timothy seed 44 lb., and of wheat 60 lb. What will be the total weight of 5 bushels of each of the products named ?

15. In freighting, lime and flour are each estimated to weigh 200 lb. per barrel; pork and beef each 320 lb.; apples and potatoes 150 lb. each; cider, whisky, and vinegar each 350 lb. What will be the freight at 20¢ per hundred pounds, on a car containing 15 barrels of each of these products ?

16. I bought 10 acres of land at \$2250 per acre and laid it out in 75 city lots, expending \$4725 for grading and streets, \$680 for sidewalks, and \$87 for ornamental trees. I then sold 40 of my lots at \$500 each, 20 at \$450 each, and exchanged the remainder for a farm of 110 acres, the cash value of which was \$65 per acre. How much was gained or lost ?

17. A gardener rented 5 acres of land for \$20 per acre and paid \$63 for seeds, \$20 for fertilizers, \$246 for labor, and \$52 for freight. He sold 2145 bushels of turnips for \$429, 1710 bushels of beets for \$513, 4350 bunches celery for \$174, and 800 heads cabbage for \$40. What was his gain ?

18. A man earning \$2.50 per day, works 306 days per year for five years. His annual expenses are, for board \$156, for clothing \$47, for charity \$12, and he expends \$2 per week for incidentals. If he deposit his surplus each year in a Savings Bank, what amount will he deposit during the time ?

19. The U. S. coupon bonds are in denominations of \$50, \$100, \$500, and \$1000, and the registered bonds in denominations of \$50, \$100, \$500, \$1000, \$5000, and \$10000. Of the 4½'s of 1891, and the 4's of 1907, there are registered bonds of the denominations of \$20,000 and \$50,000. What would be the aggregate face value of twelve of each of the bonds above named ?

20. A man rented a farm of 132 acres of grain land, 67 acres of pasture land, and 45 acres of meadow land; paying for the grain land \$7 per acre, for the pasture land \$4 per acre, and for the meadow land \$11 per acre. He produced 61 bushels of oats per acre on 45 acres, 32 bu. barley per acre on 30 acres, 75 bu. corn per acre on 15 acres, 150 bu. potatoes on 9 acres, 28 bu. buckwheat on 20 acres, and 24 bu. beans per acre on the remainder of the grain land. He re-let the pasture land for \$200, and on the meadows cut 2 tons per acre of hay worth \$13 per ton. If he paid \$695 for labor and \$467 for other expenses, did he gain or lose, estimating oats at \$275, barley at \$672, corn at \$394, potatoes at \$743, buckwheat at \$420, and beans at \$2 per bushel ?

DIVISION.

102. Division is the process of finding how many times one number is contained in another of the same kind.

103. The Dividend is the number divided.

104. The Divisor is the number by which the dividend is divided.

105. The Remainder is the part remaining when the division is not exact.

106. The Sign of Division is the character \div ; it indicates that the number before it is to be divided by the number after it. Thus, $24 \div 3 = 8$, is read 24 divided by 3 equals 8. We see by this operation that 3 is an exact divisor of 24, also that 3 and 8 are factors of 24.

REMARK.—From the above it is clear that the *dividend* in division corresponds to the *product* in multiplication, and the *divisor* and *quotient* to the *multiplier* and *multiplicand*, or the factors in multiplication.

107. General Principles.—1. *Multiplying the dividend multiplies the quotient.* Thus, $48 \div 6 = 8$; $(48 \times 2) \div 6 = 16$.

2. *Dividing the divisor multiplies the quotient.* Thus, $48 \div 6 = 8$; $48 \div (6 \div 2) = 48 \div 3 = 16$.

3. *Dividing the dividend divides the quotient.* Thus, $48 \div 6 = 8$; $(48 \div 2) \div 6 = 24 \div 6 = 4$.

4. *Multiplying the divisor divides the quotient.* Thus, $48 \div 6 = 8$; $48 \div (6 \times 2) = 48 \div 12 = 4$.

108. General Law.—I. *Any change in the dividend produces a like change in the quotient.*

II. *Any change in the divisor produces an opposite change in the quotient.*

III. *A like change in both dividend and divisor will not change the quotient.*

109. General Rules.—1. *If the dividend and divisor be given, the quotient may be found by dividing the dividend by the divisor*

2. *If the dividend and quotient be given, the divisor may be found by dividing the dividend by the quotient.*

3. *If the divisor and quotient be given, the dividend may be found by multiplying the divisor by the quotient.*

4. *If the divisor, quotient, and remainder be given, the dividend may be found by multiplying the divisor by the quotient and adding the remainder to the product.*

110. To Prove Division.—*Divide the dividend by the quotient, or multiply the divisor by the quotient. In divisions which are not exact, add the remainder to the product of the divisor and quotient; the sum thus obtained should be the dividend.*

111. The Reciprocal of a number is one, or unity, divided by that number.

A reciprocal will be produced by changing the relation of dividend and divisor; as, $28 \div 4 = 7$, while $4 \div 28 = \frac{1}{7}$; the resulting $\frac{1}{7}$ is the reciprocal of the first quotient 7.

MENTAL EXERCISES.

112. What is the quotient of

- | | |
|---------------------------------|-------------------------------------|
| 1. $16 \div 2$, 4, 8. | 11. $125 \div 5$, 25. |
| 2. $20 \div 2$, 4, 5, 10. | 12. $48 \div 4$, 12, 3, 6, 2. |
| 3. $56 \div 4$, 8, 2, 7, 14. | 13. $64 \div 8$, 4, 32, 2, 16. |
| 4. $90 \div 5$, 3, 6, 15, 9. | 14. $120 \div 20$, 3, 8, 5, 12. |
| 5. $45 \div 9$, 15, 5, 3. | 15. $80 \div 4$, 16, 10, 20, 8. |
| 6. $36 \div 4$, 18, 12, 2, 9. | 16. $144 \div 12$, 8, 6, 4, 3, 24. |
| 7. $72 \div 6$, 2, 12, 24. | 17. $175 \div 35$, 7, 5. |
| 8. $84 \div 7$, 4, 2, 12, 21. | 18. $96 \div 6$, 8, 32, 12, 16. |
| 9. $100 \div 5$, 25, 2, 4, 10. | 19. $108 \div 3$, 2, 9, 6, 12, 27. |
| 10. $24 \div 6$, 2, 4, 12, 8. | 20. $200 \div 5$, 10, 20, 8, 4. |

Operations in Division are of two kinds, *Short Division* and *Long Division*.

113. In **Short Division**, operations are restricted to those divisions in which the divisor consists of one figure, or is a number coming within one's thorough knowledge of the multiplication table.

114. When the Divisor consists of only one figure.

EXAMPLE.—Divide 6482 by 2.

OPERATION.

$$\begin{array}{r} 2 \overline{) 6482} \\ \underline{3241} \end{array}$$

EXPLANATION.—Write the divisor at the left of the dividend, separating them by a line, next draw a line below the dividend and then divide each figure of the dividend by the divisor, writing the quotient below the figure divided. Thus, 2 is contained in 6 thousands, 3 thousands times; write the 3 below the 6 in thousands' column, next, 2 is contained in 4 hundreds, 2 hundreds times; place the 2 below the 4 in hundreds' column; 2 is contained in 8 tens, 4 tens times; write the quotient in tens' column; 2 is contained in 2 units, 1 unit times, or once; write 1 in units' place, thus completing the division, and obtaining 3241 as a quotient

115. When the Divisor is a Number within one's thorough knowledge of the Multiplication Table.

EXAMPLE.—Divide 31605 by 15.

OPERATION.

$$\begin{array}{r} 15 \overline{) 31605} \\ \underline{2107} \end{array}$$

EXPLANATION.—Write the terms as before. Divide 31 by 15 and obtain 2, which write below the 1 as the first figure of the quotient; next, 15 is contained in 16, once; write 1 in hundreds' column; 15 in 10, 0, or no times; write the 0, or cipher, in tens' column; 15 in 105, 7 times; write the 7 as units of the quotient, thus completing the division, and obtaining the quotient 2107.

116. When any Figure or Figures of the Dividend will not Exactly Contain the Divisor.

EXAMPLE.—Divide 394015 by 8.

OPERATION.

$$\begin{array}{r} 8 \overline{) 394015} \\ \underline{49251\frac{7}{8}} \end{array}$$

EXPLANATION.—Write the terms as before. Since 3 hundreds of thousands is not divisible by the divisor 8, unite the 3 hundreds of thousands and the 9 tens of thousands, obtaining 39 tens of thousands; divide this by 8 and obtain for the first figure of the quotient 4 tens of thousands, with a remainder of 7 tens of thousands; write the 4 below the 9 as the tens of thousands of the quotient, and unite the 7 tens of thousands to the

4 thousands of the dividend and divide; 8 is contained in 74 thousands, or 7 tens of thousands + 4 thousands, 9 thousands times with a remainder of 2 thousands; write the 9 in the column of thousands, and unite the 2 thousands to the next figure of the dividend; 8 is contained in 20 hundreds, 2 hundreds times with a remainder of 4 hundreds; write the 2 hundreds in the column of hundreds, and unite the 4 hundreds to the next figure of the dividend; 8 is contained in 41 tens, or 4 hundreds + 1 ten, 5 tens times, with a remainder of 1 ten; write the 5 in tens' column and unite the 1 ten to the last figure of the dividend; 8 is contained in 15 units, 1 unit times, or once, with a remainder of 7 units, or 7; write the remainder over the divisor in the form of a fraction and annex the result to the entire part of the quotient, thus obtaining $49251\frac{7}{8}$ as the complete quotient of 394015 divided by 8

Rule.—I. *Write the divisor at the left of the dividend with a line separating them.*

II. *Beginning at the left, divide each figure of the dividend by the divisor, and write the resulting quotient underneath the dividend.*

III. *If after any division there be a remainder, regard this remainder as prefixed to the next figure of the dividend, and divide as before*

IV. *Should any partial dividend considered, be less than the divisor, place a cipher in the quotient and regard the undivided part as prefixed to the succeeding figure in the dividend and again divide*

V. *If the division is not exact, write the remainder over the divisor in fractional form, and annex the result to the integral part of the quotient.*

EXAMPLES FOR PRACTICE.

117. Divide

- | | | |
|----------------|--------------------|---------------------|
| 1. 646 by 2. | 8. 143258 by 11. | 15. 7600 by 16. |
| 2. 945 by 3. | 9. 81052 by 13. | 16. 240000 by 13. |
| 3. 1124 by 4. | 10. 5841226 by 14. | 17. 20416201 by 15. |
| 4. 2645 by 5. | 11. 90090 by 7. | 18. 952451 by 17. |
| 5. 31562 by 8. | 12. 163208 by 15. | 19. 200468 by 18. |
| 6. 60703 by 9. | 13. 21406 by 8. | 20. 1119306 by 10. |
| 7. 2075 by 12. | 14. 51007 by 11. | 21. 8476432 by 12. |

118. When the Divisor is a Composite Number.

When the divisor is a composite number the operation may be simplified by using the factors of the divisor

EXAMPLE.—Divide 15552 by 288.

OPERATION.		EXPLANATION.—First resolve the number 288 into the factors 3, 8, 12. Then dividing the dividend by the factor 3 obtain 5184, the first quotient; dividing this quotient, treated as a new dividend, by the factor 8 obtain 648 as the second quotient; again, dividing by the factor 12 obtain 54, the third, or final quotient, which is the quotient required. Hence 14552 divided by 288 equals 54.
3) 15552		
8) 5184	1st quotient.	
12) 648	2nd "	
54	3rd "	

Rule.—Divide the dividend by any one of the factors, and the quotient thus obtained by another of the factors, and so on until all of the factors have been used as a divisor. The last quotient will be the required result.

EXAMPLES FOR PRACTICE.

119. 1. Divide 216 by 72, using the factors 8 and 9.
2. Divide 1100 by 55, using the factors 5 and 11.
3. Divide 5280 by 480, using the factors 4, 12, and 10.
4. Divide 31248 by 144, using the factors 12 and 12.
5. Divide 31248 by 144, using the factors 9 and 16.
6. Divide 51248 by 144, using the factors 8 and 18.
7. Divide 31248 by 144, using the factors 8, 2, and 9.
8. Divide 31248 by 144, using the factors 4, 2, 3, and 6.
9. Divide 31248 by 144, using the factors 2, 2, 2, 3, 3, and 2.
10. Divide 2025 by 45, using the factors 3 and 15.
11. Divide 2025 by 45, using the factors 3, 3, and 5.
12. Divide 2025 by 45, using the factors 9 and 5.

REMARK.—The pupil will observe that the order in which the factors are used, does not vary the result.

120. To find the True Remainder after Dividing by the Factors of a Composite Number.

EXAMPLE.—Divide 1347 by 105, using the factors 5, 3, and 7.

OPERATION.	
5) 1347 units.	
3) 269 ^{5's} +	2 units.
7) 89 ^{15's} + 2 ^{5's} =	10 "
12 ^{105's} + 5 ^{15's} =	75 "
	87 true rem.
18 ⁸⁷ ₁₀₅	quotient.

EXPLANATION.—Divide the given dividend by 3, obtaining the quotient 269, with 2 units for a remainder; the quotient 269 is composed of units equal in value to 5 times those of the *given* dividend, and may be written 269^{5's}; the remainder, 2, is of the same unit value as the *given* dividend, and is, therefore, a part of the *true* remainder; next divide the quotient 269^{5's} by 3 obtaining 89 for a quotient and 2 for a remainder. The units of which the quotient 89 is composed, are equal in value to 15 times those of the *given* dividend and may be written 89^{15's}; the remainder is 2^{5's} and equals 5 × 2, or 10 units of the *given* dividend, next divide by 7 which gives the quotient 12, with 5 for a remainder; the quotient 12 is composed of units equal in value to 105 times those in the *given* dividend and may

be written $12^{105's}$; the remainder is $5^{15's}$ and equals 15×5 , or 75 units of the *given* dividend. The sum of the remainders, 2 units, $25's$, or 10 units, and $5^{15's}$, or 75 units, equals 87, the true remainder, and the result of the division, or the quotient, is 12 with a remainder of 87 : or, in another form $12\frac{87}{105}$.

EXAMPLES FOR PRACTICE.

121. 1. Divide 1121 by 25, using as factors 5 and 5.
 2. Divide 819 by 42, using as factors 3, 2, and 7.
 3. Divide 1705 by 64, using as factors 8 and 8.
 4. Divide 4600 by 135, using as factors 3, 5, 3, and 3.
 5. Divide 22406 by 125, using as factors 5, 5, and 5.
 6. Divide 53479 by 144, using as factors 12 and 12.
 7. Divide 53479 by 144, using as factors 9 and 16.
 8. Divide 53479 by 144, using as factors 8 and 18.
 9. Divide 53479 by 144, using as factors 4, 9, and 4.
 10. Divide 53479 by 144, using as factors 4, 3, 3, and 4.
 11. Divide 53479 by 144, using as factors 2, 2, 3, 3, 2, and 2.
 12. Divide 419047 by 81, using as factors 3, 3, 3, and 3.
 13. Divide 341772 by 4095, using as factors 7, 5, 9, and 13.
 14. Divide 792431 by 72, using as factors 6, 2, and 6.
 15. Divide 19111 by 24, using as factors 2, 2, 2, and 3.

122. To Divide by 10, or any one of its powers.

Since by the decimal system, numbers increase in value from right to left and decrease from left to right in a tenfold ratio, it follows that to cut off from the right of a number one place, divides the number by 10, two places by 100, three places by 1000, etc.

Rule.—*From the right of the dividend point off as many orders of units, or places, as the divisor contains ciphers. The figure or figures so cut off will express the remainder.*

123. To Divide by any multiple of 10, 100, or 1000, etc.

EXAMPLE.—Divide 16419 by 600.

FIRST OPERATION.

$$1/00 \overline{) 16419}$$

First quotient 164..... 19, first rem.

SECOND OPERATION.

$$6 \overline{) 164}$$

Second quotient 27... $2 \times 100 = 200$, second rem.

219, true rem.

$27\frac{219}{600}$ required quotient.

EXPLANATION.—6 and 100 are factors of 600. First divide 16419 by 100, by separating from it the last two figures, obtaining 164 as the first quotient and 19 as the first remainder; next divide 164 by 6 and obtain 27 as the second, or last quotient, and 2 as the second, or last remainder; multiply this remainder by 100, to obtain its true value, and to the result add the first remainder obtaining 219 for the true remainder. The result of the division is a quotient of 27 and a remainder of 219, or $27\frac{219}{600}$.

Rule.—*From the right of the dividend separate as many figures as the divisor contains ciphers; divide the figures at the left of the separatrix by the digit or digits of the divisor, and to the remainder, if there be one, annex the figures first separated from the dividend; the result will be the true remainder.*

EXAMPLES FOR PRACTICE.

124. 1. Divide 519 by 40, using as factors 4 and 10.
2. Divide 1164 by 300, using as factors 3 and 100.
3. Divide 2684 by 500, using as factors 5 and 100.
4. Divide 90406 by 1500, using as factors 15 and 100.
5. Divide 83251 by 600, using as factors 6 and 100.
6. Divide 416250 by 9000, using as factors 9 and 1000.
7. Divide 94275 by 3000, using as factors 3 and 1000.
8. Divide 730246 by 11000, using as factors 11 and 1000.
9. Divide 50640231 by 120000, using as factors 12 and 10000.
10. Divide 620974 by 41000, using as factors 41 and 1000.
11. Divide 124689011 by 5910000, using as factors 591 and 10000.
12. Divide 365021467 by 6250000, using as factors 625 and 10000.

MISCELLANEOUS EXAMPLES IN SHORT DIVISION.

125. 1. A gentleman left his estate worth \$618330 to be shared equally by his wife and five children; what was the share of each?
2. A county containing 400000 acres is divided into 25 townships of equal area. How many acres in each township?
3. \$21735 was received from the sale of a farm at \$35 per acre. How many acres did the farm contain?
4. If a speculator pays \$15730 for 715 acres of Nebraska prairie land, and sells the same for \$17875, what is his gain per acre?
5. In New York City, in February, 1882, Hazel walked 660 miles in 6 days, receiving as a prize \$20000. Allowing no time for stops, what was his average distance and the average amount earned per hour?
6. Great Britain makes 330 million pins weekly, or 9 for each inhabitant; what is the number of inhabitants?
7. The dividend is 230304561, the divisor is 15; find the quotient and the remainder.
8. The remainder is 7, the quotient 19023, and the dividend 247306; what is the divisor?
9. If 8 men can do a certain piece of work in 9 days, in how many days can 12 men do the same work?
10. I sell my village home for \$3250, my store for \$5000, my stock of goods for \$11250, receiving in part payment \$8775 cash, and for the remainder Iowa prairie land at \$15 per acre; how many acres should I receive?

11. The steamship *Servia* crosses the Atlantic from New York City to Liverpool in 150 hours, averaging for the first 24 hours, 18 miles per hour; for the next 48 hours, 17 miles per hour; for the next 30 hours, 19 miles per hour; and for the next 12 hours, 21 miles per hour. If the entire distance be 2841 miles, what was the average distance per hour traveled for the remainder of the time?

REMARK.—Short division, though a mental process, is practicable whenever the divisor is 25 or less, if the pupil has mastered the multiplication table as given.

LONG DIVISION.

126. When the divisor is a number larger than can be treated mentally, the following method, called *Long Division*, is employed.

EXAMPLE.—Divide 81437 by 37.

OPERATION.

Divisor. Dividend Quotient.

37) 81437 (2201

74

74

74

37

37

0 Remainder.

EXPLANATION.—Write the terms as in short division, and place a line after the dividend to separate it from the quotient, which is now to be written at the right. Then divide the first two figures of the dividend, 81, by the divisor, 37, and obtain 2 as the first figure of the quotient; then subtract from 81 the product of 2×37 , or 74, obtaining 7 as a remainder; to this remainder annex 4, the succeeding figure of the dividend, which gives 74 as the next partial dividend; the divisor is contained in this dividend twice, or 2 times, giving 2 as the next or second quotient figure; subtracting the product of 2×37 from 74, nothing remains; then bring down 3, the next figure of the dividend and as it is less than the divisor, place a 0 in the quotient; next bring down 7, the

remaining figure of the dividend which gives 37 as the last partial dividend; the divisor is contained in this dividend once, or 1 time; writing this 1 as the final figure of the quotient and subtracting the last partial product from the last partial dividend nothing remains, and the quotient, 2201, is the result of dividing 81437 by 37.

Rule.—1. Write the divisor at the left of the dividend with a curved line between them, and another line at the right of the dividend to separate it from the quotient when found.

II. From the left of the dividend select the least number of figures that will contain the divisor one or more times, and divide. Write the quotient figure thus obtained at the right of the dividend, multiply the divisor by this quotient figure and subtract the product from the partial dividend used. To the remainder annex the succeeding figure of the dividend and divide as before; so continue until the last partial product has been subtracted from the last partial dividend. If there be a remainder place it over the divisor with a line between, and write the resulting fraction as a part of the quotient.

Proof.—Multiply the DIVISOR by the QUOTIENT, and to the product add the REMAINDER if there be any; the result should equal the DIVIDEND.

EXAMPLES IN LONG DIVISION.

127. Divide

1. 1728 by 48.	11. 115680 by 155.	21. 375735212 by 20812.
2. 2025 by 135.	12. 29410 by 251.	22. 26800001 by 909125.
3. 625 by 125.	13. 666666 by 2144.	23. 104690955 by 5642.
4. 1920 by 160.	14. 93462007 by 1525.	24. 9000716002 by 1776.
5. 2268 by 45.	15. 500500500 by 1888.	25. 250252500 by 1562.
6. 106295 by 28.	16. 21416009 by 5407.	26. 5087910041 by 508791.
7. 52467 by 109.	17. 11460250 by 999.	27. 3641694611 by 72853.
8. 4762 by 367.	18. 87629000 by 11181.	28. 111222333456 by 370054.
9. 250000 by 793.	19. 20405701 by 820066.	29. 9876543210 by 12345.
10. 87524 by 31.	20. 72109904 by 72109.	30. 210631890048 by 840263.

MISCELLANEOUS EXAMPLES IN LONG DIVISION.

128. 1. In 1880 the total number of persons engaged in all occupations in the United States was 17392099, of which 7670493 were engaged in agriculture; how many times greater is the whole number of workers than those engaged in agriculture?

2. The 2515 miles of canal in the United States cost \$170028636; what was the average cost per mile?

3. If an elephant produces 120 lb. of ivory and the manufactories of Sheffield consume yearly 483000 lb., how many elephants must be killed each year to supply the Sheffield market alone?

4. In 1880 there were in attendance in the 177100 public schools of the United States 9705100 pupils; what was the average number in attendance in each school?

5. During the financial crisis of 1857, 7200 business houses in the United States failed for an aggregate of 111 million dollars; what was the average insolvency?

6. Dan. Lambert, at the age of 40, weighed 739 lb.; if his weight at birth was 13 lb., what was his average yearly increase of weight?

7. Between 1871 and 1884 the Kimberly diamond field of 9 acres produced 75 million dollars worth of diamonds; what average value per acre was produced each year? Each month?

8. A bottle thrown overboard into the Pacific Ocean was picked up 455 days later, 6700 miles distant from where it was thrown; what average distance did it float per day?

9. The great bell of Moscow weighs 202 tons of 2240 lb. each; if 77 parts of the metal of which it is composed are copper and the remaining 23 parts tin, how many pounds of each metal does the bell contain?

10. The log of the yacht Wanderer in circumnavigating the globe in 1880-82, showed 48490 miles run in 280 days actual running time; what was the average miles run per day?

11. An Iowa firm manufactures daily, from 5 tons of paper, 1600 barrels, of 6 lb. weight each; what number of barrels can be made, at this rate, from 10750 lb. of paper?

12. On the planet Neptune 60127 days make one year. A year on Neptune equals how many common years on the earth?

13. For the year ending September 30, 1887, the exchanges at the Clearing House at New York amounted to \$34872848786, and those of the 36 remaining important cities, \$17253855702. What was the average of the exchanges per month at the New York Clearing House? What was the average per month of the 36 remaining Clearing Houses?

14. The Spanish Armada, sent in 1588, by Phillip II. of Spain for the intended conquest of England, comprised 132 ships with 34054 seamen and soldiers. What was the average number with each ship?

15. In 1885 the total loans of the National Banks of Chicago and St. Louis were \$55171842, while those of the National Banks of New York city were \$236823598. How many times greater was the amount loaned by the banks of New York than by the banks of the other two cities named?

16. The aggregate height above sea level of the 8 highest mountains of the earth, is 174173 feet. What is the average height in miles of 5280 feet each?

17. During the year 1854, 50 banks of New York city made exchanges through the Clearing House to the amount of \$5750455987; and in the year 1887, 64 banks made exchanges to the amount of \$34872848786. Find the average clearings of each bank for each of the two years quoted.

18. The Kingdom of Belgium averages 480 inhabitants per square mile and the United States averages only 14. How many more times densely peopled is Belgium than the United States?

19. The National Banks of St. Louis in 1885 made loans to the amount of \$9182417, while those of Chicago made, during the same year, loans to the amount of \$45989425. How many times greater were the loans of the banks of Chicago than those of St. Louis?

20. The total cost of the railroads of the U. S. in 1880 was \$5425772550. If the average cost per mile was \$62522, how many miles had there been built?

21. In 1880 the total railroad freight of the United States was 290897395 tons, of which 42003504 tons was grain and 89622899 tons was coal. How many times greater was the whole freight than that of coal alone? How many times greater than that of grain alone?

22. The total expenditures of the railroads of the United States in the year 1880, were \$541950795, and their net income was \$119344596. How many times greater were the expenditures than the net income?

AVERAGE.

129. The **Average** of several numerical terms is the quotient obtained by dividing their sum by the number of terms taken. Thus, the average of 32, 40, 56, 16, 72, 24, 70, and 66, is 47, because 8 times 47 = 376, which is the sum of the numbers taken.

130. An average may be fractional; as $33\frac{1}{2}$ is the average of 59, 43, 21, 10, and 35, because the sum of these five numbers equals 5 times $33\frac{1}{2}$.

REMARK.—The average numerical value of *fractions*, either common or decimal, may be obtained by dividing the sum of all such fractional expressions by the number of such expressions taken.

Rule.—*Divide the sum of the terms by the number of terms used.*

EXAMPLES FOR PRACTICE.

131. Find the average of the following groups of numbers and prove the results:

- | | |
|------------------------------|--|
| 1. 20, 24, 52, and 88. | 3. 71, 46, 200, 11, 93, 51, and 17. |
| 2. 32, 72, 56, 108, and 144. | 4. 5, 28, 19, 72, 40, 85, 106, 29, and 54. |
5. A man walked during six days of a week, 41, 47, 36, 54, 60, and 44 miles respectively. How many miles did he average per day?
6. A merchant sold during the 12 months of a year, goods in amounts as follows: \$14216, \$10008, \$11051, \$11097, \$18241, \$16900, \$13754, \$12291, \$9267, \$12935, \$14901, and \$20518. What were his average sales per month?
7. An errand boy earned on Monday 73¢, Tuesday 91¢, Wednesday 49¢, Thursday 67¢, Friday 81¢, and Saturday 95¢. What were his average earnings per day for the week?

COMPLEMENT.

132. The **Complement** of a number is the difference between such number and a unit of the next higher order; thus, the complement of 6 is 4, because 4 is the difference between 6 and 10, or 1 *ten*, a unit of the next higher order than 6.

Again, the complement of 83 is 17, because 17 is the difference between 83 and 100, or 1 *hundred*, a unit of the next higher order than 83.

Again, the complement of 209 is 791, because their sum is equal to 1000.

EXAMPLES FOR PRACTICE.

133. Find the complement of each of the following numbers, and prove and explain results:

- | | | | | |
|--------|---------|---------|----------|------------|
| 1. 36. | 3. 115. | 5. 81. | 7. 1249. | 9. 28763. |
| 2. 71. | 4. 704. | 6. 258. | 8. 1094. | 10. 82041. |

FACTORS AND FACTORING.

134. Factors are such numbers as multiplied together will produce a required number ; as 3 and 4, also 3, 2, and 2 are factors of 12 ; 3 and 15, also 5 and 9 are factors of 45

135. A Prime Number is one that cannot be resolved into two or more factors ; or, it is a number exactly divisible only by itself and unity ; thus, 2, 3, 5, 7, 11, and 13, are prime numbers. 2 is the only *even* number that is prime.

136. A Composite Number is one that can be resolved into factors.

137. A Prime Factor is a *prime* number used as a *factor*.

To aid the pupil in determining the prime factors of a *composite* number we give the following

Table of Prime Numbers from 1 to 1000.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

REMARK.—The pupil can with little labor memorize the prime numbers from 1 to 100.

138. To Find the Prime Factors of a Composite Number.

EXAMPLE.—Find the prime factors of 4290.

OPERATION.

$$\begin{array}{r}
 5 \overline{) 4290} \\
 2 \overline{) 858} \\
 3 \overline{) 429} \\
 11 \overline{) 143} \\
 13 \overline{) 13} \\
 1
 \end{array}$$

EXPLANATION.—Observe that the given number ends with a cipher, hence is exactly divisible by the prime number 5, by which divide it; next, observe that the quotient ends with an even number, and is, therefore, exactly divisible by 2, so divide by 2; then observe that 3 will exactly divide the quotient 429; divide by it, obtaining 143, which divide by 11, obtaining 13, which divided by itself, gives a quotient of 1. All the divisors being prime numbers they together constitute the prime factors of 4290.

Rule.—*Divide by any prime number that is exactly contained in the dividend; divide the resulting quotient in the same manner, and continue this until the final quotient is 1. The prime divisors will be all the prime factors of the dividend.*

EXAMPLES FOR PRACTICE.**139. Resolve**

- | | |
|---------------------------------|------------------------------------|
| 1. 27 into its prime factors. | 6. 1050 into its prime factors. |
| 2. 117 into its prime factors. | 7. 144 into its prime factors. |
| 3. 165 into its prime factors. | 8. 15625 into its prime factors. |
| 4. 93 into its prime factors. | 9. 22464 into its prime factors. |
| 5. 2376 into its prime factors. | 10. 881790 into its prime factors. |

DIVISORS.

140. An **Exact Divisor** of a number is one which will divide it without a remainder, or which gives a whole number as a quotient; thus, 5 is an exact divisor of 15, 3 of 12, and 2 of 4.

141. 1. Any number is divisible by itself and 1.

2. Any even number is divisible by 2.

3. Any number ending with 5 or 0 is divisible by 5.

4. Any number ending with 0 is divisible by 10.

5. An even number is not an exact divisor of an odd number.

6. A composite number is an exact divisor of any number when all its factors are divisors of the same number.

142. A **Common Divisor** of two or more numbers is one that will exactly divide all the numbers considered; thus 3 is a *common* divisor of 6, 9, 12, and 15; also 7 is a *common* divisor of 14, 28, 35, and 49.

143. The **Greatest Common Divisor** of two or more numbers is the greatest number that is exactly contained in all of them, or that will divide each of them without a remainder.

144. Numbers having no common divisor, or factor, are said to be *prime* to each other.

145. To Find the Greatest Common Divisor.

I. When the numbers are readily factored.

EXAMPLE.—Find the greatest common divisor of 10, 15, and 35.

OPERATION.	EXPLANATION.—
$\begin{array}{r} 5 \overline{) 10 - 15 - 35} \\ \underline{2 - 3 - 7} \end{array}$	By inspection find that the prime number 5 is an exact divisor of each of the numbers given; using it as a divisor, gives as quotients 2, 3, and 7; these being prime numbers have no common divisor, therefore 5 is a common divisor of the numbers 10, 15, and 35, and as it is the greatest number that will exactly divide them it must be their greatest common divisor.

REMARK.—When it is determined by inspection that any *composite* number will exactly divide all the numbers of which we wish to obtain the greatest common divisor, such *composite* number may wisely be used as a divisor.

II. When numbers are less readily factored.

EXAMPLE.—Find the greatest common divisor of 140, 210, 350, 420, and 630.

OPERATION.	EXPLANATION.—
$\begin{array}{r} 2 \overline{) 140 - 210 - 350 - 420 - 630} \\ 5 \overline{) 70 - 105 - 175 - 210 - 315} \\ 7 \overline{) 14 - 21 - 35 - 42 - 63} \\ \underline{2 - 3 - 5 - 6 - 9} \end{array}$	To prevent confusion, separate the numbers by a short dash. Observe that 2 will exactly divide each of the numbers, likewise that 5 and 7 will exactly divide the successive quotients; therefore divide by 2, 5, and 7; then observe that the remaining quotients, 2, 3, 5, 6, and 9 have no common divisor; hence the divisors 2, 5, and 7 are all factors of the greatest common divisor, which is 70.

Rule.—I. Write the numbers in a horizontal line, separating them by a dash.

II. Divide by any number that will exactly divide all the numbers given, and so continue until the quotients have no common divisor.

III. Multiply together the divisors for the Greatest Common Divisor.

REMARK.—When factors cannot be readily determined by inspection the numbers may be resolved into their prime factors. The product of all the common factors of all the given numbers will be the greatest common divisor.

EXAMPLES FOR PRACTICE.

146. Find the greatest common divisor of

- | | |
|-----------------------------------|------------------------------------|
| 1. 22, 55, and 99. | 7. 252, 630, 1134, and 1456. |
| 2. 24, 36, 60, and 96. | 8. 2150, 600, 3650, 1000, and 500. |
| 3. 32, 48, 80, 112, and 144. | 9. 302, 453, 755, 1057, and 1661. |
| 4. 54, 72, 90, 126, 180, and 216. | 10. 126, 441, 567, 693, and 1071. |
| 5. 104, 156, 260, 364, and 572. | 11. 210, 350, 280, 840, and 1260. |
| 6. 135, 450, 315, and 585. | 12. 200, 325, 525, 350, and 675. |

147. When no Common Factor can be Determined by Inspection.

EXAMPLE.—What is the greatest common divisor of 182 and 858.

OPERATION.

182	4	858
130	1	728
<hr/>		
52	2	130
52	2	104
<hr/>		
0		26

EXPLANATION.—Draw two vertical lines and write the numbers on the right and left. Then divide 858 by 182, and write the quotient, 4, between the lines; then divide 182 by the remainder, 130, and write the quotient, 1, between the lines; next divide 130 by 52 and write the quotient, 2, as before; next divide 52 by 26 and write the quotient as before. As there is nothing now remaining the last divisor, 26, is the greatest common divisor of the given numbers.

REMARKS.—1. The greatest common divisor of several numbers which cannot be factored, may be obtained by taking any two of them and applying the above formula; then the divisor thus obtained and one of the remaining numbers, and so on until the last. If 1 be the final result they have no common divisor; if any number greater than 1, that number must be the greatest common divisor of all the given numbers.

2. The only practical use of the Greatest Common Divisor is in the reduction of a common fraction to its lowest terms; we thus find a number that will affect such reduction by a division of the terms but once.

Rule.—Divide the greater number by the less, the divisor by the remainder, and so continue until nothing remains. The last divisor will be the Greatest Common Divisor.

EXAMPLES FOR PRACTICE.

148. Find the greatest common divisor of

- | | | |
|-------------------|-------------------|--------------------|
| 1. 316 and 664. | 5. 1377 and 1581. | 9. 231 and 273. |
| 2. 96 and 216. | 6. 92 and 124. | 10. 1179 and 1703. |
| 3. 1226 and 2722. | 7. 679 and 1869. | 11. 1888 and 1425. |
| 4. 1649 and 5423. | 8. 2047 and 3013. | 12. 1900 and 1375. |

MULTIPLES.

149. A Multiple is a number exactly divisible by a given number; as, 12 is a multiple of 6.

150. A Common Multiple is a number exactly divisible by two or more given numbers; as, 12 is a common multiple of 6, 3, and 2.

151. The Least Common Multiple of two or more numbers is the least number exactly divisible by each of them; as, 36 is the least common multiple of 18, 9, 6, 4, 3, and 12.

152. Principles.—1. The product of two or more numbers, or any number of times their product, must be a common multiple of the numbers.

2. Two or more numbers may have any number of common multiples.

3. A multiple of a number must contain all the prime factors of that number.

4. The common multiple of several numbers must contain all the factors of all the numbers.

5. The least common multiple of two or more numbers is the least number that will contain all the prime factors of the numbers given.

153. To Find the Least Common Multiple of Two or More Numbers

EXAMPLE.—Find the least common multiple of 12, 16, 63, and 90.

EXPLANATION.—By factoring, find the prime factors of 12 which are 2, 2, and 3.

"	"	" 16	"	2, 2, 2, and 2.
"	"	" 63	"	3, 3, and 7.
"	"	" 90	"	3, 3, 2, and 5.

Since no number less than 90 can be divided by 90, it is evident that the least common multiple cannot be less than that number; hence it must contain 3, 3, 2, and 5, the factors of 90; including with these another 2, gives all the factors of 12; two more 2's all the factors of 16; and if 7 be included, all the factors of 63 are obtained; hence the product of the factors 3, 3, 2, 5, 2, 2, 2, and 7, or 5040 must be the least common multiple of the numbers 12, 16, 63, and 90.

The method of determining the least common multiple by formula given below, will be found convenient.

EXAMPLE.—Find the least common multiple of the numbers 12, 16, 63, and 90.

Write the numbers in a horizontal line to obviate confusion, and separate them by a dash.

OPERATION.

$$\begin{array}{r}
 2 \) \ 12-16-63-90 \\
 \hline
 2 \) \ 6-8-63-45 \\
 \hline
 3 \) \ 3-4-63-45 \\
 \hline
 3 \) \ 1-4-21-15 \\
 \hline
 1-4-7-5
 \end{array}$$

EXPLANATION.—First divide by 2; 63 not being divisible by 2 bring it to the lower line and divide again by 2; neither 63 nor 45 being divisible by 2, bring both to the lower, or quotient line. Next divide by 3; 4 not being divisible by 3, bring it to the quotient line and divide again by 3; the remaining numbers 4, 7, and 5 being prime to each other, are to be taken, together with the prime divisors 2, 2, 3, and 3, as factors of the least common multiple; their product is 5040, the same as before found.

REMARKS.—1. This principle has a practical value only in determining the least common denominator of common fractions, and is even then rarely used.

2. Where one of the numbers given is a factor of another, reject the smaller.

3. When it is observed that any composite number is exactly contained in *all* the numbers given, divide by such composite number rather than by its prime factors; the operation will thus be shortened.

Rule.—I. *Write the numbers in a horizontal line, separating them by a dash.*

II. *Divide by any factor common to all the numbers, or by any prime factor of any two or more of them. In the same manner divide the quotients obtained, and continue until the quotients are prime to each other.*

III. *The product of the divisors and prime remainders is the Least Common Multiple.*

154. Greatest Common Divisor and Least Common Multiple Compared.

I. The greatest common divisor is the product of *all* the *prime* factors *common* to *all* the numbers.

II. The least common multiple is the product of *all* the *prime* factors of *all* the numbers.

EXAMPLES FOR PRACTICE.

155. Find the least common multiple of

- | | | |
|-----------------------|-------------------------|-----------------------------|
| 1. 12, 20, and 32. | 4. 42, 210, 56, and 35. | 7. 18, 80, 99, and 120. |
| 2. 25, 90, and 225. | 5. 5, 30, 24, and 28. | 8. 2, 3, 4, 5, 6, 7, and 8. |
| 3. 6, 16, 26, and 36. | 6. 11, 32, 216, and 66. | 9. 21, 72, 24, and 30. |

CANCELLATION.

156. **Cancellation** is the omission of the same factor from terms sustaining to each other the relation of *dividend* and *divisor*. It is used for the purpose of saving labor in division, and is an application of the principle already given, that dividing both dividend and divisor by the same number will not alter the quotient; thus $\frac{2}{4}$ may be read 2 divided 4; divide both terms by 2 and the result is 1 divided by 2, or $\frac{1}{2}$.

Again, $\frac{2 \times 27}{4 \times 18}$ may be read 2 times 27, divided by 4 times 18; rejecting the factor 2 from the 2 in the dividend and from the 4 of the divisor, also the factor 9 from the 27 of the dividend and the 18 of the divisor, gives $\frac{2 \times 27^3}{2 \times 4 \times 18^2} = \frac{3}{2 \times 2}$, or $\frac{3}{4}$, or 3 divided by 4, as a final quotient.

The correctness of this result is easily proved by factoring the dividend and divisor, thus: $\frac{2 \times 27}{4 \times 18} = \frac{2 \times 9 \times 3}{2 \times 2 \times 9 \times 2}$, then rejecting 2 and 9 from both terms, or cancelling, obtain $\frac{2 \times 9 \times 3}{2 \times 2 \times 9 \times 2} = \frac{3}{4}$ Ans.

157. We may supplement the former definition thus: The rejection of equivalents of factors from terms sustaining to each other the relation of dividend and divisor, is cancellation.

EXAMPLE. — What is the quotient of $3 \times 2 \times 28 \times 5 \times 7 \times 51$ divided by $6 \times 11 \times 4 \times 7 \times 35 \times 17$?

OPERATION.

$$\frac{3 \times 2 \times 28 \times 5 \times 7 \times 51^3}{6 \times 11 \times 4 \times 7 \times 35 \times 17} = \frac{3}{11}.$$

EXPLANATION. — Cancel 6 from the divisor and 3×2 from the dividend; 4×7 from the divisor and 28 from the dividend; the 35 from the divisor and 5×7 from the dividend; the 17 from the divisor and the 51 from the dividend, leaving 3 in

the dividend, and 11 in the divisor; the quotient is $\frac{3}{11}$.

REMARK. — This principle can be put to frequent and valuable use in a great variety of business computations.

Rule.—I. Write the divisor below the dividend with a line separating them.

II. Cancel from the dividend and divisor all factors common to both; then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

EXAMPLES FOR PRACTICE.

158. 1. Determine by cancellation the quotient of $5 \times 9 \times 2 \times 13 \times 40 \times 6$ divided by $8 \times 3 \times 7 \times 26$.

2. Determine by cancellation the quotient of $64 \times 25 \times 3 \times 15$ divided by $45 \times 12 \times 4 \times 11 \times 36$.

In like manner,

3. Divide $210 \times 9 \times 78 \times 5 \times 23 \times 10 \times 36$ by $13 \times 144 \times 40 \times 3 \times 27 \times 5 \times 400$.

4. Divide $38 \times 4 \times 55 \times 9 \times 32 \times 30$ by $12 \times 11 \times 3 \times 16 \times 19 \times 5$.

5. Divide $51 \times 7 \times 9 \times 27 \times 40 \times 54$ by $63 \times 17 \times 9 \times 200$.

6. Divide $24 \times 25 \times 26 \times 27$ by $2 \times 4 \times 5 \times 9 \times 13$.

7. Divide $2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$ by $23 \times 45 \times 67 \times 89$.

8. Divide the product of the numbers 98, 76, 54, and 32 by the product of the numbers 9, 8, 7, 6, 5, 4, 3, and 2.

9. Divide the product of 33, 4, 42, 9, 5, and 60 by the product of 7, 15, 12, and 11.

10. Divide the product of 416, 216, and 810 by the product of 135, 52, 24, and 5.

11. How many bushels of potatoes at 60¢ per bushel will pay for 450 lb. of sugar at 6¢ per pound?

12. A farmer traded 4 hogs weighing 325 lb. each, at 6¢ per pound, for sugar at 5¢ per pound. How many entire barrels of 312 lb. each should the farmer receive?

13. I bought 18 car loads of apples of 216 barrels each, each barrel containing 3 bushels at 60¢ per bushel, and paid for the same in woolen cloth. If each bale of cloth contained 600 yd. at 30 cents per yard, how many bales and how many odd yards did I deliver?

14. How many yards of cloth at 15¢ per yard should be given for 9 barrels of pork of 200 lb. each, at 6¢ per pound?

15. A hunter traded 6 dozen coon-skins at 40¢ each, for powder at 75¢ per lb. How many 5 lb. cans of powder should he receive?

16. How many pieces of cloth of 45 yd. each, should be received for 5 baskets of eggs, each basket containing 21 dozens at 18¢ per dozen, if the cloth be valued at 8¢ per yard?

17. How many quarter sections of Kansas prairie land valued at \$9 per acre, should be received for 80 cattle worth \$78 per head?

REMARK.—A section of land, in the United States, contains 640 acres.

18. How many years' work of 12 months of 26 days each, must be given for a farm of 112 acres at \$78 per acre, if labor be worth \$2 per day?

19. A farmer exchanged 3 loads of oats, each load containing 27 sacks of 2 bushels each, worth 33¢ per bushel, for flour at 6¢ per pound. At 196 lb. per barrel, how many barrels should he have received?

20. How many sections of Texas prairie land at \$8 per acre should be given for an Ohio farm of 272 acres at \$45 per acre?

FRACTIONS.

159. A **Fraction** is one or more of the equal parts of a unit. If a unit be divided into 3 equal parts, one of the parts is called *one-third* and is written $\frac{1}{3}$, two of the parts are called *two-thirds* and are written $\frac{2}{3}$.

160. A **Fractional Unit** is *one* of the equal parts into which the number or thing is divided. $\frac{1}{7}$, $\frac{1}{4}$, $\frac{1}{3}$, are fractional units.

161. The **Numerator** is the number above the line; it numerates, or numbers the parts, and is a dividend.

162. The **Denominator** is the number below the line; it denominates, or names the value, or size, of the parts showing the number of parts into which the unit has been divided. It is a divisor.

163. The **Terms** of a fraction are the *numerator* and *denominator*, taken together.

164. The **Value** of a fraction is the *quotient* of the *numerator* divided by the *denominator*.

165. Fractions are distinguished as *Common Fractions* and *Decimal Fractions*; and common fractions are either *proper* or *improper*.

166. A **Common Fraction** is one expressed by two numbers, one written above the other, with a line between.

167. A **Proper Fraction** is one whose value is less than 1, the *numerator* being less than the *denominator*. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{13}$, $\frac{4}{11}$ are *proper* fractions.

168. An **Improper Fraction** is one whose *numerator* is either equal to or greater than its *denominator*; its value is equal to or greater than 1. $\frac{3}{2}$, $\frac{5}{3}$, $\frac{4}{1}$, $\frac{8}{6}$, $\frac{4^2}{4}$, $\frac{11^3}{16}$, $\frac{17}{17}$ are *improper* fractions.

169. A **Mixed Number** is an *entire* or *whole* number and a *fraction* united. $2\frac{1}{2}$, $5\frac{3}{4}$, $9\frac{1}{4}$, $14\frac{2}{3}$, $107\frac{3}{11}$ are *mixed* numbers.

170. A **Complex Fraction** is one having a *fraction* for its *numerator* or *denominator*, or for both of its *terms*.

As a fraction indicates a division to be performed, a *complex fraction* indicates a division of *fractions* to be performed. $\frac{\frac{2}{3}}{\frac{5}{7}}$ is a *complex* fraction and indicates that $\frac{2}{3}$ is to be divided by $\frac{5}{7}$; the expression is read $\frac{2}{3} \div \frac{5}{7}$; $\frac{5}{\frac{2}{3}}$, and $\frac{\frac{4}{8}}{\frac{5}{8}}$ are also *complex* fractions.

Principles.—1. *Multiplying the numerator multiplies the fraction; dividing the numerator divides the fraction.*

2. *Multiplying the denominator divides the fraction; dividing the denominator multiplies the fraction.*

3. *Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

REDUCTION OF FRACTIONS.

171. To Reduce a Whole Number to a Fractional Form.

EXAMPLE.—Reduce 3 to a fraction the denominator of which is 7.

EXPLANATION.—The fractional unit having 7 for a denominator is $\frac{1}{7}$; and since 1 unit equals 7 sevenths, 3 units which are 3 times 1 unit must equal 3 times 7 sevenths, or 21 sevenths; therefore, $3 = \frac{21}{7}$.

Rule.—*Multiply the whole number by the required denominator, and place the product over the denominator for a numerator.*

EXAMPLES FOR PRACTICE.

- 172.** 1. Reduce 5 to a fraction the denominator of which will be 4.
 2. Reduce 7 to a fraction the denominator of which will be 9.
 3. Reduce 4 to a fraction the denominator of which will be 13.
 4. Reduce 3 to a fraction the denominator of which will be 8.
 5. Reduce 8 to a fraction the denominator of which will be 12.
 6. Reduce 15 to a fraction the denominator of which will be 10.
 7. Reduce 14 to a fraction the denominator of which will be 5.
 8. Reduce 27 to a fraction the denominator of which will be 11.
 9. Reduce 416 to a fraction the denominator of which will be 23.
 10. Reduce 1125 to a fraction the denominator of which will be 57.

173. To Reduce a Mixed Number to an Improper Fraction.

EXAMPLE.—Reduce $5\frac{2}{3}$ to an improper fraction.

EXPLANATION.—Since 1 unit is equal to 3 thirds, 5 units, which are 5 times 1 unit, must be equal to 5 times 3 thirds, or 15 thirds; and 15 thirds plus 2 thirds equals 17 thirds; therefore, $5\frac{2}{3} = \frac{17}{3}$.

Rule.—*Multiply the whole number by the denominator of the fraction, to the product add the numerator, and place the sum over the denominator.*

EXAMPLES FOR PRACTICE.

- 174.** Reduce
- | | |
|--|--|
| 1. $3\frac{1}{2}$ to an improper fraction. | 6. $78\frac{3}{4}$ to an improper fraction. |
| 2. $7\frac{3}{5}$ to an improper fraction. | 7. $26\frac{5}{3}$ to an improper fraction. |
| 3. $10\frac{4}{3}$ to an improper fraction. | 8. $170\frac{1}{10}$ to an improper fraction. |
| 4. $43\frac{5}{7}$ to an improper fraction. | 9. $1040\frac{1}{20}$ to an improper fraction. |
| 5. $16\frac{1}{12}$ to an improper fraction. | 10. $968\frac{7}{15}$ to an improper fraction. |

175. To Reduce an Improper Fraction to a Whole or Mixed Number.

EXAMPLE.—Reduce $\frac{23}{4}$ to a whole or mixed number.

EXPLANATION.—Since 4 fourths make 1 unit, 23 fourths will make as many units as 4 is contained times in 23, or 5 times with a remainder of 3, or three-fourths; therefore, $\frac{23}{4} = 5\frac{3}{4}$.

Rule.—*Divide the numerator by the denominator, place the remainder, if any, over the denominator, and annex the fraction thus found to the entire part of the quotient*

EXAMPLES FOR PRACTICE.

176. Reduce

- | | |
|--|--|
| 1. $\frac{1}{3}$ to a whole or mixed number. | 6. $\frac{200}{71}$ to a whole or mixed number. |
| 2. $\frac{4}{9}$ to a whole or mixed number. | 7. $\frac{1021}{49}$ to a whole or mixed number. |
| 3. $\frac{2}{8}$ to a whole or mixed number. | 8. $\frac{4}{3}$ to a whole or mixed number. |
| 4. $\frac{9}{11}$ to a whole or mixed number. | 9. $\frac{6152}{183}$ to a whole or mixed number. |
| 5. $\frac{23}{29}$ to a whole or mixed number. | 10. $\frac{1280}{179}$ to a whole or mixed number. |

177. To Reduce a Fraction to its Lowest Terms.

EXAMPLE.—Reduce $\frac{6}{18}$ to its lowest terms.

EXPLANATION.—By applying the principles of factoring, change the form of the fraction $\frac{6}{18}$ to $\frac{2 \times 3}{2 \times 3 \times 3}$; then by cancellation reject the 2 and 3 from the numerator, and the same factors from the denominator, leaving 1 for the new numerator and 3 for the new denominator; the resulting fraction is $\frac{1}{3}$.

Or, observe that 6 is a factor of both the terms and that $\frac{1}{3}$ is the result of dividing both the terms by 6.

Rules.—1. *Divide both terms of the fraction by their greatest common divisor.* Or,

2. *Divide both terms of the fraction by any common factor, and continue the operation with the resulting fractions until they have no common divisor.*

REMARKS.—1. When the terms of a fraction have no common factor, the fraction is in its simplest form, or its lowest terms.

2. If both terms of a fraction be divided by their *greatest common divisor* the fraction will be reduced to its *lowest terms*. This is the only use in practical arithmetic of the theory of the greatest common divisor.

EXAMPLES FOR PRACTICE.

- | | |
|---|---|
| 178. 1. Reduce $\frac{3}{8}$ to its lowest terms. | 6. Reduce $\frac{50}{625}$ to its lowest terms. |
| 2. Reduce $\frac{1}{24}$ to its lowest terms. | 7. Reduce $\frac{840}{1728}$ to its lowest terms. |
| 3. Reduce $\frac{3}{8}$ to its lowest terms. | 8. Reduce $\frac{123}{271}$ to its lowest terms. |
| 4. Reduce $\frac{1}{8}$ to its lowest terms. | 9. Reduce $\frac{2150}{400}$ to its lowest terms. |
| 5. Reduce $\frac{78}{102}$ to its lowest terms. | 10. Reduce $\frac{320}{440}$ to its lowest terms. |

179. To Reduce a Fraction to Higher Terms.

EXAMPLE.—Reduce $\frac{5}{7}$ to a fraction the denominator of which is 21.

EXPLANATION.—Since 7 is contained in 21 three times, the given fraction may be reduced to a fraction whose denominator is 21, by multiplying both of its terms by 3; multiplying $\frac{5 \times 3}{7 \times 3}$ gives $\frac{15}{21}$, the required result. This operation does not alter the value of the given fraction.

Rule.—*Divide the required denominator by the denominator of the given fraction and multiply the numerator by the quotient thus obtained; write the product over the required denominator*

EXAMPLES FOR PRACTICE.

180. 1. Reduce $\frac{2}{3}$ to a fraction the denominator of which is 15.
 2. Reduce $\frac{4}{5}$ to a fraction the denominator of which is 36.
 3. Reduce $\frac{3}{7}$ to a fraction the denominator of which is 42.
 4. Reduce $\frac{5}{8}$ to a fraction the denominator of which is 32.
 5. Reduce $\frac{1}{11}$ to a fraction the denominator of which is 88.
 6. Reduce $\frac{7}{13}$ to a fraction the denominator of which is 52.
 7. Reduce $\frac{2}{3}$ to a fraction the denominator of which is 115.
 8. Reduce $\frac{3}{16}$ to a fraction the denominator of which is 128.
 9. Reduce $\frac{5}{24}$ to a fraction the denominator of which is 192.
 10. Reduce $\frac{1}{11}$ to a fraction the denominator of which is 147.

181. To Reduce Fractions to Equivalent Fractions Having a Common Denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$, to equivalent fractions having a common denominator.

EXPLANATION.—The product of the denominators 3, 2, 5, 7, = 210, and this number is exactly divisible by each of the several denominators; hence each of the given fractions may be reduced to an equivalent having 210 for a denominator; the desired result is then accomplished, as 210 is a denominator common to all the given fractions; $\frac{2}{3} = \frac{140}{210}$, $\frac{1}{2} = \frac{105}{210}$, $\frac{3}{5} = \frac{84}{210}$, and $\frac{4}{7} = \frac{120}{210}$.

Rule.—Multiply together the denominators of the given fractions for a common denominator. Multiply each numerator by all the denominators except its own and write the several results in turn over the common denominator.

REMARK.—Where one or more of the given denominators are factors of the others, the smaller may be rejected.

EXAMPLES FOR PRACTICE.

182. Reduce to equivalent fractions having a common denominator:

- | | |
|--|---|
| 1. $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{8}$, and $\frac{2}{5}$. | 6. $4\frac{3}{4}$, $5\frac{1}{5}$, $13\frac{1}{2}$, 6, and $11\frac{2}{3}$. |
| 2. $\frac{7}{9}$, $\frac{1}{15}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{8}{13}$. | 7. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{15}$, $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{1}{4}$. |
| 3. $\frac{1}{6}$, $\frac{5}{8}$, $\frac{1}{4}$, 2, $\frac{1}{12}$, $\frac{1}{3}$, and $\frac{3}{5}$. | 8. $2\frac{1}{2}$, $7\frac{3}{4}$, $11\frac{1}{2}$, $23\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{8}$, and 5. |
| 4. $7\frac{3}{4}$, 5, $\frac{1}{8}$, $1\frac{1}{16}$, $\frac{1}{2}$, $\frac{7}{8}$, and $\frac{3}{4}$. | 9. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{6}{10}$, 8, $\frac{2}{5}$, $\frac{5}{8}$, $14\frac{3}{4}$, and $\frac{7}{8}$. |
| 5. $\frac{1}{10}$, $5\frac{3}{4}$, $\frac{7}{10}$, $38\frac{1}{4}$, $23\frac{1}{4}$, and 12. | 10. $\frac{8}{11}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{7}{16}$, $\frac{1}{15}$, $\frac{2}{3}$, $\frac{4}{5}$, and 20. |

183. To Reduce Fractions to Equivalent Fractions Having the Least Common Denominator.

The Least Common Denominator of two or more fractions is the least denominator to which they can all be reduced, and must be the least common multiple of the given denominators.

EXAMPLE.—Reduce $\frac{1}{3}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{7}{15}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator.

OPERATION.		EXPLANATION.—Find the least common multiple of the given denominators for the least common denominator, which is 180. Then by Art. 179, reduce each of the
$\begin{array}{r} 3, 2,) 9-15-4 \\ \underline{3 \quad 5 \quad 2} \end{array}$	$\frac{1}{3} = \frac{60}{180}, \quad \frac{4}{5} = \frac{144}{180},$ $\frac{2}{3} = \frac{120}{180}, \quad \frac{1}{2} = \frac{90}{180}.$	
$3 \times 2 \times 3 \times 5 \times 2 = 180. \quad \frac{7}{15} = \frac{84}{180}, \quad \frac{3}{4} = \frac{135}{180}.$		

given fractions to a fraction whose denominator is 180.

Rule.—I. *Find the least common multiple of the given denominators.*
 II. *Divide this multiple by the denominator of each of the given fractions, and multiply its numerator by the quotient thus obtained.*

REMARKS.—1. The pupil should do as much of this work as possible by inspection.
 2. Mixed numbers should be reduced to improper fractions before applying the rule.

EXAMPLES FOR PRACTICE.

184. Reduce to equivalent fractions having the least common denominator:

- | | |
|---|--|
| 1. $\frac{2}{3}, \frac{5}{6}, \frac{1}{8},$ and $\frac{3}{4}.$ | 7. $23\frac{1}{2}, 14\frac{2}{3}, 7\frac{9}{10}, 5\frac{3}{4},$ and $\frac{8}{9}.$ |
| 2. $\frac{1}{9}, \frac{4}{7}, \frac{5}{8}, \frac{1}{2},$ and $\frac{1}{6}.$ | 8. $17, 2\frac{3}{7}, 14\frac{1}{2}, 8\frac{3}{4}, 3\frac{5}{21},$ and $5.$ |
| 3. $\frac{1}{4}, \frac{2}{11}, 7, \frac{4}{9}, 2\frac{1}{2},$ and $\frac{7}{10}.$ | 9. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8},$ and $\frac{9}{10}.$ |
| 4. $\frac{9}{13}, \frac{4}{6}, \frac{5}{8}, \frac{1}{8}, 2, 5\frac{1}{2},$ and $\frac{1}{2}.$ | 10. $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}, \frac{4}{15}, 8, \frac{4}{45}, 3\frac{1}{2}, 12\frac{3}{4},$ and $\frac{7}{10}.$ |
| 5. $\frac{1}{5}, \frac{1}{6}, \frac{1}{10}, \frac{1}{12}, \frac{1}{2}, \frac{1}{3},$ and $5.$ | 11. $\frac{1}{11}, \frac{9}{10}, \frac{8}{9}, 14, 1, \frac{1}{22}, \frac{40}{11}, \frac{21}{44}, 5\frac{2}{3},$ and $11.$ |
| 6. $\frac{4}{15}, \frac{4}{9}, 11, 7\frac{2}{3}, 1\frac{5}{6}, 9,$ and $1.$ | 12. $\frac{1}{20}, \frac{1}{10}, \frac{1}{5}, \frac{1}{4}, 3, 2\frac{1}{3}, 4\frac{3}{5}, 2, \frac{4}{15}.$ |

ADDITION OF FRACTIONS.

185. To Add Fractions having a Common Denominator.

EXAMPLE.—Find the sum of $\frac{2}{3}, \frac{7}{9}, \frac{5}{9}, \frac{1}{9},$ and $\frac{4}{9}.$

OPERATION.

$$\frac{2}{9} + \frac{7}{9} + \frac{5}{9} + \frac{1}{9} + \frac{4}{9} = \frac{19}{9}$$

$$\frac{19}{9} = 2\frac{1}{9}.$$

EXPLANATION.—As the given fractions have a common denominator, their sum may be found by adding the numerators and placing the result 19, over the common denominator; the simplest form of this sum is found by application of Art. 175.

Rule.—*Add the numerators and place the sum over the common denominator; if the result be an improper fraction reduce it to a whole or mixed number*

EXAMPLES FOR PRACTICE.

186. Add

- | | |
|--|---|
| 1. $\frac{2}{7}, \frac{6}{7}, \frac{3}{7}, \frac{5}{7}, \frac{1}{7},$ and $\frac{4}{7}.$ | 6. $\frac{1}{18}, \frac{1}{18}, \frac{5}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18},$ and $\frac{7}{18}.$ |
| 2. $\frac{1}{19}, \frac{1}{19}, \frac{3}{19}, \frac{1}{19}, \frac{2}{19},$ and $\frac{1}{19}.$ | 7. $\frac{1}{27}, \frac{5}{27}, \frac{2}{27}, \frac{2}{27}, \frac{4}{27},$ and $\frac{5}{27}.$ |
| 3. $\frac{7}{25}, \frac{2}{25}, \frac{8}{25}, \frac{1}{25}, \frac{1}{25}, \frac{1}{25},$ and $\frac{1}{25}.$ | 8. $\frac{1}{24}, \frac{5}{24}, \frac{7}{24}, \frac{1}{24}, \frac{1}{24}, \frac{1}{24}, \frac{1}{24},$ and $\frac{2}{24}.$ |
| 4. $\frac{8}{15}, \frac{4}{15}, \frac{6}{15}, \frac{2}{15}, \frac{7}{15}, \frac{1}{15},$ and $\frac{2}{15}.$ | 9. $\frac{5}{32}, \frac{1}{32}, \frac{7}{32}, \frac{3}{32}, \frac{9}{32}, \frac{1}{32}, \frac{1}{32},$ and $\frac{3}{32}.$ |
| 5. $\frac{5}{17}, \frac{3}{17}, \frac{7}{17}, \frac{1}{17}, \frac{9}{17}, \frac{1}{17},$ and $\frac{2}{17}.$ | 10. $\frac{1}{64}, \frac{6}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{3}{64}, \frac{3}{64},$ and $\frac{4}{64}.$ |

187. To Add Fractions not having a Common Denominator.

EXAMPLE.—What is the sum of $\frac{2}{3}, \frac{5}{7},$ and $\frac{1}{2}.$

OPERATION.

$$\frac{2}{3} + \frac{5}{7} + \frac{1}{2} =$$

$$\frac{28}{42} + \frac{30}{42} + \frac{21}{42} = \frac{79}{42} = 1\frac{37}{42}.$$

EXPLANATION.—Since the given fractions are not of the same unit value, reduce them to a common denominator (Art. 181), and writing their equivalents below, add their numerators, and place the sum over the common denominator; reduce this result to an improper fraction.

Rule.—*Reduce the fractions to a common denominator, or if desired, to their least common denominator; add the resulting numerators, place the sum obtained over the common denominator and reduce the fraction.*

EXAMPLES FOR PRACTICE.

188. Add

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|--|---|
| <ol style="list-style-type: none"> 1. $\frac{3}{4}, \frac{7}{8}, \frac{5}{6}, \frac{1}{2}, \frac{6}{9}, \frac{2}{5},$ and $\frac{8}{9}.$ 2. $\frac{1}{15}, \frac{7}{10}, \frac{1}{2}, \frac{3}{5}, \frac{4}{9}, \frac{3}{4},$ and $\frac{7}{15}.$ 3. $\frac{3}{11}, \frac{37}{66}, \frac{2}{5}, \frac{21}{2}, \frac{1}{3}, \frac{1}{4},$ and $\frac{23}{33}.$ 4. $\frac{6}{7}, \frac{3}{4}, \frac{9}{10}, \frac{3}{5}, \frac{5}{14},$ and $\frac{2}{3}.$ 5. $\frac{1}{6}, \frac{3}{4}, \frac{7}{8}, \frac{1}{2}, \frac{5}{6}, \frac{1}{12},$ and $\frac{1}{3}.$ | <ol style="list-style-type: none"> 6. $\frac{19}{20}, \frac{4}{5}, \frac{7}{10}, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{11}{15},$ and $\frac{3}{10}.$ 7. $\frac{6}{7}, \frac{3}{5}, \frac{97}{105}, \frac{31}{15}, \frac{18}{5}, \frac{1}{4}, \frac{1}{3},$ and $\frac{1}{2}.$ 8. $\frac{12}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{15}, \frac{17}{20}, \frac{5}{6}, \frac{2}{3},$ and $\frac{5}{8}.$ 9. $\frac{8}{3}, \frac{3}{4}, \frac{2}{3}, \frac{5}{18}, \frac{6}{15}, \frac{3}{7},$ and $\frac{5}{12}.$ 10. $\frac{7}{8}, \frac{1}{2}, \frac{13}{15}, \frac{3}{4}, \frac{5}{6}, \frac{1}{3}, \frac{3}{10},$ and $\frac{7}{20}.$ |
|--|---|

189. To Add Mixed Numbers.

EXAMPLE.—Find the sum of $2\frac{1}{2}, \frac{7}{3}, 4,$ and $\frac{2}{5}.$

OPERATION.

$$2\frac{1}{2} + \frac{7}{3} + 4 + \frac{2}{5} =$$

$$\frac{5}{5} + \frac{7}{3} + \frac{4}{1} + \frac{2}{5} =$$

$$\frac{25}{50} + \frac{70}{50} + \frac{360}{50} + \frac{20}{50} = \frac{691}{50} = 7\frac{41}{50}.$$

EXPLANATION.—Write the expressions in a horizontal line; then change such of the expressions as are in mixed or entire form to fractional equivalents, and place them, together with the simple fractions, in a line below. Next find by inspection that 90 is the least common multiple of the denominators, or the least common denominator of the expressions; then apply Art. 179, add, and reduce results.

For convenience the fractions may be written in a vertical line and only the fractional parts of the expressions reduced; then adding the integers and the fractions separately, unite the results.

OPERATION.

2	$\frac{1}{2}$	45
	$\frac{7}{3}$	70
4		
	$\frac{2}{5}$	36
$7\frac{61}{50}$		

EXPLANATION.—Separate, mentally, or by a vertical line, the integers from the fractions. By inspection reduce the fractions to equivalents having the common denominator 90; now, keeping this in mind, write only the numerators 45, 70, and 36; the sum of these is 151, which placed over the common denominator in the form of a fraction, gives $\frac{151}{90}$, reducible to $1\frac{61}{90}$; this added to the integers gives $7\frac{61}{50}$, the sum as before found.

Rules.—1. *Reduce mixed numbers and integers to common fractional forms and then to common denominators. Add their numerators, place the result over the common denominator in the form of a fraction, and reduce to simplest form.* Or,

2. *Find the sum of the integers and the fractional expressions separately, and add the results.*

EXAMPLES FOR PRACTICE.

190. Add

- | | |
|--|--|
| <ol style="list-style-type: none"> 1. $4, \frac{3}{4}, \frac{2}{5}, \frac{1}{2}, \frac{7}{8}, 3, \frac{4}{5}, \frac{1}{10},$ and $11.$ 2. $\frac{3}{4}, 2\frac{1}{5}, \frac{5}{6}, \frac{1}{3}, 6, \frac{1}{2}, 1, \frac{9}{10},$ and $4\frac{1}{4}.$ 3. $\frac{3}{5}, \frac{5}{8}, \frac{1}{10}, 2\frac{3}{5}, 5, 3\frac{4}{5}, \frac{5}{6}, 7\frac{7}{8},$ and $\frac{8}{9}.$ 4. $2, 5\frac{3}{5}, \frac{2}{3}, 11\frac{1}{5}, \frac{1}{9}, 14, 20\frac{1}{2},$ and $4.$ 5. $\frac{4}{5}, 3, 6\frac{1}{4}, \frac{2}{3}, 19, 75\frac{1}{3}, \frac{1}{15},$ and $\frac{9}{10}.$ | <ol style="list-style-type: none"> 6. $3\frac{5}{8}, 10, 21\frac{1}{2}, 42\frac{1}{4}, 84\frac{1}{2},$ and $168\frac{1}{8}.$ 7. $6\frac{5}{6}, 5\frac{2}{3}, 50\frac{5}{6}, 29\frac{2}{3},$ and $86\frac{1}{3}.$ 8. $59\frac{5}{6}, 103\frac{2}{3}, 55\frac{1}{2}, 400,$ and $96\frac{1}{4}.$ 9. $103\frac{1}{15}, 119\frac{1}{15}, 297\frac{1}{4},$ and $188\frac{3}{4}.$ 10. $33\frac{1}{3}, 15, 12\frac{1}{2}, 6\frac{1}{4}, 25,$ and $16\frac{2}{3}.$ |
|--|--|

REMARK.—In invoices of cloth, &c., account of fractional parts is made only in quarters and merely the numerators are written; as, $5^2 = 5\frac{2}{4}$, $3^1 = 3\frac{1}{4}$, $13^3 = 13\frac{3}{4}$, etc.

Miscellaneous Examples in Addition of Fractions.**EXAMPLES FOR MENTAL PRACTICE.****191.** What is the sum of

- | | | |
|---|--|---|
| 1. $\frac{1}{2}, \frac{2}{3}, \frac{1}{2},$ and $\frac{4}{5}$. | 5. $\frac{5}{8}, \frac{2}{4}, \frac{7}{8},$ and $\frac{3}{8}$. | 9. $\frac{2}{3}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{2},$ and $\frac{1}{4}$. |
| 2. $\frac{2}{3}, \frac{2}{4}, \frac{4}{5},$ and $\frac{5}{6}$. | 6. $\frac{2}{3}, \frac{1}{5}, \frac{8}{9},$ and $\frac{1}{10}$. | 10. $\frac{2}{3}, \frac{1}{6}, \frac{1}{10}, \frac{1}{15}, \frac{1}{2}, \frac{2}{3},$ and $\frac{2}{3}$. |
| 3. $\frac{1}{3}, \frac{4}{5}, \frac{5}{6},$ and $\frac{6}{7}$. | 7. $\frac{1}{2}, \frac{2}{4}, \frac{5}{8}, \frac{2}{3}, \frac{2}{3},$ and $\frac{1}{12}$. | 11. $\frac{1}{4}, \frac{5}{6}, \frac{2}{3}, \frac{1}{4},$ and $\frac{3}{4}$. |
| 4. $\frac{4}{5}, \frac{5}{6}, \frac{2}{4},$ and $\frac{6}{7}$. | 8. $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12},$ and $\frac{1}{2}$. | 12. $\frac{1}{4}, \frac{2}{4}, \frac{2}{4}, \frac{2}{6}, \frac{1}{4}, \frac{2}{4}, \frac{2}{6}, \frac{1}{4},$ and $\frac{2}{4}$. |

EXAMPLES FOR WRITTEN PRACTICE,**192.** Add

- | | |
|---|--|
| 1. $130\frac{3}{4}, 69\frac{5}{8}, 600\frac{6}{11}, 204\frac{1}{11},$ and 46. | 4. 900, $47\frac{1}{2}, 3\frac{3}{8}, 4, 29\frac{2}{5}, 66\frac{2}{3},$ and 4. |
| 2. $80\frac{1}{4}, 2\frac{1}{4}, 5\frac{3}{8}, 17, 41\frac{1}{8}, 83\frac{1}{4},$ and $14\frac{3}{8}$. | 5. $16\frac{2}{3}, 33\frac{1}{3}, 66\frac{2}{3}, 88\frac{1}{3}, 100,$ and $116\frac{2}{3}$. |
| 3. $28\frac{1}{4}, 85\frac{1}{11}, 60\frac{5}{11}, 400, 20\frac{3}{8},$ and 11. | 6. $18\frac{3}{4}, 65\frac{3}{8}, 161\frac{3}{4}, 67\frac{5}{8}, 23\frac{3}{4},$ and 75. |
7. The six fields of a farm measure respectively, 10, $12\frac{1}{2}$, $19\frac{1}{4}$, $26\frac{4}{10}$, $30\frac{1}{2}$, and $2\frac{1}{2}$ acres. How many acres in the farm?
8. Ten sheep weighed as follows: $90\frac{1}{16}$, $110\frac{1}{2}$, $89\frac{5}{8}$, 100, $106\frac{3}{4}$, $101\frac{1}{16}$, 96, 99, $113\frac{3}{8}$, and $198\frac{1}{2}$ lb. respectively. What was their aggregate weight?
9. A farmer sold $360\frac{1}{2}$ pounds of pork, $167\frac{3}{8}$ lb. of turkey, $241\frac{1}{8}$ lb. of chicken, $690\frac{1}{8}$ lb. of butter, 475 lb. of lard, a cow's hide weighing $97\frac{3}{4}$ lb., $71\frac{1}{2}$ lb. of tallow, and three quarters of beef weighing respectively, $161\frac{1}{4}$, $187\frac{1}{2}$, and 190 lb. How many pounds in all had he to deliver?

10. For $341\frac{2}{3}$ bushels of wheat I received $\$375\frac{1}{4}$,
 For $597\frac{3}{8}$ bushels of barley I received $\$500\frac{1}{8}$,
 For $1120\frac{1}{2}$ bushels of oats I received $\$619\frac{3}{4}$,
 For $316\frac{1}{4}$ bushels of buckwheat I received $\$200\frac{1}{10}$,
 For 250 bushels of beans I received $\$525\frac{1}{10}$,
 For $1386\frac{1}{2}$ bushels of potatoes I received $\$755\frac{1}{10}$,
 For $1050\frac{1}{4}$ bushels of apples I received $\$301\frac{1}{10}$,
 For $630\frac{1}{2}$ bushels of turnips I received $\$63\frac{1}{10}$.

How many bushels did I sell and what sum was received for all?

SUBTRACTION OF FRACTIONS.**193.** To Subtract Fractions having a Common Denominator.**EXAMPLE.**—Subtract $\frac{2}{7}$ from $\frac{5}{7}$.

OPERATION.	EXPLANATION.
$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$.	—Since the fractions have a common denominator, their difference may be found by taking the numerator 3 from the numerator 5, and placing the difference 2, over their common denominator 7.

Rule.—*Subtract the numerator of the subtrahend from that of the minuend, and place the difference over the common denominator.*

REMARK.—A proper fraction may be subtracted from 1 by writing the difference between its numerator and denominator over the denominator. Results should always be reduced to their lowest terms. Improper fractions may be treated the same as if proper.

EXAMPLES FOR MENTAL PRACTICE.

194. What is the difference between

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|--|---|--|---|--|
| 1. $\frac{4}{5}$ and $\frac{1}{6}$. | 4. $\frac{14}{15}$ and $\frac{4}{15}$. | 7. $\frac{1}{8}$ and $\frac{3}{8}$. | 10. $\frac{2}{11}$ and $\frac{2}{11}$. | 13. $\frac{11}{18}$ and $\frac{7}{18}$. |
| 2. $\frac{7}{8}$ and $\frac{4}{8}$. | 5. $\frac{5}{6}$ and $\frac{1}{6}$. | 8. $\frac{1}{18}$ and $\frac{1}{18}$. | 11. 1 and $\frac{1}{16}$. | 14. $\frac{1}{2}$ and $\frac{4}{8}$. |
| 3. $\frac{1}{11}$ and $\frac{6}{11}$. | 6. 1 and $\frac{1}{3}$. | 9. $\frac{2}{9}$ and $\frac{1}{11}$. | 12. 1 and $\frac{1}{9}$. | 15. 1 and $\frac{3}{5}$. |

EXAMPLES FOR WRITTEN PRACTICE.

195. Subtract

- | | | | |
|--|---|--|--|
| 1. $\frac{2}{3}$ from $\frac{5}{8}$. | 5. $\frac{1}{10}$ from 1. | 9. $\frac{4}{5}$ from $\frac{7}{11}$. | 13. $\frac{2}{3}$ from $\frac{14}{31}$. |
| 2. $\frac{7}{11}$ from $\frac{11}{11}$. | 6. $\frac{5}{6}$ from $\frac{6}{6}$. | 10. $\frac{7}{10}$ from $\frac{1}{10}$. | 14. $\frac{1}{10}$ from $\frac{2}{10}$. |
| 3. $\frac{1}{6}$ from $\frac{4}{6}$. | 7. $\frac{1}{10}$ from $\frac{5}{10}$. | 11. $\frac{1}{5}$ from $\frac{2}{5}$. | 15. $\frac{1}{4}$ from 1. |
| 4. $\frac{1}{10}$ from $\frac{2}{10}$. | 8. $\frac{1}{12}$ from $\frac{6}{12}$. | 12. $\frac{5}{6}$ from $\frac{1}{6}$. | 16. $\frac{1}{10}$ from 3. |

196. To Subtract Fractions not having a Common Denominator.

EXAMPLE.—From $\frac{3}{4}$ take $\frac{2}{5}$.

OPERATION.

$$\frac{3}{4} - \frac{2}{5} =$$

$$\frac{15}{20} - \frac{8}{20} = \frac{7}{20}.$$

EXPLANATION.—As the denominators indicate the kind of parts, and only *like* things can be taken the one from the other, it follows that before the subtraction can be performed, the fractions must be reduced to a common denominator; then, the difference between the resulting numerators, placed over the common denominator gives $\frac{7}{20}$ as a result.

Rule.—Reduce the given fractions to equivalent fractions having a common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and write the result over the common denominator.

REMARK.—Improper fractions may be treated in like manner.

EXAMPLES FOR MENTAL PRACTICE.

197. What is the difference between

- | | | | |
|--------------------------------------|---------------------------------------|--|---|
| 1. $\frac{1}{2}$ and $\frac{2}{6}$? | 4. $\frac{5}{6}$ and $\frac{1}{6}$? | 7. $\frac{3}{8}$ and $\frac{1}{2}$? | 10. $\frac{7}{12}$ and $\frac{4}{3}$? |
| 2. $\frac{2}{3}$ and $\frac{5}{6}$? | 5. $\frac{2}{11}$ and $\frac{1}{2}$? | 8. $\frac{4}{5}$ and $\frac{5}{6}$? | 11. $\frac{1}{11}$ and $\frac{2}{11}$? |
| 3. $\frac{7}{8}$ and $\frac{3}{4}$? | 6. $\frac{1}{4}$ and $\frac{3}{4}$? | 9. $\frac{1}{12}$ and $\frac{1}{14}$? | 12. $\frac{2}{8}$ and $\frac{1}{6}$? |

EXAMPLES FOR WRITTEN PRACTICE.

198. Subtract

- | 1. $\frac{3}{4}$ from $\frac{7}{8}$. | From | Find the difference between |
|--|--|--|
| 2. $\frac{2}{5}$ from $\frac{8}{5}$. | 5. $\frac{8}{9}$ take $\frac{1}{2}$. | 9. $\frac{1}{12}$ and $\frac{2}{3}$. |
| 3. $\frac{1}{14}$ from $\frac{5}{7}$. | 6. $\frac{1}{12}$ take $\frac{2}{3}$. | 10. $\frac{5}{4}$ and 1. |
| 4. $\frac{2}{12}$ from $\frac{1}{6}$. | 7. $\frac{2}{7}$ take $\frac{1}{4}$. | 11. $\frac{2}{5}$ and $\frac{1}{10}$. |
| | 8. $\frac{1}{15}$ take $\frac{1}{4}$. | 12. $\frac{1}{10}$ and $\frac{5}{8}$. |
| | | 13. $\frac{4}{9}$ and $\frac{1}{2}$. |
| | | 14. $\frac{1}{3}$ and $\frac{2}{5}$. |
| | | 15. $\frac{3}{4}$ and $\frac{4}{5}$. |
| | | 16. $\frac{5}{6}$ and $\frac{7}{8}$. |

199 To Subtract Mixed Numbers.

EXAMPLE.—From $16\frac{2}{3}$ take $11\frac{3}{4}$.

OPERATION.

$$16\frac{2}{3} - 11\frac{3}{4} =$$

$$16\frac{8}{12} - 11\frac{9}{12} =$$

$$16\frac{8}{12} = 15\frac{20}{12},$$

$$15\frac{20}{12} - 11\frac{9}{12} =$$

$$4\frac{11}{12}$$

EXPLANATION.—Reduce the fractions to a common denominator. Observing that the $\frac{3}{4}$ of the subtrahend is greater than the $\frac{2}{3}$ of the minuend, take 1 from the 16 of the minuend, reduce it to twelfths ($\frac{12}{12}$), and adding it to the $\frac{8}{12}$ obtain $\frac{20}{12}$; from this take the $\frac{9}{12}$ and the fractional remainder is found to be $\frac{11}{12}$. Having taken 1 from the 16 in the minuend, there remains 15 from which to take the 11 of the subtrahend; therefore the integral remainder is 4, and the entire result $4\frac{11}{12}$.

REMARK.—In case the minuend is integral subtract 1 and reduce it to a fractional form of the required denominator.

Rule.—Write the subtrahend underneath the minuend. Reduce the fractional parts to like denominators. Subtract fractional and integral parts separately and unite the results.

REMARK.—In case the lower fraction be greater than the upper, take 1 from the upper whole number, reduce it and add to the upper fraction; from this sum take the lower fraction.

EXAMPLES FOR MENTAL PRACTICE.

200. What is the difference between

- | | | | |
|--|---|--|---|
| 1. $6\frac{1}{2}$ and $2\frac{1}{2}$? | 5. $3\frac{1}{2}$ and $5\frac{1}{2}$? | 9. $3\frac{1}{3}$ and $12\frac{1}{3}$? | 13. $17\frac{1}{3}$ and $22\frac{5}{6}$? |
| 2. $5\frac{3}{4}$ and $3\frac{1}{4}$? | 6. $8\frac{2}{3}$ and $11\frac{7}{10}$? | 10. $17\frac{7}{10}$ and $5\frac{1}{2}$? | 14. $12\frac{5}{8}$ and $31\frac{1}{16}$? |
| 3. $12\frac{3}{8}$ and $3\frac{3}{8}$? | 7. $14\frac{5}{8}$ and $21\frac{1}{8}$? | 11. $21\frac{1}{4}$ and $11\frac{3}{4}$? | 15. $113\frac{5}{8}$ and $14\frac{1}{2}$? |
| 4. $17\frac{11}{12}$ and $1\frac{1}{12}$? | 8. $6\frac{7}{8}$ and $14\frac{11}{12}$? | 12. $9\frac{7}{10}$ and $23\frac{2}{10}$? | 16. $215\frac{9}{10}$ and $45\frac{3}{5}$? |

EXAMPLES FOR WRITTEN PRACTICE.

201. From

Subtract

Find the difference between

- | | | | |
|---|---|--|---|
| 1. $4\frac{1}{3}$ take $1\frac{1}{3}$. | 5. $9\frac{1}{10}$ from 11. | 9. $240\frac{2}{3}$ and $89\frac{1}{10}$. | 13. $117\frac{3}{8}$ and $57\frac{7}{8}$. |
| 2. $18\frac{1}{8}$ take $5\frac{1}{8}$. | 6. 20 from $56\frac{3}{8}$. | 10. $210\frac{1}{11}$ and 250. | 14. $95\frac{4}{9}$ and $183\frac{5}{9}$. |
| 3. $79\frac{7}{8}$ take $49\frac{1}{8}$. | 7. $17\frac{1}{10}$ from $80\frac{1}{10}$. | 11. 200 and $1\frac{1}{2}$. | 15. $1050\frac{2}{3}$ and $2020\frac{2}{3}$. |
| 4. $104\frac{8}{10}$ " $84\frac{4}{5}$. | 8. $41\frac{1}{11}$ from $50\frac{2}{11}$. | 12. $11\frac{1}{3}$ " $54\frac{0}{1}$. | 16. $2016\frac{2}{3}$ and $2503\frac{1}{3}$. |

EXAMPLES REQUIRING THE USE OF THE PRECEDING EXPLANATIONS.

202. From the sum of

Subtract the sum of

- | | |
|--|---|
| 1. $\frac{2}{3}$ and $\frac{1}{2}$ take $\frac{5}{6}$. | 7. $\frac{5}{11}$ and $\frac{1}{2}$ from $14\frac{2}{11}$. |
| 2. $\frac{7}{8}$ and $\frac{4}{5}$ take $\frac{1}{2}$. | 8. $8\frac{1}{4}$ and $2\frac{5}{8}$ from 20. |
| 3. $\frac{2}{3}$ and $\frac{3}{8}$ take $\frac{1}{2}$. | 9. $\frac{3}{8}$ and 5 from $11\frac{1}{8}$. |
| 4. $\frac{7}{8}$ and $2\frac{1}{4}$ take 4. | 10. $18\frac{2}{3}$ and $15\frac{1}{11}$ from $100\frac{9}{10}$. |
| 5. $\frac{1}{10}$ and $9\frac{5}{8}$ take $3\frac{1}{2}$. | 11. $20\frac{1}{10}$ and $15\frac{1}{5}$ from $40\frac{9}{10}$. |
| 6. $5\frac{3}{4}$ and $4\frac{1}{8}$ take 9. | 12. $201\frac{1}{2}$ and $87\frac{5}{8}$ from $304\frac{1}{2}$. |
13. Take the sum of $\frac{4}{5}$ and $\frac{2}{3}$ from the sum of $2\frac{1}{2}$ and $\frac{5}{6}$.
14. Take the sum of $3\frac{1}{2}$ and $\frac{1}{3}$ from the sum of 4 and $8\frac{1}{4}$.
15. Take the sum of 20 and $14\frac{3}{10}$ from the sum of $18\frac{2}{3}$ and $19\frac{1}{3}$.
16. Take the sum of $28\frac{5}{8}$ and $60\frac{1}{8}$ from the sum of $50\frac{1}{4}$ and $40\frac{1}{4}$.
17. Take the sum of $100\frac{1}{2}$ and $28\frac{5}{8}$ from the sum of $66\frac{6}{8}$ and $69\frac{1}{8}$.
18. Take the sum of $13\frac{2}{3}$ and 46 from the difference between 125 and $1\frac{5}{11}$.
19. Take the sum of $216\frac{1}{2}$ and $101\frac{1}{2}$ from the difference between 1000 and $87\frac{7}{8}$.
20. Take the sum of $45\frac{1}{6}$ and $25\frac{1}{6}$ from the difference between $305\frac{2}{3}$ and $425\frac{1}{10}$.
21. Take the sum of 23 and $41\frac{9}{10}$ from the difference between $21\frac{1}{4}$ and $93\frac{1}{4}$.
22. Take the sum of $9\frac{3}{8}$ and $5\frac{3}{8}$ from the difference between $1\frac{2}{5}$ and $19\frac{7}{8}$.
23. From $216\frac{1}{2}$ acres of land, lots of 21 A, $16\frac{3}{4}$ A, $261\frac{5}{8}$ A, $41\frac{1}{2}$ A, and $63\frac{3}{4}$ acres were sold. How many acres remained unsold?

24. A lady went shopping with $\$35\frac{3}{4}$ in her purse. She expended for car fare, $\frac{1}{5}$ of a dollar; for thread, $\frac{1}{10}$ of a dollar; for gloves, $1\frac{1}{4}$ dollars; for a hat, $5\frac{1}{3}$ dollars; for a clock, $21\frac{1}{2}$ dollars; and invested the remainder in linen. How much was paid for the linen?

25. A dealer bought a farm for \$3685 $\frac{3}{4}$, the crops for \$887 $\frac{1}{4}$, the stock for \$1015 $\frac{1}{2}$, and the utensils for \$602 $\frac{1}{2}$. He sold the entire property for \$6425 $\frac{9}{10}$. Did he gain or lose and how much?

26. From the difference between 280 $\frac{1}{10}$ and 1200 $\frac{1}{3}$ take the sum of 20 $\frac{1}{4}$, 16 $\frac{3}{8}$, 51 $\frac{9}{11}$, 86, 141 $\frac{1}{2}$, 1 $\frac{9}{10}$, 4 $\frac{6}{8}$, and 100.

27. From the sum of 80 $\frac{3}{8}$, 70 $\frac{7}{8}$, 60 $\frac{6}{8}$, and 1 take the difference between 1 $\frac{7}{8}$ and 101.

28. From the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{9}{10}$ take the difference between $\frac{1}{9}$ and 2.

29. Having \$1302 $\frac{2}{3}$ in bank I drew checks for \$204 $\frac{4}{8}$, \$189 $\frac{9}{10}$, \$640 $\frac{1}{4}$, \$82 $\frac{1}{2}$, \$20, \$30 $\frac{3}{4}$, and \$100. How much remained to my credit in the bank?

30. A town owing \$38246 $\frac{1}{3}$, paid, in '85, \$9304 $\frac{2}{3}$; in '86, \$12000 $\frac{1}{3}$; in '87, \$4250 $\frac{1}{3}$; and in '88 the remainder. How much was the payment of 1888?

31. If I pay \$3500 for a house, \$346 $\frac{3}{8}$ for repairs, \$1126 $\frac{3}{4}$ for furniture, \$400 $\frac{1}{2}$ for carpets and curtains, and sell the entire property for \$5000, how much will I lose?

MULTIPLICATION OF FRACTIONS.

203. To Multiply a Fraction by a Whole Number.

EXAMPLE.—Multiply $\frac{3}{8}$ by 4.

OPERATION I.

$$\frac{3}{8} \times 4 = \frac{3 \times 4}{8} = \frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}.$$

OPERATION II.

$$\frac{3}{8} \times 4 = \frac{3}{8 \div 4} = \frac{3}{2} = 1\frac{1}{2};$$

$$\text{or, } \frac{3}{8} \times 4 = \frac{3}{2} = 1\frac{1}{2}.$$

EXPLANATION.—Since the numerator is the dividend, the fraction may be multiplied by multiplying the numerator 3 by the multiplier 4; the product is 12, which reduced gives 1 $\frac{4}{8}$, or 1 $\frac{1}{2}$. Or, since the denominator 8, is the divisor, the fraction may be multiplied by dividing this divisor 8, by the multiplier 4, which will give the reduced form, $\frac{3}{2} = 1\frac{1}{2}$. This introduces the principle of cancellation into fractional operations.

For examples containing concrete numbers, reason as follows:

EXAMPLE.—If one pound of wool costs $\frac{2}{3}$ of a dollar, what will be the cost of 21 pounds?

EXPLANATION.—Since one pound costs $\frac{2}{3}$ of a dollar, 21 pounds which are 21 times 1 pound will cost 21 times $\frac{2}{3}$ of a dollar, or 7 $\frac{2}{3}$ dollars.

Rule.—Multiply the numerator, or divide the denominator, by the whole number.

REMARK.—To economize time and space divide the denominator or cancel when it can be done, as the numbers to be treated are thus put into simpler form.

EXAMPLES FOR MENTAL PRACTICE.

204. What is the product of

- | | | |
|-------------------------------------|--------------------------------------|---------------------------------------|
| 1. $\frac{2}{3}$ multiplied by 3 ? | 6. $\frac{2}{3}$ multiplied by 3 ? | 11. $\frac{1}{6}$ multiplied by 12 ? |
| 2. $\frac{4}{5}$ multiplied by 2 ? | 7. $\frac{11}{12}$ multiplied by 5 ? | 12. $\frac{2}{3}$ multiplied by 5 ? |
| 3. $\frac{5}{8}$ multiplied by 4 ? | 8. $\frac{7}{8}$ multiplied by 8 ? | 13. $\frac{2}{3}$ multiplied by 6 ? |
| 4. $\frac{7}{12}$ multiplied by 3 ? | 9. $\frac{3}{10}$ multiplied by 15 ? | 14. $\frac{7}{15}$ multiplied by 30 ? |
| 5. $\frac{8}{11}$ multiplied by 7 ? | 10. $\frac{5}{8}$ multiplied by 6 ? | 15. $\frac{9}{16}$ multiplied by 32 ? |

EXAMPLES FOR WRITTEN PRACTICE.

205. Multiply

- | | | | |
|--------------------------|--------------------------|----------------------------|-----------------------------|
| 1. $\frac{5}{17}$ by 85. | 5. $3\frac{1}{4}$ by 16. | 9. $5\frac{1}{10}$ by 10. | 13. $5\frac{2}{3}$ by 115. |
| 2. $\frac{1}{3}$ by 8. | 6. $4\frac{1}{7}$ by 11. | 10. $2\frac{0}{19}$ by 57. | 14. $2\frac{2}{1}$ by 49. |
| 3. $\frac{1}{5}$ by 12. | 7. $1\frac{0}{4}$ by 40. | 11. $\frac{8}{17}$ by 21. | 15. $3\frac{1}{15}$ by 105. |
| 4. $2\frac{0}{9}$ by 9. | 8. $2\frac{5}{8}$ by 28. | 12. $1\frac{1}{8}$ by 20. | 16. $3\frac{3}{8}$ by 156. |

REMARK.—It is sometimes desirable to reduce the whole number to fractional form by placing 1 for its denominator.

206. To Multiply a Whole Number by a Fraction.

EXAMPLE.—Multiply 6 by $\frac{7}{12}$.

OPERATION I.

$$6 \times \frac{7}{12} = 4\frac{2}{2} = 3\frac{6}{2} = 3\frac{1}{2}.$$

OPERATION II.

$$\frac{7}{12} \times 6 = \frac{7}{2} = 3\frac{1}{2}.$$

OPERATION III.

$$6 \times \frac{7}{12} = \frac{7}{2} = 3\frac{1}{2}.$$

EXPLANATIONS.—1st. If the multiplicand 6, be multiplied by $\frac{7}{12}$, the numerator of the fraction $\frac{7}{12}$, the product 42 will be 12 times too large, because the multiplier was not 7, but one-twelfth of 7; hence this product must be divided by 12, which gives $4\frac{2}{2} = 3\frac{6}{2} = 3\frac{1}{2}$.

2d. Since 6 and $\frac{7}{12}$ are the factors of the product, and as it matters not which term is multiplied, reverse the order of the factors and proceed as in Art. 203.

3d. Place 1 as a denominator for the multiplicand, then cancel and reduce.

Rule.—Multiply the whole number by the numerator of the fraction, and divide the product by the denominator. Cancel when possible.

EXAMPLES FOR MENTAL PRACTICE.

207. What is the product of

- | | | |
|-------------------------------------|--------------------------------------|---------------------------------------|
| 1. 5 multiplied by $\frac{2}{3}$. | 6. 22 multiplied by $\frac{1}{11}$. | 11. 15 multiplied by $\frac{4}{5}$. |
| 2. 7 multiplied by $\frac{7}{8}$. | 7. 40 multiplied by $\frac{3}{8}$. | 12. 21 multiplied by $\frac{1}{11}$. |
| 3. 6 multiplied by $\frac{5}{8}$. | 8. 9 multiplied by $\frac{2}{3}$. | 13. 12 multiplied by $\frac{5}{8}$. |
| 4. 9 multiplied by $\frac{1}{11}$. | 9. 4 multiplied by $\frac{1}{11}$. | 14. 18 multiplied by $\frac{2}{3}$. |
| 5. 14 multiplied by $\frac{5}{8}$. | 10. 7 multiplied by $\frac{2}{5}$. | 15. 42 multiplied by $\frac{4}{5}$. |

EXAMPLES FOR WRITTEN PRACTICE.

208. Multiply.

- | | | | |
|----------------------------|----------------------------|-----------------------------|------------------------------|
| 1. 81 by $\frac{4}{5}$. | 5. 19 by $2\frac{4}{5}$. | 9. 27 by $\frac{7}{8}$. | 13. 71 by $\frac{1}{2}$. |
| 2. 56 by $\frac{1}{5}$. | 6. 22 by $1\frac{3}{5}$. | 10. 210 by $\frac{3}{10}$. | 14. 203 by $1\frac{2}{11}$. |
| 3. 61 by $\frac{5}{8}$. | 7. 240 by $\frac{1}{10}$. | 11. 48 by $\frac{1}{5}$. | 15. 415 by $\frac{3}{5}$. |
| 4. 105 by $\frac{1}{11}$. | 8. 8 by $3\frac{1}{4}$. | 12. 91 by $\frac{1}{13}$. | 16. 672 by $\frac{3}{8}$. |

REMARK.—Entire or mixed number can be treated with facility by reducing them to fractional forms and cancelling when possible.

209. To Multiply a Fraction by a Fraction.EXAMPLE.—1. (*Abstract*). Multiply $\frac{2}{5}$ by $\frac{3}{7}$.

OPERATION.

$$\frac{2}{5} \times \frac{3}{7} = \frac{2}{5} \times \left\{ \begin{array}{l} 3 \text{ times } \frac{1}{7}, \\ \text{or} \\ \frac{1}{7} \text{ of } 3. \end{array} \right.$$

$$\frac{2}{5} \times 3 = \frac{6}{5}; \quad \frac{6}{5} \times \frac{1}{7} = \frac{6}{35}.$$

EXPLANATION.—The multiplier, $\frac{3}{7}$, is equal to 3 times $\frac{1}{7}$ or $\frac{1}{7}$ of 3. By application of 203, multiply $\frac{2}{5}$ by 3, which gives $\frac{6}{5}$, which must be seven times too large, since the multiplier was not 3, but one-seventh of 3; hence, the correct result will be obtained by dividing the product, $\frac{6}{5}$, by 7, which gives $\frac{6}{35}$.

EXAMPLE.—2. (*Concrete*). If a pound of tea costs $\frac{4}{5}$ of a dollar, what will $\frac{2}{3}$ of a pound cost?

EXPLANATION.—If a pound of tea costs $\frac{4}{5}$ of a dollar, $\frac{1}{5}$ of a pound will cost $\frac{1}{5}$ of $\frac{4}{5}$ of a dollar, or $\frac{4}{25}$ of a dollar; if $\frac{1}{5}$ of a pound costs $\frac{4}{25}$ of a dollar, $\frac{2}{3}$ which is 2 times $\frac{1}{3}$, must cost 2 times $\frac{4}{25}$ of a dollar, or $\frac{8}{25}$ of a dollar.

EXAMPLE.—3. If a yard of cloth costs $1\frac{1}{3}$ (or $1\frac{5}{6}$) dollars, what will $\frac{4}{5}$ of a yd. cost?

EXPLANATION.—If 1 yd. costs $1\frac{5}{6}$ dollars, $\frac{1}{6}$ of a yd. will cost $\frac{1}{6}$ of $1\frac{5}{6}$, or $\frac{5}{36}$ of a dollar; and if $\frac{1}{6}$ of a yd. costs $\frac{5}{36}$ of a dollar, $\frac{4}{5}$ which is 4 times $\frac{1}{5}$, must cost 4 times $\frac{5}{36}$, or $\frac{2}{3} = 1\frac{1}{3}$ dollars.

REMARKS—1. Observe that the numerator of the product is the product of the numerators of the factors, and that the denominator of the product is the product of the denominators of the factors.

2. This will apply to the product of any fractions, proper or improper, or to the product of continued fractions.

Rule.—1. *Cancel all equivalent factors from the numerators and denominators.*

II. *Multiply together the remaining numerators for the numerator of the product, and the remaining denominators for the denominator of the product.*

EXAMPLES FOR MENTAL PRACTICE (ABSTRACT).**210.** Find the product of

Multiply

- | | | | |
|--------------------------------------|---------------------------------------|--------------------------------------|---------------------------------------|
| 1. $\frac{1}{6}$ and $\frac{4}{5}$. | 5. $\frac{3}{4}$ and $\frac{7}{8}$. | 9. $\frac{5}{14}$ by $\frac{7}{8}$. | 13. $\frac{2}{3}$ by $\frac{5}{7}$. |
| 2. $\frac{3}{8}$ and $\frac{2}{3}$. | 6. $\frac{5}{8}$ and $\frac{3}{18}$. | 10. $\frac{1}{6}$ by $\frac{7}{8}$. | 14. $\frac{1}{6}$ by $1\frac{1}{2}$. |
| 3. $\frac{5}{8}$ and $\frac{1}{2}$. | 7. $1\frac{1}{2}$ and $\frac{3}{4}$. | 11. $\frac{2}{3}$ by $\frac{7}{8}$. | 15. $1\frac{1}{6}$ by $\frac{3}{4}$. |
| 4. $\frac{5}{6}$ and $\frac{7}{8}$. | 8. $\frac{2}{5}$ by $1\frac{1}{4}$. | 12. $\frac{1}{2}$ by $\frac{7}{8}$. | 16. $\frac{1}{10}$ by $\frac{5}{6}$. |

EXAMPLES FOR MENTAL PRACTICE (CONCRETE).

211. 1. What will be the cost of $\frac{2}{3}$ of a pound of tea, if the cost of a pound be $\frac{4}{5}$ of a dollar?

2. I bought $\frac{5}{8}$ of an acre of land and sold $\frac{4}{5}$ of my purchase. What part of an acre did I sell?

3. What will be the cost of $\frac{1}{3}$ of $\frac{3}{4}$ of a cord of wood at \$6 per cord?

4. A girl having $\frac{7}{8}$ of a yard of ribbon used $\frac{5}{6}$ of what she had. What part of a yard had she left?

5. John was given $\frac{2}{3}$ of a farm and James $\frac{3}{4}$ as much. What part had James?

6. If $\frac{1}{4}$ of a stock were lost by fire and the remainder sold at $\frac{3}{4}$ of its cost, what part of the first cost was received.

7. Divide 20 into two parts, one of which shall be $\frac{3}{4}$ of the other.
8. So divide \$150 between two persons that one may have $\frac{1}{2}$ of the whole more than the other.
9. Tea costing $\frac{5}{8}$ of a dollar per pound is sold for $\frac{3}{4}$ of its cost. For what price per pound is the tea sold?
10. Coffee costing $\frac{5}{16}$ of a dollar per pound is sold for $\frac{4}{5}$ of its cost. What price is obtained for the coffee?
11. What is $\frac{1}{2}$ of $\frac{2}{3}$ of a yard of cloth worth, if the entire yard costs $\frac{2}{3}$ of a dollar?
12. If a pound of steak costs $\frac{3}{16}$ of a dollar, what will $\frac{1}{8}$ of $\frac{1}{2}$ of a pound cost?
13. After paying $\frac{1}{2}$ a dollar for a pound of nuts, I sold $\frac{2}{3}$ of $\frac{3}{4}$ of my purchase at the same rate. How much did I receive?
14. From a gallon of oil $\frac{2}{3}$ of $\frac{3}{4}$ of a gallon leaked away. What part of a gallon was left?

EXAMPLES FOR WRITTEN PRACTICE.

212. Multiply together
- | | |
|--|---|
| 1. $\frac{13}{20}, \frac{2}{11}, \frac{43}{8},$ and $\frac{2}{5}.$ | 8. $206\frac{1}{2}, 187\frac{5}{8}, 40\frac{7}{16},$ and $51\frac{1}{8}.$ |
| 2. $3\frac{2}{3}, \frac{8}{9}, \frac{5}{6}, 21,$ and $11\frac{2}{3}.$ | 9. $500, 186\frac{2}{11}, 63\frac{3}{4}, 41, 19\frac{2}{3},$ and $4.$ |
| 3. $\frac{5}{6}, \frac{5}{6}, 8\frac{1}{2}, 11\frac{2}{3}, \frac{5}{4},$ and $20.$ | 10. $23\frac{5}{12}, 16\frac{5}{6}, 8\frac{3}{4},$ and $12\frac{1}{2}.$ |
| 4. $12\frac{3}{4}, 16\frac{1}{2}, 21\frac{2}{3}, 60\frac{1}{8},$ and $\frac{1}{2}.$ | 11. $\frac{5}{8}, \frac{7}{11}, \frac{2}{15}, \frac{8}{7}, \frac{2}{9},$ and $\frac{1}{4}.$ |
| 5. $5\frac{3}{8}, 8, 11\frac{1}{10}, 17\frac{1}{2}, 25\frac{3}{8},$ and $6.$ | 12. $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{16},$ and $60.$ |
| 6. $9\frac{1}{2}, 19\frac{3}{4}, 29\frac{5}{8}, 39\frac{1}{8},$ and $49.$ | 13. $13\frac{5}{18}, 21\frac{4}{9}, 12, 4, 2\frac{1}{2},$ and $3\frac{1}{3}.$ |
| 7. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10},$ and $10.$ | 14. $\frac{1}{20}, \frac{1}{10}, 5, \frac{7}{15}, \frac{1}{11},$ and $\frac{1}{2}.$ |
15. What will be the cost of $7\frac{1}{2}$ tons of hay at $\frac{3}{4}$ of $15\frac{1}{2}$ dollars per ton?
 16. Having bought $\frac{4}{5}$ of a farm of $106\frac{1}{4}$ acres, I sold $\frac{2}{3}$ of my purchase. How many acres do I sell?
 17. I bought a house for \$2165 $\frac{1}{2}$ and sold it for $\frac{1}{16}$ of its cost. How many dollars did I lose?
 18. If $7\frac{1}{2}$ barrels of flour be consumed by a family in ten months, how many barrels would fifteen such families consume in $13\frac{1}{2}$ months?
 19. Having bought $2\frac{1}{2}$ tons of coal at $\frac{2}{3}$ of $16\frac{2}{3}$ dollars per ton, I gave in payment a twenty dollar bill. How much change should I have received?
 20. If $17\frac{1}{2}$ cords of wood are bought at $\frac{4}{5}$ of $13\frac{2}{3}$ dollars per cord, and sold at $\frac{7}{8}$ of $9\frac{1}{4}$ dollars per cord, what is the gain or loss?

DIVISION OF FRACTIONS.

213. To Divide a Fraction by a Whole Number.

EXAMPLE—1. Divide $\frac{4}{5}$ by 2.

OPERATION. EXPLANATION.—By the General Principles of Fractions, dividing the numerator (dividend) divides the fraction; hence, divide the numerator 4 of the $\frac{4}{5} \div 2 = \frac{2}{5}$. fraction $\frac{4}{5}$ by 2, and the quotient is $\frac{2}{5}$.

EXAMPLE.—2. If a pound of tea costs $\frac{4}{5}$ of a dollar what will $\frac{1}{2}$ of a pound cost?

OPERATION.

$$\frac{4}{5} \div 2 = \frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$$

EXPLANATION.—If one pound costs $\frac{4}{5}$ of a dollar, $\frac{1}{2}$ of a pound will cost $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{2}{5}$ of a dollar. Observe that in this operation the multiplier $\frac{1}{2}$ is the reciprocal of the divisor 2, or 1 has been written under the divisor as a denominator, the divisor inverted and the work performed as in multiplication.

EXAMPLE.—3. Divide $\frac{3}{5}$ by 7.

EXPLANATION.—By the General Principles of Fractions, if we multiply the denominator we divide the fraction. Therefore, $\frac{3}{5} \div 7 = \frac{3}{35}$.

Rule.—*Divide the numerator or multiply the denominator by the whole number.*

REMARKS.—1. Divide the numerator if it be divisible, as the numbers will thus be made smaller.

2. If the dividend be a mixed number and the divisor an integer, it is not necessary to reduce the dividend to an improper fraction; divide the integral part of the dividend by the divisor, and if there be a remainder from such division, reduce it to a fraction of the same denomination as the fractional part of the dividend, add it to this fractional part and divide as before shown.

EXAMPLE.—4. Divide $2815\frac{2}{3}$ by 8.

OPERATION.

$$\begin{array}{r} 8 \overline{) 2815\frac{2}{3}} \\ 351\frac{2}{3} \end{array}$$

EXPLANATION.—(Short Method).—Write as in Short Division and divide; 8 is contained in 2815 $\frac{2}{3}$, 351 times with a remainder of $7\frac{2}{3}$ not divided; reduce the 7 to thirds and to the result add the $\frac{2}{3}$ making $2\frac{2}{3}$, which divide by 8, obtaining $\frac{2}{3}$ as the fractional part of the quotient; annex this to the integral part which gives $351\frac{2}{3}$.

EXAMPLES FOR MENTAL PRACTICE.

214. What is the quotient of

- | | | |
|----------------------------------|--|---|
| 1. $\frac{2}{3}$ divided by 3? | 5. $1\frac{5}{6}$ divided by 5? | 9. $\frac{1}{2}$ divided by 4? |
| 2. $1\frac{2}{3}$ divided by 4? | 6. $\frac{2}{3}\frac{2}{1}$ divided by 7? | 10. $\frac{2}{3}\frac{1}{5}$ divided by 7? |
| 3. $\frac{9}{11}$ divided by 9? | 7. $1\frac{10}{11}$ divided by 15? | 11. $\frac{5}{6}\frac{4}{1}$ divided by 18? |
| 4. $1\frac{6}{11}$ divided by 8? | 8. $\frac{4}{3}\frac{2}{3}$ divided by 13? | 12. $\frac{6}{5}\frac{6}{5}$ divided by 22? |

13. If a pound of powder costs $\frac{3}{8}$ of a dollar, what will $\frac{1}{4}$ of a pound cost?

14. Having $1\frac{1}{5}$ of a yard of cloth, I divided it into 7 equal pieces. How much cloth was there in each piece?

15. If $\frac{2}{3}\frac{1}{6}$ of a farm be grain land, and evenly divided into 3 fields, what part of the farm will each field contain?

EXAMPLES FOR WRITTEN PRACTICE.

215. Divide

- | | | | |
|---|-------------------------------------|-------------------------------|----------------------------|
| 1. $\frac{2}{3}\frac{4}{5}$ by 9. | 5. $\frac{5}{11}\frac{6}{11}$ by 8. | 9. $308\frac{7}{11}$ by 40. | 13. $205\frac{5}{6}$ by 6. |
| 2. $2\frac{5}{12}$ by 17. | 6. $21\frac{1}{4}$ by 5. | 10. $1000\frac{1}{10}$ by 41. | 14. $185\frac{5}{6}$ by 9. |
| 3. $\frac{3}{5}\frac{0}{5}\frac{6}{5}$ by 12. | 7. $41\frac{3}{4}$ by 11. | 11. $16\frac{2}{3}$ by 5. | 15. $112\frac{1}{2}$ by 8. |
| 4. $3\frac{1}{7}\frac{1}{7}$ by 21. | 8. $2096\frac{2}{5}$ by 21. | 12. $108\frac{1}{3}$ by 25. | 16. $321\frac{2}{3}$ by 6. |

216. To Divide a Whole Number by a Fraction.

EXAMPLE.—Into how many pieces $\frac{2}{3}$ of a yard each, may 5 yards of ribbon be cut?

OPERATION.

$5 \div \frac{2}{3} =$ many pieces, each containing $\frac{2}{3}$ yd. as $\frac{2}{3}$ is contained times in $\frac{1}{5}$ which is $\frac{1}{5} \div \frac{2}{3} = 7\frac{1}{2}$. $7\frac{1}{2}$ times.

REMARK.—Since the denominator *names* or tells the *kinds*, or value of the parts taken, when fractions are reduced to the same denomination, or to equivalent fractions having a common denominator, their numerators compare as whole numbers. We may consequently ignore the denominators.

Rules.—1. *Multiply the denominator of the fraction by the whole number, and divide the result by the numerator.* Or,

2. *Reduce the whole number to a fraction of the same denomination as the divisor, and divide the numerator of the dividend by that of the divisor.*

EXAMPLES FOR MENTAL PRACTICE.**217. Divide**

- | | | | |
|-----------------------------|-------------------------------------|---------------------------|----------------------------|
| 1. 17 by $\frac{3}{4}$. | 6. 14 by $\frac{1}{20}$. | 11. 31 by $\frac{2}{3}$. | 16. 24 by $\frac{3}{8}$. |
| 2. 11 by $\frac{5}{6}$. | 7. 20 by $\frac{9}{10}$. | 12. 50 by $\frac{4}{3}$. | 17. 30 by $\frac{1}{10}$. |
| 3. 20 by $1\frac{1}{3}$. | 8. 51 by $\frac{2}{3}\frac{5}{6}$. | 13. 21 by $\frac{7}{8}$. | 18. 12 by $\frac{5}{6}$. |
| 4. 86 by $\frac{5}{6}$. | 9. 39 by $1\frac{1}{3}$. | 14. 60 by $\frac{8}{9}$. | 19. 33 by $1\frac{1}{2}$. |
| 5. 101 by $1\frac{0}{88}$. | 10. 25 by $\frac{5}{4}$. | 15. 18 by $\frac{2}{3}$. | 20. 15 by $\frac{5}{4}$. |

EXAMPLES FOR WRITTEN PRACTICE.

218. 1. If $\frac{2}{3}$ of an acre of land sell for 45 dollars, what will an acre sell for at the same rate?

2. A farm of 471 acres is divided into shares of $94\frac{1}{2}$ acres each. How many shares are there?

3. A church collection of 232 dollars was divided among poor families to each of which was given $5\frac{1}{3}$ dollars. How many families shared the bounty?

4. When potatoes are worth $\frac{3}{4}$ of a dollar per bushel and apples $\frac{2}{3}$ of a dollar per bushel, how many bushels of potatoes will pay for a load of apples measuring 30 bushels?

5. A woman buys $\frac{3}{4}$ of a cord of wood worth $6\frac{2}{3}$ dollars per cord and pays for it in work at $\frac{1}{4}$ of a dollar per day. How many days must she work to make full payment?

6. A dealer paid $\frac{1}{4}$ of $15\frac{3}{4}$ dollars for $\frac{2}{3}$ of $14\frac{1}{2}$ cords of wood. What was the cost per cord?

7. If $\frac{3}{16}$ of a farm of $67\frac{1}{2}$ acres be divided into 63 village lots, what part of an acre will each lot contain?

8. 1760 bushels of wheat, 2100 bushels of barley, 2758 bushels of oats, and 696 bushels of beans were put into sacks; those for the wheat contained each $2\frac{1}{4}$ bushels, for the barley $2\frac{1}{2}$ bushels, for the oats $2\frac{3}{4}$ bushels, and for the beans $1\frac{1}{2}$ bushels. How many sacks in all were required?

219. To Divide a Fraction by a Fraction.**EXAMPLE.**—Divide $\frac{3}{5}$ by $\frac{4}{7}$.**OPERATION.**

$$\frac{4}{7} = \begin{cases} 4 \text{ times } \frac{1}{7} \\ \text{or} \\ \frac{1}{7} \text{ of } 4. \end{cases} \quad \frac{3}{5} \div 4 = \frac{3}{20};$$

$$\frac{3}{5} \times 7 = \frac{21}{5} = 1\frac{1}{5}.$$

$$\frac{3}{5} \times \frac{7}{4} = \frac{21}{20} = 1\frac{1}{20}.$$

EXPLANATION.—The divisor, $\frac{4}{7}$, is equal to 4 times $\frac{1}{7}$, or $\frac{1}{7}$ of 4. Applying the explanation of Art. 213, and dividing the dividend, $\frac{3}{5}$, by 4, gives $\frac{3}{20}$ as a quotient; but since the given divisor was $\frac{1}{7}$ of 4, and the divisor used was 4, a number 7 times too great, $\frac{3}{20}$, the quotient obtained, is 7 times too small; to correct this error multiply $\frac{3}{20}$ by 7, obtaining $\frac{21}{20}$ as an answer. Observe that 21 (the numerator of the quotient), is obtained by multiplying the numerator of the dividend

by the denominator of the divisor, and that 20 (the denominator of the quotient), is obtained by multiplying the denominator of the dividend by the numerator of the divisor, or by effecting a cross multiplication as shown by the connecting or tracing lines in the *operation*.

Rules.—1. *Multiply the numerator of the dividend by the denominator of the divisor for the numerator of the quotient, and multiply the denominator of the dividend by the numerator of the divisor for the denominator of the quotient; Or,*

2. *Invert the terms of the divisor and proceed as in multiplication of fractions.*

REMARK.—Reduce mixed numbers to improper fractions before applying the rule.

EXAMPLES FOR MENTAL PRACTICE.**220. Divide**

- | | | | |
|-------------------------------------|---------------------------------------|--|---|
| 1. $\frac{2}{3}$ by $\frac{3}{4}$. | 5. $\frac{1}{8}$ by $\frac{5}{6}$. | 9. $\frac{11}{14}$ by $\frac{2}{3}$. | 13. $\frac{14}{15}$ by $\frac{4}{5}$. |
| 2. $\frac{5}{6}$ by $\frac{2}{3}$. | 6. $\frac{5}{13}$ by $\frac{2}{3}$. | 10. $\frac{7}{8}$ by $\frac{5}{7}$. | 14. $\frac{3}{10}$ by $\frac{11}{13}$. |
| 3. $\frac{5}{8}$ by $\frac{1}{2}$. | 7. $\frac{1}{8}$ by $\frac{11}{12}$. | 11. $\frac{2}{3}$ by $\frac{11}{13}$. | 15. $\frac{14}{15}$ by $\frac{4}{5}$. |
| 4. $\frac{1}{6}$ by $\frac{2}{3}$. | 8. $\frac{3}{10}$ by $\frac{5}{6}$. | 12. $\frac{5}{6}$ by $\frac{5}{8}$. | 16. $\frac{11}{12}$ by $\frac{3}{4}$. |

EXAMPLES FOR WRITTEN PRACTICE.**221. Divide**

- | | | | |
|--|--|--|--|
| 1. $\frac{14}{15}$ by $\frac{2}{3}$. | 5. $\frac{207}{160}$ by $\frac{1}{18}$. | 9. $\frac{240}{11}$ by $\frac{16}{13}$. | 13. $\frac{4}{7}$ by $\frac{1}{17}$. |
| 2. $\frac{80}{81}$ by $\frac{7}{11}$. | 6. $\frac{1}{6}$ by $\frac{13}{31}$. | 10. $\frac{8}{9}$ by $\frac{1}{4}$. | 14. $\frac{3}{8}$ by $\frac{2}{30}$. |
| 3. $\frac{100}{100}$ by $\frac{7}{16}$. | 7. $\frac{333}{100}$ by $\frac{2}{10}$. | 11. $\frac{3}{10}$ by $\frac{2}{5}$. | 15. $\frac{14}{15}$ by $\frac{7}{8}$. |
| 4. $\frac{11}{10}$ by $\frac{1}{11}$. | 8. $\frac{3}{8}$ by $\frac{10}{16}$. | 12. $\frac{11}{12}$ by $\frac{16}{20}$. | 16. $\frac{2}{3}$ by $\frac{5}{13}$. |

17. If a boy earns $\frac{1}{5}$ of a dollar in a day, how long will it take him to earn \$15 $\frac{3}{4}$?

18. How many fields of 9 $\frac{5}{8}$ acres each can be made from a farm containing 125 $\frac{1}{2}$ acres?

19. If a wheelman runs 93 $\frac{2}{3}$ miles per day, how long time will he require to run 1167 $\frac{1}{2}$ miles?

20. If 12 $\frac{3}{4}$ acres produce 982 $\frac{1}{2}$ bu. of corn, how many bu. will 15 $\frac{1}{2}$ acres produce?

21. If ten men cut 132 $\frac{1}{2}$ cords of wood in six days, how many cords can eighteen men cut in twenty-one days?

22. If a man bought 1150 $\frac{3}{4}$ bushels of wheat with $\frac{5}{8}$ of his money, how many bushels could he have bought had all his money been invested?

23. After traveling $\frac{1}{11}$ of the distance between two cities, a pedestrian finds that there are 101 $\frac{1}{2}$ miles still before him. How far apart are the cities?

COMPLEX FRACTIONS.

222. A fraction is complex when either or both of its terms are fractional.

Thus $\frac{5}{\frac{4}{5}}$ is a complex fraction and is read $5 \div \frac{4}{5}$; it indicates that 5 is to be divided by $\frac{4}{5}$. $\frac{\frac{5}{6}}{8}$ is read $\frac{5}{6} \div 8$ and indicates what is thus expressed. $\frac{\frac{3}{4}}{\frac{2}{5}}$ is read $\frac{3}{4} \div \frac{2}{5}$ and indicates what is thus expressed.

REMARK.—The entire subject of complex fractions will, on account of its lack of practical value, be dismissed with the full illustration of one example of each of two forms.

EXAMPLE 1.—What is the value of $\frac{\frac{3}{6}}{\frac{2}{7}}$?

OPERATION: $\frac{\frac{3}{6}}{\frac{2}{7}} = \frac{3}{6} \div \frac{2}{7} = \frac{3}{6} \times \frac{7}{2} = \frac{21}{12} = 2\frac{1}{4}$.

EXAMPLE 2.—What is the value of $\frac{\frac{5}{6} \text{ of } 2\frac{3}{5}}{\frac{1}{3} \text{ of } 1\frac{1}{4}}$?

OPERATION: $\frac{\frac{5}{6} \text{ of } 2\frac{3}{5}}{\frac{1}{3} \text{ of } 1\frac{1}{4}} = (\frac{5}{6} \times 1\frac{3}{5}) \div (\frac{1}{3} \times 1\frac{1}{4}) = \frac{5}{6} \times \frac{13}{5} \times \frac{3}{1} \times \frac{4}{1\frac{1}{4}} = \frac{64}{14} = 4\frac{4}{7}$.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

223. 1. From the sum of $\frac{3}{4}$ and $5\frac{3}{8}$, take the difference between $17\frac{1}{2}$ and 21.
2. How much will remain after the product of $\frac{2}{3}$, $\frac{7}{11}$, 2, $1\frac{3}{10}$, and $3\frac{1}{4}$ is taken from $10\frac{7}{8}$.
3. Divide into six equal parts the product of $11\frac{1}{10}$ multiplied by $3\frac{1}{2}$.
4. Find the remainder after subtracting the product of $3\frac{3}{4}$, $\frac{4}{7}$, $7\frac{3}{8}$, 5, $\frac{2}{3}$, and 1, from the product of 3, $\frac{3}{4}$, $\frac{4}{7}$, 7, $\frac{3}{8}$, 5, $\frac{2}{3}$, and 14.
5. An estate is so divided among A, B, and C, that A gets $\frac{2}{5}$, B $\frac{3}{10}$, and C the remainder, which was \$4200. What was the amount of the estate?
6. My bank deposit is \$5605, which is $4\frac{1}{2}$ times the amount in my purse. How much money have I in all?
7. If 14 bu. of apples can be bought for $\$3\frac{1}{2}$, how many bushels can be bought for $\$4\frac{1}{2}$?
8. A woman having \$1, gave $\frac{2}{3}$ of it for coffee at $33\frac{1}{3}\phi$ per pound. How many pounds did she buy?
9. Having bought $\frac{2}{3}$ of a ship, I sold $\frac{2}{3}$ of my share for \$12000. What was the value of the ship at that rate?
10. If the ingredients are $\frac{3}{10}$ sulphur, $\frac{1}{10}$ saltpeter, and $\frac{2}{5}$ charcoal, what is the number of pounds of each, in $2154\frac{3}{11}$ pounds of gunpowder?
11. What must be the amount of an estate which, if divided into three parts, the first will be double the second, the second double the third, and the difference between the second and the third be \$7500?
12. Having paid \$115 for a watch and chain, I discover that the cost of the chain was only $\frac{1}{8}$ of the cost of the watch. What was the cost of each?

13. I gave two 20-dollar gold coins to a dealer, of whom I bought 2 cords of wood at $5\frac{1}{4}$ dollars per cord, and $3\frac{1}{2}$ tons of coal at $6\frac{2}{3}$ dollars per ton. How much change should I have received.

14. A and B, working equally, can mow a meadow in 10 days of 9 hours per day. In how many days of 12 hours can A alone do the work? 15

15. An estate valued at \$120000 was so distributed that A received $\frac{2}{3}$, B $\frac{1}{10}$ of the estate more than A, C as much as A and B together less \$6000, and two charities the remainder in equal parts. How much did each charity receive? 9000

16. Brown owned $\frac{1}{10}$ of a stock of goods, $\frac{2}{3}$ of which was destroyed by fire and $\frac{1}{10}$ of the remainder so damaged by water that it was sold at half its cost. If the uninjured goods when sold at cost brought \$10800, what must have been the amount of Brown's loss?

17. A grocer bought a cask of molasses containing $65\frac{1}{2}$ gal., from which he sold at one time $\frac{1}{3}$ of it, at another $\frac{1}{6}$ of it, at another 5 gal. less than $\frac{1}{4}$ of what remained, and the remainder was sold with the cask for $20\frac{1}{2}$ dollars. If the value of the cask was one dollar, at what price per gallon was the last sale made? $51\frac{1}{2}$

18. A painter worked $17\frac{1}{2}$ days, and after expending $\frac{4}{5}$ of his wages for board, had \$15 left. How much did he earn per day? 2.70

19. A farmer having 650 bu. of wheat, kept for his own use $52\frac{3}{4}$ bu. less than $\frac{1}{4}$, sold to his neighbors for seed $45\frac{1}{2}$ bu. more than $\frac{1}{3}$, and marketed the remainder at 80¢ per bushel. How much money was received from the market sales? 248

20. Of a journey of 160 miles, a walker accomplished $\frac{1}{4}$ of the distance the first day, $\frac{1}{4}$, less $15\frac{3}{4}$ miles, the second day, $\frac{1}{10}$, plus $4\frac{3}{8}$ miles, the third day, and finished his journey on the two following days by traveling fifteen hours each day. What must have been his average distance per hour for those two days?

21. A mechanic worked $21\frac{2}{3}$ days, and after paying his board with $\frac{2}{3}$ of his earnings, had $66\frac{2}{3}$ dollars left. How much did he earn per day?

22. So place a sum of money that $\frac{1}{6}$ of it shall be in the first package, $\frac{2}{3}$ in the second, $\frac{1}{12}$ in the third, and the remainder, which is \$550, in the fourth package. What amount of money will be required? 13700

23. If $\frac{1}{2}$ the trees of an orchard are apple, $\frac{1}{4}$ peach, $\frac{1}{8}$ pear, $\frac{1}{10}$ plum, and the remaining 21 trees cherry, how many trees in all?

24. John's weight is $\frac{3}{4}$ as much as mine, and Ben's is $\frac{1}{2}$ of John's. What is my weight if John is 15 pounds heavier than Ben?

25. If 12 boys earn \$54 in a week, how much will 15 boys earn in the same time at the same rate?

26. A, B, and C rented a pasture for \$37. A put in 3 cows for 4 months, B, 5 for 6 months, and C, 8 for 4 months. How much ought each to pay?

27. Henry, when asked his age, replied, "If $7\frac{1}{2}$ years be added to $13\frac{1}{2}$ years, the sum will represent $\frac{1}{3}$ of my age." How old was he?

28. Silas, Harvey, and Robert have together \$2210. Silas has $2\frac{1}{2}$ times as much as Harvey, who has $\frac{2}{3}$ as much as Robert. How many dollars has each? 547

29. Theodore's age is $7\frac{1}{4}$ years, and Herbert's $9\frac{5}{8}$ years; three times the sum of their ages is 8 years more than the age of their mother, who is $5\frac{1}{2}$ years younger than their father. What is the united age of the parents? 119

30. A farmer sold two cows for \$75, receiving for one only $\frac{1}{2}$ as much as for the other. What was the price of each?

31. After selling 45 turkeys, a dealer had $\frac{1}{4}$ of his stock remaining. How many had he at first?

32. If 8 horses consume $4\frac{1}{2}$ bushels of oats in $3\frac{1}{2}$ days, how many bushels will 12 horses consume in the same time? $6\frac{3}{4}$

33. A and B can do a piece of work in 10 days, which A alone can do in 18 days. In what time can B alone do the work?

34. John and Calvin have agreed to build a wall for \$86. If Calvin can work only $\frac{1}{2}$ as fast as John, how shall the money be divided?

35. A flagstaff stands $\frac{1}{3}$ of its length above and $7\frac{1}{2}$ ft. below the surface of the ground. What is the length of the staff? 40

36. What is the length of a pole that stands $\frac{2}{3}$ in the mud, $\frac{1}{3}$ in the water, and $25\frac{1}{2}$ feet above the water? $74\frac{1}{2}$

37. A colt and cow cost \$124. If the colt cost \$4 more than three times the cost of the cow, what was the cost of each?

38. What is the hour when the time past noon equals $\frac{1}{3}$ of the time to midnight?

39. A tree 84 ft. high was so broken in a storm that the part standing was $\frac{2}{3}$ the length of the part broken. How many feet were standing? 36

40. A farmer has $\frac{2}{3}$ of his sheep in one pasture, $\frac{1}{3}$ in another, and the remainder of his flock, 72 sheep, in the third pasture. How many sheep had he?

41. For a horse and carriage I paid \$540. What was the cost of each, if the cost of the carriage was $1\frac{1}{2}$ times the cost of the horse? $216-324$

42. Calvin is $8\frac{1}{2}$ years old, Leo $6\frac{1}{2}$ years less than three times as old as Calvin, and John's age is 3 years more than the sum of the ages of Calvin and Leo. What is John's age? $30\frac{1}{2}$

43. Peter can do a piece of work in 12 days and Charles in 15 days. How many days will be required for its completion, if both join in the work? $6\frac{2}{3}$

44. If A can do a piece of work in 21 days, B in 18 days, and C in 15 days, in how many days can the three working together perform the work?

REMARK.—In the above and similar examples, reason in general as follows: If A can build a wall in 4 days, he can build $\frac{1}{4}$ of it in 1 day; and if B can build the same wall in 5 days, he can build $\frac{1}{5}$ of it in 1 day. Since in 1 day A can build $\frac{1}{4}$ of the wall, and B $\frac{1}{5}$ of it, the two can, if they work together, build in 1 day the sum of $\frac{1}{4}$ and $\frac{1}{5}$ or $\frac{9}{20}$ of it; and since they can together do $\frac{9}{20}$ in 1 day, it will take them as many days to do the whole work, or $\frac{20}{9}$, as $\frac{9}{20}$ is contained times in $\frac{20}{9}$, or $2\frac{2}{9}$.

45. John and his father have joint work, which they can do working together in 25 days. If it require 60 days for John working alone to complete the work, how many days will it require for the father to complete it? $42\frac{2}{3}$

46. A man and boy can in 16 days complete a job that can be done by the man alone in 21 days. How long would it take the boy alone to complete the work? $67\frac{1}{2}$

47. Smith said to Brown, " $\frac{2}{3}$ of my money is equal to $\frac{1}{3}$ of yours, and the difference between your money and mine is \$30." How much money had each? $150-150$

48. Izaak Walton having lost $\frac{2}{3}$ of his trolling line, added 65 ft., when he found it was just $\frac{1}{3}$ of its original length. What was its length at first? $\$150\text{ ft.}$

49. A cistern sprung a leak by which $\frac{2}{3}$ of its contents ran out, but during the same time $\frac{2}{3}$ as much ran in. What part of the cistern was filled? $\frac{1}{3}$

50. A dog pursuing a rabbit which has 32 rods the start, runs 11 rods while the rabbit runs but 9. How far must the dog run before he can overtake the rabbit?

51. A cistern has two faucets, by the larger of which it can be emptied in 24 minutes, and by the smaller in 36 minutes. If both be opened at once, what length of time will be required to empty the cistern? $14\frac{2}{3}$

52. Ben and John bought a cocoanut for 8 cents, of which Ben paid 5¢ and John 3¢. Henry offered 8¢ for one-third of the cocoanut, which offer was accepted, each taking and eating one-third of it. How should Ben and John divide the 8¢ received from Henry?

53. There are 108 bu. of corn in two bins, and in one of the bins there are 12 bushels less than one-half as many bushels as in the other. How many bushels in each?

54. At what time between one and two o'clock will the hour and minute hands of a clock be together?

55. At what time between 6 and 7?

56. At what time between 9 and 10?

57. At what time between 10 and 11?

58. Nick bought a basket of oranges at the rate of 3 for 2 cents, and gained 50¢ by selling them at the rate of 2 for 3 cents. How many oranges did he buy?

59. If you buy 60 lemons at the rate of 6 for 10 cents, and twice as many more at the rate of 5 for 8 cents, and sell the entire lot at the rate of 3 for 4 cents, will you gain or lose, and how much? $57\frac{1}{2}$

60. So divide \$15,000 among A, B, C, and D, that their portions shall be to each other as 1, 2, 3, and 4. What is the portion of each.

61. I wish to line the carpet of a room that is $7\frac{1}{2}$ yd. long and $5\frac{2}{3}$ yd. wide, with duck $\frac{3}{4}$ of a yd. wide. How many yards of duck will be required if it shrink $\frac{1}{8}$ in length and $\frac{1}{16}$ in width?

62. A and B are engaged to perform a certain work for \$35 $\frac{5}{10}$. It is supposed that A does $\frac{1}{4}$ more work than B, and they are to be paid proportionally. How much should each receive?

63. A tank has an inlet by which it can be filled in 10 hours, and an outlet by which when filled it can be emptied in 6 hours. If both inlet and outlet be opened when the tank is full, in what time will it be emptied?

64. A cistern has two faucets, by the larger of which its contents may be emptied in 12 minutes and by the smaller in 15 minutes; the cistern being full, the smaller faucet is left open for 6 minutes, after which both are opened. How long before the cistern will be emptied?

65. A man being asked his age replied, "My mother was born in 1800 and my father in 1801; the sum of their ages at the time of my birth was two and one-third times my age in 1846." How old was the man in 1880?

66. Three men dig a well for \$36. If during the time A and B working together could do $\frac{4}{5}$ of the work, B and C together could do $\frac{3}{4}$ of the work, and A and C together could do $\frac{2}{3}$ of the work, how should the money be divided?

67. Brown and Smith have joint work for 16 days. In any given time Brown does only $\frac{2}{3}$ as much work as Smith. How many days would each working alone require to complete the work? If they work together, how should the \$45 paid for the work be divided?

68. Coe, Hall, Tell, and Lee have a contract to dig a ditch which Coe can dig in 35 days, Hall in 45 days, Tell in 50 days, and Lee in 60 days. How long will it take all together to do the work? If \$100 be paid for the work and all join till it is completed, how much should each get?

69. A and B have joint work for 21 days, but B can in a day do only $\frac{3}{4}$ as much as A; after B has worked alone for 3 days and A for 5 days, they unite and complete the work. How many days will they require? If \$75 be paid for the work, what part of it should each receive?

70. An estate was left to A, B, and C, so that A's part was $\frac{1}{3}$ of the whole increased by a sum equal to $\frac{1}{10}$ of C's part; B's was $\frac{1}{3}$ of the whole increased by a sum equal to $\frac{1}{3}$ of C's part; and to C was given the remainder, which was \$700 less than B's share. What was the value of the estate and of each one's share?

71. Hill, Mann, and Benton have joint work for 36 days, for which they are to receive \$200. If Hill can do only $\frac{2}{3}$ as much as Mann and Benton does twice as much as Mann, in how many days could each working alone complete the work?

72. How long would it take Hill and Mann?

73. How long would it take Hill and Benton?

74. How long would it take Mann and Benton?

75. If all work together until the job is completed, how should the money be divided?

76. A, B, C, and D, having joint work for 30 days, A begins and works alone for 2 days, when he is joined by B; after the two have worked together for 3 days, they are joined by C; the three work together for 4 days, when D joins them, and all working together complete the work. If A can do but $\frac{1}{2}$ as much as D, B $\frac{2}{3}$ as much as A, and C $\frac{3}{10}$ as much as B, how long would each alone require to do the entire work.

77. How long would it take A and B; A and C; A and D?

78. How long would it take A, B, and C; A, C, and D; B, C, and D?

79. How long after D began did it take for all to do it?

80. If \$300 was paid for the work and the men worked according to conditions given in Example 76, how should the money be divided?

DECIMALS.

224. A **Decimal Fraction** or a **Decimal** is a fraction having for its denominator ten or some power of ten ; as 10, 100, 1000, 10000. It expresses one or more of the decimal divisions of a unit.

225. Decimals may be expressed in the same form as common fractions; that is, with the denominator written. Practically, however, this is never done.

REMARK.—The two points of difference between *common* and *decimal* fractions are,

1. The denominator of a common fraction is always written, while that of a decimal is only indicated.

2. The denominator of a common fraction may be any number, while that of a decimal must be 10 or some power of 10.

226. The **Decimal Point** (.) is a period, and is used to limit the value of of a decimal expression, and to determine the denominator; in this latter relation it takes the place of the unit 1 of the denominator when fully written; as, in the decimal expression .3, read 3 tenths, the decimal point considered as 1 and placed before a cipher, represents the order of its units, and shows that the indicated denominator is 10.

REMARK.—When the decimal point is used to separate the integral from the fractional part in mixed decimals, or dollars and cents in decimal currency, it is called a *separatrix*.

227. Decimals are either *pure* or *mixed*.

228. A **Pure Decimal** corresponds to a *proper* fraction, the value being less than the unit 1 ; as, .3, .17, .206, .5191.

A **Mixed Decimal** corresponds to an *improper* fraction, the value being greater than the unit 1 ; as, 17.4, 5.192, 32.3017.

229. The **Value of a Decimal** is computed from the *decimal point*, and the orders have the same scale as integers. A removal of the decimal point *one* place to the right, multiplies the expression by ten ; removing it *two* places, by 100 ; *three* places, by 1000, and so on. A removal of the decimal point *one* place to the left, divides the expression by 10 ; *two* places, by 100 ; *three* places, by 1000, and so on.

230. From the above it will be observed that if a cipher be placed between the numerical expression of the decimal and the *point*, the expression being thereby removed *one* place further from the point, will be divided by 10. But as the value of the decimal expression is computed from the *point* to the right, it follows that *one* or *more* ciphers, placed *after* the decimal, will not alter its value. $\frac{3}{10}$ is expressed decimally .3 ; a cipher annexed to the decimal gives $.30 = \frac{30}{100}$; *two* ciphers annexed gives $.300 = \frac{300}{1000}$.

By this it will be observed that the expressions, though unlike in form are of equal value. Each of the expressions .5, .50, .500, .5000, .50000, .500000, is equal to $\frac{1}{2}$.

Rule.—I *Numerate from the decimal point, to determine the denominator.*

II. *Read the decimal as a whole number, and give to it the denomination of the right-hand figure.*

EXAMPLES FOR PRACTICE.

235. 1. Read .297, .1471, .20442, .56007.
 2. Read .105, .6931, .214698, .4003755.
 3. Read .19005, .3050408, .690004003.
 4. Read .2, .20, .200, .2000, .20000, .200000.
 5. Read 18.3, 29.75, 460.215, 80.03465.
 6. Read 270.01, 5960.030506, 8205.506007.
 7. Read 10002.200001, 38960041.100086341.
 8. Read 27000.000027, 8100081.81000081.
 9. Read 1001001.1000100001, 9003009.009009.
 10. Read 39864125.86954769, 919101.01919.
 11. Read 50000000.00000050, 1000.1000.
 12. Read 123456.10203040506, 801.00801.
 13. Read 46000046.004600046.
 14. Read 37538651.0352615093.
 15. Read 45316255.83715632650.

236. Read the following decimals:

- | | | |
|---------------|--------------------|-----------------------------|
| 1. .206. | 9. 5320.008641. | 17. 2000.00020002. |
| 2. 1.423. | 10. 6000.58302. | 18. 564636.002616. |
| 3. 7.005. | 11. 9001.00901. | 19. 202020.20202. |
| 4. 19.11035. | 12. 340006.583. | 20. 21212121.51210021. |
| 5. 170.2093. | 13. 75075.07507. | 21. 30560078.0124861. |
| 6. 1050.0501. | 14. 560.00020201. | 22. 503760.2000463. |
| 7. 300.003. | 15. 53200.56931. | 23. 37564.03060507. |
| 8. 1000.0001. | 16. 214600.086005. | 24. 10023580021.1809010724. |

NOTATION OF DECIMALS.

237. The doubt which often arises in the mind of the pupil as to *how* a decimal should be written, may be entirely dispelled by keeping in mind the following facts:

1st. That they are fractions.

2nd. That both terms should be written or indicated.

3rd. That the denominator of any decimal (if written) would be 1, with as many ciphers to the right as the decimal contains places.

4th. When the numerator (or decimal) does not contain as many places as the denominator (if written) would contain ciphers, prefix ciphers to make the number of places equal.

EXAMPLE.—Write as a decimal *three-tenths*.

EXPLANATION.—Observe that in writing three-tenths as a common fraction, the mental operation is as follows: after writing 3, the numerator, you ask yourself *3 what?* the answer is, *3 tenths*; then the ten is written below as a denominator, thus obtaining $\frac{3}{10}$. Now reason in the same way regarding the decimal, and after writing 3, the numerator, ask yourself *3 what?* and answer, *3 tenths*; and *indicate* it by placing before the three, a decimal point, which represents the 1 of the decimal denominator; notice that the 3 occupies one place corresponding to the one cipher in the denominator.

Again, express decimally 416 thousandths.

EXPLANATION.—Write the 416 and ask *what?* answer, *thousandths*, which is determined by numerating from the right; units, tenths, hundredths, and (the point answering to the figure 1 of the denominator) thousandths; then *place* the *point*.

REMARK.—By extending and developing this method of writing decimals, the pupil can in a few minutes master the entire matter, so that he can write *any* decimal as readily and with as great certainty as if it were a whole number.

Rule.—I. *Write the decimal the same as a whole number, prefixing ciphers when necessary, to give to each figure its true local value.*

II. *Place the decimal point before the left-hand figure of the decimal.*

EXAMPLES FOR PRACTICE.

238. Express by figures the following decimals:

1. Twenty-six thousandths.
2. Twenty-seven hundredths.
3. Sixteen ten-thousandths.
4. Four hundredths.
5. Twenty-two hundred-thousandths.
6. Five and seven tenths.
7. Eighty-three and five hundred four ten-thousandths.
8. Seven hundred ten and two hundred forty-three hundred thousandths.
9. Five hundred and five hundredths.
10. Forty-five and forty-six thousandths.
11. One thousand one and one hundred ten-thousandths.
12. One thousand eight hundred ninety and ninety thousandths.
13. Eight hundred fifty and five hundredths.
14. Ten hundred and ten hundredths.

ADDITIONAL EXERCISES.

239. Write as decimals

1. Eleven and one hundred seven thousandths.
2. Fifteen and fourteen ten-thousandths.
3. Seven hundred twenty-six millionths.
4. Eleven hundred six and twelve ten-thousandths.
5. Sixteen hundred and sixteen hundredths.
6. Ten million and ten millionths.
7. Three hundred and sixty-five hundredths.

8. Twenty-five thousand four hundred and eleven hundredths.
9. Twenty-one and fifteen thousand fifteen ten-millionths.
10. Eighteen thousand eighteen ten-billionths.
11. Five hundred thousandths.
12. Five hundred-thousandths.
13. Nine hundred millionths.
14. Nine hundred-millionths.
15. Fifty-four million, fifty-four thousand, fifty-four and fifty-four million fifty thousand fifty-four ten-billionths.
16. One hundred three thousand five hundred eighty-seven thousandths.
17. Sixty-four thousand sixty-four hundredths.
18. Two million six hundred four thousand two hundred-thousandths.
19. Nine billion nineteen million twenty-nine thousand thirty-nine millionths.
20. Seventy-seven tenths.
21. Eighty-seven thousand one hundredths.
22. Four hundred seventy-nine million twenty seven thousand four and ninety-nine thousand four ten-billionths.
23. Seventy trillion and seven trillionths.
24. Eleven hundred and eleven ten-thousandths.
25. Three thousand one billionths.
26. One thousand three millionths.
27. One hundred-thousand eleven ten-millionths.
28. Six hundred five hundred-millionths.
29. Eighteen hundred ninety and eighteen hundred ninety hundred-billionths.

240. Write as decimals the following:

- | | | | |
|----------------------------|------------------------------------|----------------------------------|--------------------------------------|
| 1. $\frac{25}{100}$. | 6. $\frac{193024}{1000000}$. | 11. $\frac{3325481}{100}$. | 16. $\frac{35700007}{10000000000}$. |
| 2. $\frac{106}{10000}$. | 7. $\frac{504}{100000}$. | 12. $\frac{1876}{10000000000}$. | 17. $\frac{2197600801}{10000}$. |
| 3. $\frac{256}{1000000}$. | 8. $\frac{21469003}{100000}$. | 13. $\frac{10007}{10000}$. | 18. $\frac{46700004}{10000000000}$. |
| 4. $\frac{901}{100}$. | 9. $\frac{56773805}{10000}$. | 14. $\frac{97}{1000000}$. | 19. $\frac{68001}{10000000000}$. |
| 5. $\frac{1476}{10000}$. | 10. $\frac{1934675}{1000000000}$. | 15. $\frac{150015}{1000000}$. | 20. $\frac{110110011}{1000000}$. |

REDUCTION OF DECIMALS.

241. To Reduce Decimals to a Common Denominator.

EXAMPLE 1.—Reduce .021, .64, .03705, .5, .17272538, to equivalent decimals having the least common denominator.

OPERATION.

.02100000
.64000000
.03705000
.50000000
.17272538

EXPLANATION.—Since the decimal having the greatest number of decimal places is hundred-millionths its denominator is the *least common denominator* of the given expressions; this highest decimal contains 8 places, and by adding 5 places, or ciphers, to the first, 6 to the second, 3 to the third, and 7 to the fourth, all are reduced to 8 places, or to hundred-millionths, which is the least common denominator of the given expressions.

Rule. *By annexing ciphers make the number of decimal places equal.*

REMARKS—1. Decimals, like other fractions, can be neither added nor subtracted until reduced to a common denominator; but the scale in decimals being in the uniform ratio of *ten*, it is only necessary to write decimals for addition or subtraction so that the decimal points are in the same vertical line; the columns will then be of the same orders of units; in other words the decimals will be *practically* reduced to a common denominator.

2 The denominator of that expression containing the highest number of places is the least common denominator of the decimals; therefore the least common denominator may in all cases be determined by inspection, and decimals reduced to their least common denominator by simply supplying decimal ciphers until all have the same number of places.

3. In practice, however, this is never done, being rendered unnecessary by observing to write decimals so that the *points* stand under each other.

EXAMPLE 2.—Reduce .7, .23, .18765, and .175 to a common denominator.

OPERATION.	EXPLANATION. —As shown in the preceding operation, the effect of reducing
.7	decimals to a common denominator by annexing ciphers, is to cause the decimal
.23	points to fall in the same vertical column. Since annexing ciphers to decimals
.18765	does not alter their value, omit the ciphers and write the decimals so that the
.175	points are in the same vertical column.

Rule.—*Write the expressions so that the decimal points will stand in the same vertical line.*

REMARK.—This Rule applies equally to Pure and to Mixed Decimals.

EXAMPLES FOR PRACTICE.

- 242.** 1. Reduce .26, .423, 7.05, .56931 to their least common denominator.
 2. Reduce 21.18, .20463, 4636.02 to their least common denominator
 3. Reduce 56 hundredths, 75 millionths, 3 tenths, and 41 thousandths to their least common denominator.
 4. Reduce 2.36, .0005, .1, .62053, and 15.2 to their least common denominator.
 5. Reduce 19.0043, 3.87, 38.7 and .387 to their least common denominator

243. To Reduce Decimals to Common Fractions.

It has already been demonstrated

1st. That *Decimals* are fractions.

2d. That their denominators are merely indicated, and that the denominator may be expressed by writing 1, with as many ciphers at its right as the decimal contains places.

EXAMPLE.—Reduce .17 to a common fraction.

OPERATION.	EXPLANATION. —Since the decimal contains two places, its indicated denom-
.17 = $\frac{17}{100}$	inator must be 100.

Rule.—*Omit the decimal point and write for a denominator 1 with as many ciphers as the decimal contains places.*

REMARK.—Mixed Decimals may be reduced in a similar manner.

EXAMPLES FOR PRACTICE.

244.—Reduce to fractions in their lowest terms

1. .3	5. .4625	9. .42504	13. .114608
2. .63	6. .2244	10. .28828	14. .315264
3. .105	7. .1878	11. .08004	15. .2000534
4. .372	8. .1900	12. .24042	16. .983004752

Reduce to an ordinary mixed number

17. 5.16	20. 3005.1258	23. 1234500.0012345
18. 13.205	21. 1600.0016	24. 6540000.0002697
19. 117.602	22. 1000000.00000001	25. 188900.00001888

245. To Reduce a Common Fraction to a Decimal.

EXAMPLE.—Reduce $\frac{3}{5}$ to an equivalent decimal.

FIRST OPERATION. EXPLANATION.—From the definition of decimals, observe that the denominator must be 10 or some power of 10, and that $\frac{3}{5}$ may be reduced to a fraction the denominator of which is 10 by multiplying both its terms by 2. To change this fraction to an equivalent decimal, omit the denominator, and place a decimal point before the numerator.

SECOND OPERATION. EXPLANATION.—Place a decimal point and cipher after the numerator 3. This does not alter its value, though in form it becomes 3.0 = thirty tenths; and since this numerator is a dividend and the divisor is 5, divide 3.0 (thirty-tenths) by 5, and obtain .6 (six-tenths), as a result, an equivalent in decimal form as required.

$$\begin{array}{r} 5 \overline{) 3.0} \\ \underline{.6} \end{array}$$

Rule.—Place a decimal point and ciphers at the right of the numerator; divide by the denominator, and from the right of the quotient point off for decimals as many places as there have been ciphers annexed.

EXAMPLES FOR PRACTICE.

246. Reduce to equivalent decimals

1. $\frac{1}{16}$.	6. $\frac{13}{26}$.	11. $\frac{15}{16}$.	16. $\frac{31}{32}$.	21. $\frac{47}{60}$.
2. $\frac{13}{20}$.	7. $\frac{21}{400}$.	12. $\frac{1}{84}$.	17. $\frac{63}{64}$.	22. $\frac{113}{160}$.
3. $\frac{11}{16}$.	8. $\frac{15}{32}$.	13. $\frac{7}{140}$.	18. $\frac{7}{250}$.	23. $\frac{31}{320}$.
4. $\frac{3}{32}$.	9. $\frac{125}{128}$.	14. $\frac{11}{80}$.	19. $\frac{77}{800}$.	24. $\frac{11}{16}$.
5. $\frac{11}{80}$.	10. $\frac{3}{1250}$.	15. $\frac{3}{12500}$.	20. $\frac{1}{20}$.	25. $\frac{3}{320}$.

CIRCULATING DECIMALS.

247. Certain common fractions, as $\frac{1}{3}$, $\frac{7}{6}$, $\frac{5}{6}$, and $\frac{11}{12}$ cannot be reduced to an equivalent decimal, because the denominator (divisor) is not an exact divisor of any power of 10. Such expressions cannot be reduced to exact decimal forms, and are termed *repeating*, or *circulating* decimals; if used in the decimal form they are followed by the sign + to indicate inexactness. The repeated part is called a *repetend*; as, .3333+ is called the repetend 3; .171717+ is called the repetend 17; .206206+ is called the repetend 206.

248. To Express the Exact Value of a Repetend.

The exact value of any repetend is a common fraction, the numerator of which is the repetend and the denominator as many 9's as the repetend contains places; thus $.333+ = \frac{3}{9}$. $.171717+ = \frac{17}{99}$. $.206206+ = \frac{206}{999}$.

Rule.—Take the repetend for the numerator of a common fraction, and for its denominator write as many 9's as the repetend has orders of units.

EXAMPLES FOR PRACTICE.

249. Express the exact value of the following repetends

- | | | |
|-----------|----------------|-------------------|
| 1. .2222+ | 3. .232323+ | 5. .613613613+ |
| 2. .7777+ | 4. .105105105+ | 6. .201120112011+ |

REMARKS.—1. Limiting marks are sometimes used; as, 234234; they are, however, of no importance.

2. In business, final results are carried to three places, the fourth being rejected if less than one-half, but if one-half or more than one-half, 1 is added.

3. In interest rates or other multipliers, it is generally safest to use a common fractional equivalent.

ADDITION OF DECIMALS.

250. EXAMPLE.—Add .7, 2.43, .865, 11.5, 113.2075, and 200.00165.

OPERATION.

```

    .7
  2.43
   .865
  11.5
113.2075
200.00165
-----
328.70415

```

EXPLANATION.—Since by the decimal system numbers increase in value from right to left in ten-fold ratio, and the decimal point separates integral from fractional orders, observe to write decimals so that the *points* fall in the same vertical line, as units of the same order will thus fall in the same column; the result of the addition is then obtained in the same manner as in simple numbers.

REMARK.—As before shown, the decimals added could be reduced to a common denominator, but this being practically accomplished by the order in which they are written, the actual reduction by supplying ciphers is entirely unnecessary.

Rule.—Write the decimals so that the points will fall in the same vertical line. Add as in whole numbers, and place the point in the sum directly below the points in the numbers added.

EXAMPLES FOR PRACTICE.

- 251.** 1. Add 4, .37, 2.46, 19.301, and 103.21.
 2. Add 3.04, 25.001, .67, .2146, and 819.256.
 3. Add 30.1257, 605.2146, 1000.864532, and 16.25694.
 4. Add 896.111, 9530.216753, 1111.230004, and 1100.960005.
 5. Add 265.4203, 1129.000111, 8.005, .0060008, and 1200.12000014.
 6. Add 8046.0012, 250.0000001, 311.00555, and 81.0081001.
 7. Add 11000.4604, 7652.0000004, 5000.500005, and 365.50053004.

8. Add 14.0000864, .0096, 250.4, 700.0007, 1000.00000001, 563.3001468, 20.2001, 10000.001001 and 896.707075.

9. Find the sum of seventeen and forty-six ten-thousandths, eighty-three and one thousand four millionths, five hundred two and seventy-five hundred-thousandths, three thousand eleven and three hundred eleven thousandths, one million six hundred six and one ten-thousandths.

10. Add fifty-six thousand twelve and one thousand twenty millionths, six and ninety-seven million five billionths, one thousand five hundred seventy-nine and twenty-six thousand twenty-one hundred-thousandths.

11. Add one and one thousandths, ten and eleven hundred-thousandths, one hundred ten and nine millionths, eleven hundred eleven and ninety-nine billionths, one thousand eight hundred ninety and ninety-seven hundred-billionths, seven millions and seven hundred-thousandths.

12. A farmer having 315.625 acres of land, added at different times by purchase, 505.85 acres, 115.75 acres, 469.2 acres and 220.9 acres. How many acres had he in all?

13. What is the sum of 16.5 acres, 21.125 acres, 86.0625 acres 111.45 acres, 216.05 acres, $37\frac{1}{2}$ acres, $426\frac{7}{16}$ acres, $89\frac{5}{8}$ acres, and $13\frac{3}{32}$ acres?

14. What is the number of bushels in ten bins of 93.625 bu., 111.025 bu., 306.005 bu., $81\frac{1}{2}$ bu., $193\frac{3}{8}$ bu., $200\frac{3}{4}$ bu., 300.0625 bu., $125\frac{1}{8}$ bu., $250\frac{1}{4}$ bu., and $136\frac{7}{16}$ bu. respectively?

15. I bought ten bales of cloth as follows: $32\frac{1}{4}$, $411\frac{1}{16}$, $39\frac{1}{32}$, $46\frac{1}{4}$, $29\frac{1}{2}$, $38\frac{1}{16}$, $43\frac{1}{8}$, $41\frac{3}{32}$, $421\frac{3}{8}$, and 40.625 yd. respectively. How many yards in my purchase?

REMARK.—In invoices of goods only *fourths* are usually counted, and these are written as follows: $3^1 = 3\frac{1}{4}$, $15^3 = 15\frac{3}{4}$, $12^2 = 12\frac{1}{2}$. By the omission of the denominator time is saved. In additions, find the sum of the small figures first as so many fourths, reduce to units and carry as in other addition.

16. Add 21^1 , 54^2 , 17^1 , 30^3 , 46^1 , 61^2 , 80^1 , 39^3 , and 24^2 .

17. Add 121^1 , 97^3 , 46^2 , 111^3 , 43 , 71^2 , 86^3 , 50^1 , 103^3 , 72^1 , 71^3 , and 50.

252. To Add Repetends.

REMARK.—In addition of repetends, bear in mind their equivalents; thus, in adding $.6\overline{4}$ to $.3\overline{4}$ remember that the value of the first is $\frac{2}{3}$, and of the second $\frac{1}{3}$; their sum is $\frac{3}{3}$, or 1. In all examples in addition of repetends, before beginning the operation, continue the repetends so that all have the same number of places, and in the right-hand column add each 9 as 10

EXAMPLES FOR PRACTICE.

253. Add

1. $.33333\overline{3} +$	2. $.11111\overline{4} +$	3. $.2222\overline{2} +$	4. $.561561561\overline{4} +$
$.17171\overline{7} +$	$.77777\overline{7} +$	$.3333\overline{3} +$	$.202202202\overline{2} +$
$.306306\overline{6} +$	$.46207\overline{7} +$	$.8787\overline{7} +$	$.333333333\overline{3} +$
<hr/>	$.55555\overline{5} +$	$.0101\overline{1} +$	$.504300542\overline{7} +$
$.811357\overline{7} +$	$.33333\overline{3} +$	$.3467\overline{7} +$	$.306306306\overline{6} +$

5. Find the sum of the following expressions: $105.333\overline{3} +$, $86.1919\overline{9} +$, $53.103103\overline{3} +$, $17.66\overline{6} +$, $204.77\overline{7} +$, $29.11\overline{1} +$, $815.201201\overline{2} +$ and $73.11081108\overline{8} +$.

6. Add $.66\overline{6} +$, $1.2121\overline{2} +$, $50.55\overline{5} +$, $89.99\overline{9} +$, $2046.33\overline{3} +$, $38.22\overline{2} +$, $106.77\overline{7} +$, $1593.44\overline{4} +$, $11.230230\overline{3} +$, $528.60916091\overline{9} +$ and $1102.300300\overline{0} +$.

SUBTRACTION OF DECIMALS.

254. EXAMPLE.—Subtract .17 from .56.

OPERATION.

.56

.17

— .39

EXPLANATION.—For reasons heretofore explained, place the subtrahend below the minuend, so that the decimal points shall fall in the same vertical line. Subtract as in simple numbers, and place the *point* in the remainder below the points in the terms above.

REMARKS.—1. In case the number of decimal places of the subtrahend be greater than those of the minuend, consider decimal ciphers as annexed to the minuend, and subtract as before.

2. Mixed decimals may be treated in the same manner.

Rule.—Write the terms in decimal order and subtract as in integers, placing the point in the remainder below the points in the other terms.

EXAMPLES FOR PRACTICE.

255. Subtract

- | | | |
|-----------------------|---------------------------|------------------------------|
| 1. .573 from .985. | 5. .754352 from 2.3. | 9. 24.6852 from 25. |
| 2. .13823 from .668. | 6. 46.2906 from 100.52. | 10. 286.111 from 500.000625. |
| 3. .8627 from 1.549. | 7. 3491.5 from 4246.1005. | 11. .09 from .900. |
| 4. 1.232 from 6.7584. | 8. .0001 from 10000.1. | 12. 250.98754 from 386.245. |

MULTIPLICATION OF DECIMALS.

256. EXAMPLE.—Multiply .17 by .5.

FIRST OPERATION.

.17 = $\frac{17}{100}$ (Com. frac'l form.)

.5 = $\frac{5}{10}$ “ “ “

$\frac{17}{100} \times \frac{5}{10} = \frac{85}{1000} = .085$.

EXPLANATION.—Write .17 as $\frac{17}{100}$ and .5 as $\frac{5}{10}$, and apply the rule for multiplication of common fractions. Multiplying these fractional equivalents, obtain $\frac{85}{1000}$ as the common fractional expression of the product; by Art. 245, this may be written .085 as the decimal expression of the product, or the product required.

REMARK.—Observe that the denominator of the product is, as in other fractions, the product of the denominators; also that the denominator of the multiplicand contains *two* ciphers, or two places, and that of the multiplier *one* cipher, or one place; these taken together contain *three* ciphers, or three places, the same number of ciphers, or places, as are found in the product. Then by applying the theories of decimals already explained, the expression is changed to decimal form.

SECOND OPERATION.

.17

.5

— .085

EXPLANATION.—Write and multiply the expressions as in whole numbers. Since the numerator is 17 hundredths and the denominator 5 tenths, the product must be 85 thousandths. Hence, change the product 85 to 85 thousandths by prefixing a cipher and a decimal point, thus: .085.

Rule.—Multiply as in whole numbers; then, from the right of the product, point off for decimals a number of places equal to the number in both factors, prefixing ciphers if needed to obtain the required number.

EXAMPLES FOR PRACTICE.

257. Multiply

- | | | |
|-------------------|------------------------|-------------------------|
| 1. .78 by .7. | 5. 2085.109 by 11.256. | 9. 25000 by .000025. |
| 2. .123 by .16. | 6. 1000.87 by 4621.5. | 10. 8.76 by .100. |
| 3. 1.45 by .875. | 7. 10000 by .0001. | 11. 716.0025 by 10.1005 |
| 4. 26.08 by 1.53. | 8. 300 by .03. | 12. 7000 by .007. |

13. 1000000 by .0000001.

14. 3400000.0081 by 81.000034.

15. What will be the cost of 187.0625 acres of land at \$108.08 per acre?

16. I sold 14.4 bales of cloth of 61.625 yd. each, at \$.60½ per yd. How much did I receive?

17. What will be the cost of 5.75 cases of paper, the average weight of which is 403.625 pounds, at \$.40375 per pound?

18. From 10.85 acres of wheat a farmer harvested 31.875 bushels per acre, and sold his crop at \$.9725 per bushel. How much was received for the crop?

REMARKS.—1. The contraction of multiplication of decimals by restricting the number of places to appear in the product, is not deemed of sufficient practical importance to justify presentation.

2. As has been previously explained, decimal expressions, either *pure* or *mixed*, may be multiplied by 10 or by any power of 10, by removing the *point* as many places to the right as the multiplier contains ciphers. In such cases annex ciphers to the multiplicand if there is not already a sufficient number of decimal places.

DIVISION OF DECIMALS.

258. EXAMPLE.—Divide .085 by .17.

FIRST OPERATION.

$$.085 = \frac{85}{1000}$$

$$.17 = \frac{17}{100}$$

$$\frac{85}{1000} \div \frac{17}{100} =$$

$$\frac{85}{1000} \times \frac{100}{17} = \frac{5}{10} = .5.$$

SECOND OPERATION.

$$.17 \overline{) .085} \quad (.5$$

$$\begin{array}{r} 85 \\ \underline{00} \end{array}$$

EXPLANATION.—Since $.085 = \frac{85}{1000}$ and $.17 = \frac{17}{100}$, proceed as in Division of Common Fractions; that is, invert the terms of the divisor and multiply.

Observe now, that cancelling 17 and 100 from opposite terms of the fractional multiplicand and multiplier there is left only the factor 5 for the numerator and the factor 10 for the denominator of the quotient, or the fraction $\frac{5}{10} = .5$.

EXPLANATION.—Divide as in whole numbers. The dividend has 3 decimal places; the divisor has 2 decimal places; the dividend having one more decimal place than the divisor, point off one place from the right of the quotient.

REMARK.—It will be seen from the first operation that the number of decimal places of the divisor cancels, or offsets, the same number in the dividend. If the number of places in the terms be equal, it is obvious that the quotient will be a whole number.

Rule.—I. When needed, annex ciphers to the dividend to make its places equal in number to those of the divisor.

II. Divide as in integers, and, from the right of the quotient, point off for decimals as many places as the number of places in the dividend exceeds those in the divisor.

259. Decimals may be readily divided if, in connection with the above explanations, attention be given to the following

SUGGESTIONS.—1. Do not commence the division until the number of decimal places in the dividend is at least equal to the number of decimal places in the divisor. Supply any deficiency in the dividend by annexing ciphers.

2. If the divisor and dividend have the same number of decimal places, the quotient obtained, to the limit of the dividend as given, will be a whole number.

3. If the number of decimal places in the dividend be greater than the number of decimal places in the divisor, point off from the right of the quotient for decimals, a number of places equal to such excess, prefixing ciphers to the quotient if necessary.

4. If after division there be a remainder, ciphers may be annexed to it and the division continued to exactness, or to the discovery of a repetend, or to the two or three places ordinarily demanded in business computations. All such added ciphers should be considered as parts of the dividend.

REMARKS.—1. Inasmuch as the main difficulty experienced by pupils with decimals is found in division, and as that difficulty increases when the principles of decimals are applied to practice in percentage, it is advised that most thorough and repeated drill in division of decimals be given to all grades of pupils in all stages of class work.

2. From pleasant experience in teaching this subject, it is suggested that *ten* or more examples be grouped as a single exercise, and so arranged that the numerical quotient be the same for all. The pupil thus relieved from effort to determine this feature of the quotient, finds the requirement narrowed down to the *placing of the decimal point*, and soon fully masters all difficulty.

EXAMPLES FOR PRACTICE.

260. Divide

1. .625 by 2.5.
2. 15.25 by .05.
3. 1100 by 4.4.
4. 9.5 by 19.
5. 9.5 by 190.
6. .95 by .019.
7. 36.5 by .073.
8. 250 by .0625.

(25.)

- 1 \div 1 = ?
- 1 \div .1 = ?
- 1 \div .01 = ?
- 10 \div .1 = ?
- 10 \div .01 = ?
- .1 \div 1 = ?
- .1 \div .1 = ?
- .1 \div .01 = ?
- .1 \div .001 = ?
- .1 \div 10 = ?

9. 1750 by .875.
10. 3.6 by 1800.
11. .005 by 200.
12. 27.465 by .00015.
13. 1396.875 by 250.
14. 131300 by .025.
15. 62.5 by 1.25.
16. .00875 by 125.

(26.)

- 1 \div 10 = ?
- 1 \div 100 = ?
- .1 \div 1000 = ?
- .001 \div 100 = ?
- .0001 \div .1 = ?
- 100 \div .00001 = ?
- 1000 \div .01 = ?
- .00001 \div 1000 = ?
- 10 \div 100000 = ?
- 10000 \div .0001 = ?

17. 17.5 by 17500.
18. .44 by .00011.
19. 10000 by .00001.
20. .001 by 1000.
21. 1.6 by .064.
22. 6400 by .0000016.
23. .0081 by .054.
24. 1860 by .000031.

(27.)

- .22 \div 11 = ?
- 2.2 \div .011 = ?
- 220 \div 11000 = ?
- .022 \div 110 = ?
- .00022 \div 11000 = ?
- 2.2 \div .000011 = ?
- 2200 \div .00011 = ?
- .022 \div 110000 = ?
- .0000022 \div 1100000 = ?
- 220000 \div .000022 = ?

(28.)

$1.6 \div 2.5 = ?$
 $160 \div .25 = ?$
 $.0016 \div 250 = ?$
 $16 \div .00025 = ?$
 $160 \div 250000 = ?$
 $16000 \div .000025 = ?$
 $.0016 \div .00025 = ?$
 $.000016 \div 2500000 = ?$
 $1600 \div .00025 = ?$
 $1600000 \div .00000025 = ?$

Find the sum of the quotients.

(29.)

$6.25 \div 2.5 = ?$
 $62.5 \div .025 = ?$
 $6250 \div .0025 = ?$
 $.0625 \div 250 = ?$
 $.00625 \div .00025 = ?$
 $6.25 \div 25000 = ?$
 $.0000625 \div .00025 = ?$
 $625000 \div .0000025 = ?$
 $.0000625 \div 2500000 = ?$
 $625 \div .0000025 = ?$

Find the sum of the quotients.

(30.)

$2.5 \div 625 = ?$
 $.025 \div 62.5 = ?$
 $.0025 \div 6250 = ?$
 $.00025 \div .625 = ?$
 $.000025 \div .000625 = ?$
 $.0000025 \div 62500 = ?$
 $2500 \div .0625 = ?$
 $2500000 \div .0000625 = ?$
 $.00025 \div 6250 = ?$
 $.000025 \div 6250000 = ?$

Find the sum of the quotients.

(31.)

$440 \div 1.1 = ?$
 $.00044 \div 1100 = ?$
 $4400 \div .11 = ?$
 $440 \div .0011 = ?$
 $.0044 \div 110000 = ?$
 $44000000 \div 1100000 = ?$
 $4400000 \div .000011 = ?$
 $44000 \div .011 = ?$
 $.00000044 \div 110000 = ?$
 $4400 \div .00011 = ?$

Find the sum of the quotients.

(32.)

$.375 \div 1250 = ?$
 $375 \div .0125 = ?$
 $.0375 \div 12.5 = ?$
 $37.5 \div .000125 = ?$
 $37500 \div .00125 = ?$
 $3.75 \div 1250000 = ?$
 $.00375 \div 125000 = ?$
 $.0000375 \div .125 = ?$
 $3750000 \div .000125 = ?$
 $.000375 \div 12500 = ?$

Find the sum of the quotients.

(33.)

$2.25 \div .015 = ?$
 $225 \div 1500 = ?$
 $.0225 \div 150 = ?$
 $.00225 \div .015 = ?$
 $2250 \div .0015 = ?$
 $22500 \div 15000000 = ?$
 $.000225 \div .00015 = ?$
 $.0000225 \div 1500000 = ?$
 $2.25 \div .000015 = ?$
 $22500000 \div .00015 = ?$

Find the sum of the quotients.

REMARK.—Any decimal may be divided by 1 with any number of ciphers annexed, as 10, 100, 1000, 10000, by removing the *decimal point* as many places to the left as the divisor contains ciphers.

THE GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE OF FRACTIONS, COMMON AND DECIMAL.

261. All explanations given in finding either the Greatest Common Divisor or Least Common Multiple of integers apply equally to fractions, common or decimal.

262. To Find the Greatest Common Divisor of a set of Common Fractions.

EXAMPLE.—What is the Greatest Common Divisor of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$?

OPERATION.

$5 \overline{) 15 - 20 - 25}$
 $3 - 4 - 5$

EXPLANATION.—First reduce the given fractions to a common denominator and obtain as a result, $\frac{15}{30}$, $\frac{20}{30}$, $\frac{25}{30}$; then arrange the *numerators* of the resulting fractions in a horizontal line. Proceeding as by previous explanations find the Greatest Common Divisor of the numbers to be 5; but since these numbers are

numerators of fractions whose common denominator is 30, and 30 is the Least Common Multiple of this common denominator, the Greatest Common Divisor of the given fractions must be $5 \div 30$, or $\frac{5}{30} = \frac{1}{6}$. Notice that the numerator of the resulting $\frac{1}{6}$ is the Greatest Common Divisor of the numerators, and that the denominator 6 is the Least Common Multiple of the denominators, of the given fractions.

Rule.—Write a fraction the numerator of which shall be the Greatest Common Divisor of the numerators of the given fractions, and the denominator the Least Common Multiple of the denominators of the given fractions.

263. To Find the Least Common Multiple of a set of Common Fractions.

EXAMPLE.—Find the Least Common Multiple of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{10}$.

OPERATION.

$$2 \) \ 24 \text{---} 50 \text{---} 18$$

$$3 \) \ 12 \text{---} 25 \text{---} 9$$

$$4 \text{---} 25 \text{---} 3$$

EXPLANATION.—Reduce the given fractions to a common denominator as before, and obtain $\frac{24}{60}$, $\frac{50}{60}$, $\frac{18}{60}$; the Least Common Multiple of the numerators is found to be 1800; but the terms were not 24, 50, and 18, but $\frac{24}{3}$, $\frac{50}{5}$, and $\frac{18}{3}$, and 60 is the Greatest Common Divisor of 60, the common denominator, therefore the Least Common Multiple is not 1800, but $\frac{1800}{60}$, or 30 ; therefore 30 is the Least Common Multiple of the given fractions. Observe

that the numerator of $\frac{30}{1}$ is the Least Common Multiple of the numerators, and the denominator of the $\frac{30}{1}$ is the Greatest Common Divisor of the denominators, of the given fractions.

Rule.—Write a fraction the numerator of which shall be the Least Common Multiple of the numerators of the given fractions, and the denominator of which shall be the Greatest Common Divisor of the denominators of the given fractions.

264. To Find the Greatest Common Divisor of a set of Decimal Fractions.

EXAMPLE.—Find the Greatest Common Divisor of .5, .25, and .375.

OPERATION.

$$5 \) \ 500 \text{---} 250 \text{---} 375$$

$$5 \) \ 100 \text{---} 50 \text{---} 75$$

$$5 \) \ 20 \text{---} 10 \text{---} 15$$

$$4 \text{---} 2 \text{---} 3$$

EXPLANATION.—Reduce the expressions to equivalents having a common denominator, obtaining .500, .250, .375. For convenience omit the decimal points, find the Greatest Common Divisor of the numerators, and obtain 125. Since 500, 250, and 375 were not whole numbers, but .500, .250, and .375, the result is not 125, but .125.

Rule.—Reduce the expressions to the same decimal order, then write the Greatest Common Divisor of the expressions as a whole number, and make it of the decimal order common to all.

265. To Find the Least Common Multiple of a set of Decimal Fractions.

EXAMPLE.—Find the Least Common Multiple of .4, .72, and .416.

OPERATION.

$$4 \) \ 400 \text{---} 720 \text{---} 416$$

$$4 \) \ 100 \text{---} 180 \text{---} 104$$

$$5 \) \ 25 \text{---} 45 \text{---} 26$$

$$5 \text{---} 9 \text{---} 26$$

EXPLANATION.—Reduce to decimals of the same order, obtaining .400, .720, and .416; find the Least Common Multiple of the numerators, which is 93600. But since the expressions were not integers, but thousandths, the result is 93600 thousandths, or $93.600 = 93.6$, the Least Common Multiple.

Rule.—*Treat the expressions as integers and obtain their Least Common Multiple; then make it of the same decimal order as that one of the given decimals which has the greatest number of decimal places.*

REMARK.—These illustrations, when presented before a class, may properly be combined.

MISCELLANEOUS EXAMPLES IN DECIMALS.

266. 1. Add 51.01, 8.1006, 67.00102, 14.5, 1750.5072003, 100.0010041.
2. Add 137 thousandths, 41 hundredths, 13 millionths, 5011 ten-millionths, 608 ten-thousandths, 200600 hundred-millionths.
3. Reduce $\frac{13}{80}$ to a decimal fraction.
4. Reduce .015025 to a common fraction.
5. Divide 38.462 by 10000.
6. From 3006.01 take 889.01546.
7. From 540.123 take the sum of 81.625, 126.0972, 45.001, and 100.1002.
8. If $60\frac{1}{2}$ bushels of corn cost \$26.785, how much will 17.65 bushels cost.
9. Take the sum of nineteen millionths, five and two ten-thousandths, and sixty, from one hundred six and three tenths.
10. Multiply the sum of sixty-five and one hundred seven millionths, by the product of nine hundred millionths and one hundred twenty and seventeen hundredths.
11. From one billion take two billionths.
12. From six and fifty-hundredths take five and sixty hundredths.
13. Divide nine hundred sixteen and two thousand four millionths by sixteen ten thousandths.
14. Find the cost of 11.6 bales of cloth, each bale containing $61\frac{5}{8}$ yards, at \$1.54 per yard.
15. What is the cost of six barrels of sugar, weighing 301, 314, 297, 309, 313, and 315 pounds respectively, at $6\frac{7}{8}\phi$ per pound?
16. How many tons of phosphate, at \$34.88 per ton, will pay for 296.48 bushels of beans, at \$1.25 per bushel?
17. A contractor received \$354.06 for excavating a cellar, at 35¢ per cubic yard. How many yards of earth were removed?
18. If a wheelman travels 10.3 hours per day, how many days will be required for him to travel 558.0025 miles, at the rate of 7.88 miles per hour?
19. A teacher's salary is \$1500 per annum. If he pays \$650.50 for board, \$119.25 for books, \$31.85 for other literature, \$63.40 for charity, \$209.25 for clothes, \$109.90 for traveling expenses, and \$41.27 for incidental expenses, how much of his salary has he left?
20. I sold a lumberman 381.25 pounds of butter at \$.2875 per pound, 2468.375 pounds of cheese at \$.114 per pound, and 2356.5 pounds of dressed beef at $$.07\frac{1}{2}$ per pound, and received pay in lumber at \$23.125 per thousand feet. How many thousand feet of lumber should I have received?

UNITED STATES MONEY.

267. United States Money is the legal *currency* of the United States, adopted in 1786 and changed by various Acts of Congress since that date ; it is sometimes called *Federal Money*.

268. Money is the measure of value.

269. Legal Tender is the term applied to such money as may be legally offered in the payment of debts.

270. Bullion is pure gold or silver in bars, or ingots, and “bullion value” is the value of such metal, which varies from coin value only by the charges for coinage made by the mint.

271. Coin is the standard money of the mints, its value being established by law.

272. Currency is coin, treasury notes, bank-bills, or any substitute for money, in circulation as a medium of trade.

273. A Decimal Currency is a currency whose denominations increase and decrease by the decimal scale. United States money is a decimal currency.

274. The Dollar is the *unit* of United States money. Dollars are written as integers, with the sign (\$) prefixed ; the lower denominations are written as *decimals*, dimes being *tenths*, cents *hundredths*, and mills *thousandths* of a dollar. Thus, 15 dollars; 1 dime, 5 cents, 5 mills, is written \$15.155.

In business records and papers, *cents* are often written as fractions of a dollar ; the *half-cent* is expressed either as a fraction ($\frac{1}{2}$), or as 5 mills. Thus, \$15.75 may be written \$15 $\frac{75}{100}$; 12 $\frac{1}{2}$ cents, \$.12 $\frac{1}{2}$, or \$.125.

275. The denominations and scale of United States money are shown in the following

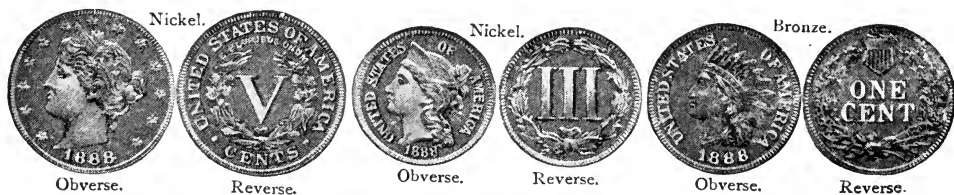
Table.

10 mills = 1 cent (c. or ct.).	10 dimes = 1 dollar (\$).
10 cents = 1 dime (d.).	10 dollars = 1 eagle (E.).

SCALE.—Descending, 10, 10, 10, 10. Ascending, 10, 10, 10, 10.

REMARKS.—1. The scale being a decimal one, all operations in United States money are performed the same as with common decimal expressions.

2. The *Dime* is a coin, but its name is never used in reading United States money. The *Mill* is not coined; it is used only as a decimal of the cent, which is the smallest money of the mint and the smallest recognized in business.



Coins of the United States.

UNITED STATES COINS.

276. The **Coins** of the United States, authorized by various Acts of Congress, are of *gold, silver, copper-nickel, and bronze.*

277. The **Gold Coins** of the United States are as follows.

1. The *Double Eagle*; value, \$20; weight, 516 Troy grains.
2. The *Eagle*; value, \$10; weight, 258 Troy grains.
3. The *Half-Eagle*; value, \$5; weight, 129 Troy grains.
4. The *Three Dollar* piece; value, \$3; weight, 77.4 Troy grains.
5. The *Quarter-Eagle*; value, \$2.50; weight, 64.5 Troy grains.
6. The *One Dollar* piece; value, \$1; weight, 25.8 Troy grains.

REMARKS.—1. All United States gold coins are made of $\frac{9}{10}$ pure gold, and $\frac{1}{10}$ alloy of copper and silver, the alloy being used to toughen the metal so as to reduce the loss from abrasion. The alloy used is never more than $\frac{1}{10}$ part silver.

2. United States gold coins of standard weight are legal tender for all debts.

278. The **Silver Coins** of the United States are as follows:

1. The *Dollar*; value, \$1.00; weight, 412.5 Troy grains.
2. The *Half-Dollar*; value, 50¢; weight, 192.9 Troy grains.
3. The *Quarter-Dollar*; value, 25¢; weight, 96.45 Troy grains.
4. The *Dime*; value, 10¢; weight, 38.58 Troy grains.

REMARKS.—1. The value of gold and silver coins is based mainly on their weight and fineness, or the amount of *pure metal* used. Silver coins are made of $\frac{9}{10}$ pure silver and $\frac{1}{10}$ alloy of copper.

2. United States silver dollars are *legal tender* for all sums not otherwise provided for by contract. The smaller silver coins are *legal tender* for all sums not exceeding ten dollars.

279. The **Copper-Nickel Coins** of the United States are as follows:

1. The *Five-Cent* piece, called the *nickel*; weight, 77.16 Troy grains.
2. The *Three-Cent* piece; weight, 30 Troy grains.

REMARK.—The 5¢ and 3¢ coins are composed of $\frac{2}{3}$ copper and $\frac{1}{3}$ Nickel.

280. The **Bronze Coin**.—The only bronze coin now issued from the mint is the one cent piece, weighing 48 Troy grains, and composed of $\frac{9.5}{100}$ copper and $\frac{5}{100}$ tin and zinc.

REMARK.—The 5¢ and 3¢ nickel coins, and the 1¢ bronze coin, are called *minor coins*; and while they are legal tender for all sums not exceeding twenty-five cents, their value is not a *bullion value*, as in case of coins of gold and silver, but an arbitrary value fixed for commercial convenience.

UNITED STATES PAPER MONEY.

281. The **Paper Money** of the United States consists of *Treasury Notes, Treasury Certificates, and National Bank Bills.*

282. **Silver Certificates**.—Any holder of silver dollars, to the amount of ten dollars or more, may deposit the same with the Treasurer or Assistant Treasurer of the United States and obtain therefor *Silver Certificates*, which are receivable for duties, taxes, and all public debts; and any holder of the smaller silver coins to the amount of twenty dollars, or any multiple thereof, may obtain therefor lawful money at the office of the Treasurer or of any Assistant Treasurer.

283. United States Treasury Notes.—Treasury Notes, or Greenbacks, are in the same denominations as the Bills of National Banks, with the addition of those of \$5,000 and \$10,000 value respectively. They are *legal tender* for all debts except customs or duties, and interest on the public debt, and are usually receivable for these also, being convertible into coin on demand when presented in sums of fifty dollars or more.

284. National Bank Bills.—National Bank Bills are the notes issued by National Banks, under the supervision of Government, and these bills are in denominations of \$5, \$10, \$20, \$50, \$100, \$500, and \$1000, and being secured by deposits of Government Bonds with the United States Treasurer, and redeemable on demand with lawful money, are usually received for all dues, but yet *are not legal tender*; and a debt cannot be paid with these notes if the creditor states as his reason for their rejection that they are not lawful money.

REDUCTION OF UNITED STATES MONEY.

285. To Reduce Dollars to Cents.

EXAMPLE.—Reduce 5 dollars to cents.

EXPLANATION.—Since there are 100 cents in 1 dollar, in 5 dollars there are 5 times 100 cents, or 500 cents.

Rule.—*Add two ciphers to the dollars.*

286. To Reduce Cents to Dollars.

EXAMPLE.—Reduce 1500 cents to dollars.

EXPLANATION.—Since 100 cents make 1 dollar, there are as many dollars in 1500 cents as 100 cents is contained times in 1500 cents, or 15 times, equal to 15 dollars.

Rule.—*Divide the cents by 100, by pointing off two places from the right.*

EXAMPLES FOR PRACTICE.

- | | | |
|---------------------------|----------------------------|------------------------|
| 287. Reduce | | |
| 1. 6 dollars to cents. | 5. 21468 cents to dollars. | 9. \$100.98 to cents. |
| 2. 111 dollars to cents. | 6. 1800 cents to dollars. | 10. \$3.75 to cents. |
| 3. 241 cents to dollars. | 7. 51000 cents to dollars. | 11. \$26.53 to cents. |
| 4. 1044 cents to dollars. | 8. 9876 cents to dollars. | 12. \$157.32 to cents. |

ADDITION AND SUBTRACTION OF UNITED STATES MONEY.

288. To Add or Subtract United States Money.

Rule.—*Write dollars under dollars and cents under cents; then add or subtract as in simple numbers.*

EXAMPLES FOR PRACTICE.

289. 1. Add ten dollars twenty cents, six dollars forty-eight cents, fourteen dollars twenty-six cents, eleven dollars eighty cents, and forty-six dollars ten cents.

2. Subtract seven hundred sixty-five dollars nineteen cents from nine hundred ten dollars eight cents.

3. A farmer sold produce as follows: wheat, for \$761.25; oats, \$382.40; barley, \$816.09; buckwheat, \$186.92; corn, \$1127.50; potatoes, \$663.11; hay, \$400.50. What were his entire sales?

4. A lady bought groceries to the amount of \$6.85; meats, \$2.11; dry goods, \$31.75; carpets, \$167.25; millinery, \$13.57. What was the total amount of her purchases?

5. A student expends for tuition and supplies, \$118.75; for board, \$167.50; for clothes, \$57.25; for entertainment and church, \$28.42; for charity, \$6.15. What amount does he expend?

6. The expenses of my house are as follows: for interest, \$167.50; taxes, \$103.29; repairs, \$56.82; insurance, \$11.35; water rent, \$11.25; and gas, \$27.08. What are my total expenses?

REMARKS.—1. Under some circumstances it is desirable to write United States money, expressed in dollars and cents, without the \$ sign and the decimal point, with the decimal part placed slightly above that expressing the integers or dollars; as \$5.25 may be written 5²⁵; thirteen dollars and eight cents may be written 13⁰⁸. This is advisable only where the sum of several items is to be found by horizontal addition.

2. The amount in each of the following examples is to be found by horizontal addition.

7. Add 15¹⁶, 2⁰⁷, 11⁴⁶, 107⁹⁰, 9²³, 81⁴⁵, 123⁹², 6⁰¹, 15³⁰, and 11¹¹.

8. Add 346⁵⁰, 291⁷⁵, 100³¹, 269¹¹, and 80⁹³.

9. What is the sum of 2165⁸⁴, 724³, 90²⁰, 117⁶⁵, 500⁵⁰, and 1127¹⁴?

10. What is the sum of 667⁴¹, 328¹⁰, 97⁷⁵, 600, 20⁶⁷, 155¹⁰, 1101, 28¹³, and 67⁵⁹?

11. My bills for a year are: for groceries, 283⁵¹; meats, 135¹¹; miller's products, 76⁷⁵; coal, 41²⁰; kindling, 4⁵⁰; milk, 47²⁵; servant, 217; incidentals, 91⁵⁴. What are my expenses?

12. A merchant bought cottons, for 3467²⁵; linens, for 1326¹⁵; woollens, for 4215⁷⁵; delaines, for 1025⁴⁵; brocades, for 1127⁵⁰. If all were sold for 13256²⁵, how much was gained?

MULTIPLICATION OF UNITED STATES MONEY.

290. To Multiply United States Money.

Rule.—*Multiply as in abstract decimals.*

REMARK.—*Money* is a concrete expression; therefore in critical analysis of its multiplication, the money *cost* or *price* of an article is a concrete multiplicand, the number of things bought or sold is an abstract multiplier, and their product is concrete and of the denomination of the multiplicand. But since the money scale is *decimal*, these terms may be interchanged for convenience.

EXAMPLES FOR PRACTICE.

291. 1. What will be the amount of the following purchases: 147 $\frac{3}{4}$ cd. hard wood, at \$5.75 per cd.; 206 $\frac{3}{4}$ cd. soft wood, at \$4.25 per cd.; 4 car loads slab wood, each containing 16 $\frac{3}{4}$ cd., at \$2.75 per cd.; 816 $\frac{1}{2}$ tons hard coal, at \$5.15 ton; and 536 $\frac{1}{4}$ tons soft coal, at \$3.85 per ton?

2. Bought 215 bar. superfine flour, at \$4.85 per bar.; 355 bar. extra flour at \$5.15 per bar.; 132 bar. rye flour, at \$4.90 per bar.; 210 bar. corn meal, at \$3.70 per bar.; and 642 sacks graham flour, at 88¢ per sack. What was the total cost?

3. A retailer bought 35 overcoats, at \$9.75 each; 160 black suits, at \$17.25 each; 125 plaid suits, at \$14.65 each; 84 jean suits, at \$6.90 each; and 50 pairs trousers, at \$3.15 each. Find the total cost.

4. An invoice of six pieces of gingham of 51^1 , 49^3 , 50^2 , 54^1 , 49^2 , and 51^2 yd. respectively, was sold at \$.09 $\frac{3}{4}$ per yd. What was the amount of the sale?

5. Six men worked $19\frac{3}{4}$ days each, at \$1.90 per day; $24\frac{1}{2}$ days each, at \$1.80, $11\frac{1}{4}$ days each, at \$1.65; and $31\frac{3}{4}$ days each, at \$1.25. How much was earned by all in the entire time?

6. A laborer received \$184.55 as a balance due him for his season's work. He paid a debt of \$19.25; bought 8^3 yd. cloth, at \$1.25 per yd.; 2 suits of clothes, at \$13.25 per suit; hosiery and gloves for \$2.85; $4\frac{3}{4}$ tons coal, at \$5.65 per ton; 2 cd. wood, at \$3.90 per cd.; 3 bar. flour, at \$4.75 per bar.; 628 pounds of pork, at $6\frac{3}{4}$ ¢ per lb.; and loaned the remainder of his money. How much did he loan?

DIVISION OF UNITED STATES MONEY.

292. To Divide United States Money.

Rule.—*Divide as in abstract decimals.*

EXAMPLES FOR PRACTICE.

293. 1. If \$11421.75 be divided equally among five persons, what will be the share of each?

2. B sold $187\frac{1}{2}$ acres of land at \$105.25 per acre, and divided the proceeds equally among fifteen persons. What sum did each receive?

3. A charitable farmer gave $15\frac{3}{4}$ bushels of apples worth \$.50 per bu., $21\frac{3}{4}$ bushels of potatoes worth \$.75 per bu., and 30 bushels of turnips worth \$.62 $\frac{1}{2}$ per bu., in equal shares to six families. What was the value of each share?

4. A dealer bought wheat at \$.95 per bu., oats at \$.45 per bu., and corn at \$.65 per bu. He paid \$332.50 for the wheat, \$191.25 for the oats, and \$113.75 for the corn. How many bushels did he buy in all?

5. C invested \$9659.50 in coal, at \$5.85 per ton; \$2645.30 in sand, at \$2.80 per cubic yd.; \$658.40 in lime, at \$1.60 per barrel. If he sold the coal at \$6.05 per ton, the sand at \$2.75 per cubic yd., and the lime at \$1.75 per barrel, what was the gain or loss?

6. Having sold my mill for \$17250, and 316 barrels of flour in stock at \$5.15 per barrel, I invested of the proceeds, \$1185.85 in furnishing a house, \$1259.30 in utensils, \$1582.25 in live stock, and with the remainder paid in full for a farm of 163 acres. What was the cost of the farm per acre?

REMARK.—In case exact quotients are not obtained in division of dollars, add two decimal ciphers and continue the quotient to cents; if not then exact add one cent if the mills be 5 or more, but if less than 5, reject the mills.

ANALYSIS.

294. Arithmetical Analysis is the process of solving problems independently of set rules, by deducing, from the terms stated, the conditions and relations required in their solution.

REMARK.—The general subject of Analysis will be treated only as auxiliary to the subject of Common Fractions, and the SPECIAL APPLICATIONS of the Fundamental Rules.

EXAMPLE 1. If 5 men earn \$30 in 4 days, how many dollars will 7 men earn in 9 days ?

FIRST EXPLANATION (*extended*).—If 5 men earn \$30 in 4 days, 1 man, or $\frac{1}{5}$ of 5 men will earn in 4 days $\frac{1}{5}$ of \$30, or \$6; and if 1 man earns \$6 in 4 days, in 1 day, which is $\frac{1}{4}$ of 4 days, he will earn $\frac{1}{4}$ of \$6, or \$1 $\frac{1}{2}$. Then, since 1 man in 1 day earns \$1 $\frac{1}{2}$, in 9 days, which are 9 times 1 day, he will earn 9 times \$1 $\frac{1}{2}$, or \$13 $\frac{1}{2}$; and if 1 man in 9 days earns \$13 $\frac{1}{2}$, 7 men, which are 7 times 1 man, will earn 7 times \$13 $\frac{1}{2}$, or \$94 $\frac{1}{2}$.

SECOND EXPLANATION (*abbreviated*).—If 5 men earn \$30 in 4 days, they will earn \$7 $\frac{1}{2}$ in 1 day; and if 5 men earn \$7 $\frac{1}{2}$ in 1 day, 1 man will earn $\frac{1}{5}$ of \$7 $\frac{1}{2}$, or \$1 $\frac{1}{2}$; since 1 man in 1 day earns \$1 $\frac{1}{2}$, 7 men in 1 day will earn 7 times \$1 $\frac{1}{2}$, or \$10 $\frac{1}{2}$; and if 7 men in 1 day earn \$10 $\frac{1}{2}$, in 9 days they will earn 9 times \$10 $\frac{1}{2}$, or \$94 $\frac{1}{2}$, the same as before found.

THIRD EXPLANATION (*more abbreviated*).—If 5 men in 4 days, doing 20 days' work, earn \$30, \$1 $\frac{1}{2}$ would equal 1 day's work; 7 men in 9 days do 63 days' work, and since 1 day's work equals \$1 $\frac{1}{2}$, 63 days' work will equal \$94 $\frac{1}{2}$, as before found.

EXAMPLE 2. If 6 men can cut 45 cords of wood in 3 days, how many cords can 8 men cut in 9 days ?

FIRST EXPLANATION (*extended*).—If 6 men cut 45 cd. in 3 days, in 1 day, which is $\frac{1}{3}$ of 3 days, they can cut $\frac{1}{3}$ of 45 cd., or 15 cd.; and if 6 men can in 1 day cut 15 cd., 1 man in 1 day can cut $\frac{1}{6}$ of 15 cd., or 2 $\frac{1}{2}$ cd.; since 1 man in 1 day can cut 2 $\frac{1}{2}$ cd., 8 men can in 1 day cut 8 times 2 $\frac{1}{2}$ cd., or 20 cd.; and if 8 men in 1 day can cut 20 cd., in 9 days they can cut 180 cd.

SECOND EXPLANATION (*abbreviated*).—6 men in 3 days, doing 18 days' work, cut 45 cd., hence 2 $\frac{1}{2}$ cd. can be cut by 1 man in 1 day; then 8 men in 9 days, doing 72 days' work, can cut 72 times 2 $\frac{1}{2}$ cd., or 180 cd., as before found.

EXAMPLE 3. If a post 4 ft. high casts a shadow 13 ft. in length, what must be the height of a post that will cast a shadow 125 ft. in length?

EXPLANATION.—If a post 4 ft. high casts a shadow 13 ft., a post 1 ft. high would cast a shadow 3 $\frac{1}{4}$ ft.; since a shadow 3 $\frac{1}{4}$ ft. is cast by a post 1 ft. high, a post that will cast a shadow 125 ft. in length must be as many times 1 ft. in height as 3 $\frac{1}{4}$ ft. are contained times in 125 ft., or 38 $\frac{1}{4}$ ft.

EXAMPLE 4. If the hour and minute hands of a clock are together at noon, at what times after noon will they again be together? At what time between 4 and 5 o'clock?

EXPLANATION.—Since the minute hand passes the hour hand 11 times in 12 hours, it will pass it the first time in $\frac{1}{11}$ of 12 hours; the second time in $\frac{2}{11}$ of 12 hours; the third time in $\frac{3}{11}$ of 12 hours; the fourth time in $\frac{4}{11}$ of 12 hours. $\frac{4}{11}$ of 12 hours equals 4 hours, 21 minutes, and $49\frac{1}{11}$ seconds; therefore the hands will be together between 4 and 5 o'clock at 21 minutes $49\frac{1}{11}$ seconds after 4 o'clock.

REMARK.—Apply the same reasoning to all examples of this class.

EXAMPLE 5. If Grace were $\frac{1}{5}$ older than she is, her age would equal $\frac{1}{4}$ of her grandmother's. What is the age of each, if the age of both is 87 years?

EXPLANATION.—If Grace were $\frac{1}{5}$ older than she is, she would be $\frac{6}{5}$ of her present age; and since if she were $\frac{6}{5}$ her present age, she would be only $\frac{1}{4}$ as old as her grandmother, the age of grandmother must be 4 times $\frac{6}{5}$ or $\frac{24}{5}$ of the age of Grace, and the age of both must be $\frac{6}{5} + \frac{24}{5}$ or $\frac{30}{5}$ of the age of Grace; since the age of both is 87 years, 87 years must be $\frac{30}{5}$ of the age of Grace, who must be 15 years old. If Grace's age be increased by $\frac{1}{5}$ of itself, or 3 years, she will be 18 years of age; and since her age would then be only $\frac{1}{4}$ of grandmother's age, the age of grandmother must be 4 times 18 years, or 72 years.

EXAMPLE 6. A man being asked his age, replied: "My father was born in 1805 and my mother in 1806; the sum of their ages at the time of my birth was two and one-third times my age in 1851." How old was the man in 1888?

EXPLANATION.—If the father was born in 1805 and the mother in 1806, the sum of their ages in 1851 was 91 years; and since the sum of their ages at the time of the birth of the son was $2\frac{1}{3}$ times his age in 1851, and the parents each increased in years after the son's birth as fast as he did, in 1851 the sum of their ages must have been $4\frac{1}{3}$ times the age of the son; hence the son, in 1851, was 91 years $\div 4\frac{1}{3}$, or 21 years of age, and he must have been born in 1830, and in 1888 would be 58 years old.

7. The sum of two numbers is 65, and their difference is equal to $\frac{1}{3}$ of the greater number. Find the two numbers.

8. How long after noon will it be when the minute hand passes the hour hand the third time?

9. How long after noon will it be when the minute hand passes the hour hand the eleventh time?

10. A's age is $2\frac{1}{3}$ times the age of B, and the age of C is $2\frac{1}{3}$ times the age of both A and B. If the sum of their ages is 116 years, what is the age of each?

11. A man bought 15 bushels of barley, and 36 bushels of oats, for \$28.80, and 25 bushels of barley, 18 bushels of oats, for \$29.10. How much per bushel did he give for each kind of grain?

12. Charles, when asked his age, replied: "My father was born in 1843, and my mother in 1847. The sum of their ages at the time of my birth was 5 times my age in 1887." In what year will Charles be 25 years of age?

SPECIAL APPLICATIONS.

295. Special Applications, as here treated, embraces the use, in the solution of problems, of any or all explanations heretofore given, and the consideration of *cost*, *price*, and *quantity*, as being the *elements* of every business transaction; it also treats of such contracted methods as may be employed in dealing with aliquot parts of the powers of 10, or of other numbers.

General Rules.—1. *If the price and quantity be given, the cost may be found by multiplying the price by the quantity.*

2. *If the cost and quantity be given, the price may be found by dividing the cost by the quantity.*

3. *If the cost and price be given, the quantity may be found by dividing the cost by the price.*

ALIUOT PARTS.

296. The **Aliquot Parts** of a number are the even parts of that number. 25, $33\frac{1}{3}$, $12\frac{1}{2}$, are aliquot, or even, parts of 100.

REMARK—The *component factors* of a number must be integral, while the *aliquot parts* of a number may be either integral or mixed.

297. The even parts of other even parts may be called *parts of parts*; as, $\frac{1}{3} = \frac{1}{3}$ of $\frac{1}{3}$; or, since $33\frac{1}{3}$ is a part of 100, $\frac{1}{3}$ of $33\frac{1}{3}$, or $11\frac{1}{3}$, must be a part of the part $33\frac{1}{3}$.

REMARK—Full illustrations of the use of aliquot parts will follow. Those of \$1, equal to 100¢, being the most valuable for use, will be mainly considered.

Aliquot Parts of One Dollar.

1. 50 cents = $\frac{1}{2}$ of \$1.	5. $16\frac{2}{3}$ cents = $\frac{1}{6}$ of \$1.	9. $6\frac{1}{4}$ cents = $\frac{1}{16}$ of \$1.
2. $33\frac{1}{3}$ cents = $\frac{1}{3}$ of \$1.	6. $12\frac{1}{2}$ cents = $\frac{1}{8}$ of \$1.	10. $3\frac{1}{3}$ cents = $\frac{1}{30}$ of \$1.
3. 25 cents = $\frac{1}{4}$ of \$1.	7. 10 cents = $\frac{1}{10}$ of \$1.	11. $2\frac{1}{2}$ cents = $\frac{1}{40}$ of \$1.
4. 20 cents = $\frac{1}{5}$ of \$1.	8. $8\frac{1}{3}$ cents = $\frac{1}{12}$ of \$1.	12. $1\frac{2}{3}$ cents = $\frac{1}{60}$ of \$1.

Aliquot Parts of Aliquot Parts of One Dollar.

$6\frac{1}{4}$ cents = $\frac{1}{4}$ of 25 cents.	5 cents = $\frac{1}{10}$ of 50 cents.	$12\frac{1}{2}$ cents = $\frac{1}{4}$ of 50 cents.
$12\frac{1}{2}$ cents = $\frac{1}{2}$ of 25 cents.	$6\frac{1}{4}$ cents = $\frac{1}{8}$ of 50 cents.	25 cents = $\frac{1}{2}$ of 50 cents.

SUGGESTION TO TEACHER.—Let each one of the following conditions be given to the class as a question, the required answer to which is the rule.

INSTRUCTIONS FOR PRACTICE WITH ALIQOT PARTS.

298. 1. To find the *cost* of a quantity when the *price* of 1 is 50 cents. RULE.—*Consider the quantity as dollars, and divide by 2.*

2. To find the *cost* when the *price* of 1 is $33\frac{1}{3}\phi$. RULE.—*Divide the quantity, considered as dollars, by 3.*

3. To find the *cost* when the *price* of 1 is 25ϕ . RULE.—*Divide the quantity, considered as dollars, by 4.*

4. To find the *cost* at 20ϕ . RULE.—*Divide the quantity, considered as dollars, by 5.*

5. To find the *cost* at $16\frac{2}{3}\phi$. RULE.—*Divide the quantity, considered as dollars, by 6.*

6. To find the *cost* at $12\frac{1}{2}\phi$. RULE.—*Divide the quantity, considered as dollars, by 8.*

7. To find the *cost* at $8\frac{1}{3}\phi$. RULE.—*Divide the quantity, considered as dollars, by 12.*

8. To find the *cost* at $6\frac{1}{4}\phi$. RULE.—*Divide the quantity, considered as dollars, by 16.*

9. To find the *cost* at 10ϕ . RULE.—*Point off from the right one place in the quantity, and consider as dollars.*

10. To find the *cost* at 5ϕ . RULE.—*Point off one place in the quantity, consider as dollars, and divide by 2.*

11. To find the *cost* at $3\frac{1}{3}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and divide by 3.*

12. To find the *cost* at $2\frac{1}{2}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and divide by 4.*

13. To find the *cost* at $1\frac{3}{4}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and divide by 6.*

14. To find the *cost* at $1\frac{1}{4}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and divide by 8.*

MISCELLANEOUS CONTRACTIONS.

299. 1. To find the *cost* when the *price* of 1 is 75 cents. RULE.—*From the quantity, considered as dollars, take $\frac{1}{4}$ of itself.*

2. To find the *cost* when the *price* of one is 80ϕ . RULE.—*From the quantity, considered as dollars, take $\frac{1}{5}$ of itself.*

3. To find the *cost* when the *price* of one is $66\frac{2}{3}\phi$. RULE.—*From the quantity, considered as dollars, take $\frac{1}{3}$ of itself.*

4. To find the *cost* when the *price* of one is \$1.25. RULE.—*To the quantity, considered as dollars, add $\frac{1}{4}$ of itself.*

5. To find the *cost* when the *price* of one is \$1.50. RULE.—*To the quantity, considered as dollars, add $\frac{1}{2}$ of itself.*

6. To find the *cost* when the *price* of one is \$2.50. RULE.—*Annex a cipher to the quantity, consider as dollars, and divide by 4.*

7. To find the *cost* when the price of one is \$7.50. RULE.—*Annex a cipher to the quantity, consider as dollars, and subtract $\frac{1}{4}$.*
8. To find the *cost* when the price of one is $6\frac{3}{4}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and subtract $\frac{1}{4}$.*
9. To find the *cost* when the price of one is $13\frac{1}{2}\phi$. RULE.—*Point off one place in the quantity, consider as dollars, and add $\frac{1}{4}$.*
10. To find the *cost* when the price of one is \$1.33 $\frac{1}{3}$. RULE.—*Add $\frac{1}{3}$ to the quantity, and consider as dollars.*
11. To find the *cost* when the price of one is \$1.10. RULE.—*Add $\frac{1}{10}$ to the quantity, and consider as dollars.*
12. To find the *cost* when the price of one is \$1.20. RULE.—*Add $\frac{1}{5}$ to the quantity, and consider as dollars.*
13. To find the *cost* when the price of one is \$1.35. RULE.—*Add $\frac{1}{4}$ and $\frac{1}{10}$ to the quantity, and consider as dollars.*
14. To find the *cost* when the price of one is \$1.75. RULE.—*Add $\frac{1}{2}$ and $\frac{1}{4}$ to the quantity, and consider as dollars.*
15. To find the *cost* when the price of one is \$3.33 $\frac{1}{3}$. RULE.—*Annex a cipher to the quantity, consider as dollars, and divide by 3.*

INSTRUCTIONS FOR FINDING QUANTITY.

300. 1. To find the *quantity* of articles that any given sum of money will purchase, when the price of one is 50 ϕ . RULE.—*Multiply the dollars, considered as quantity, by 2.*
2. To find the *quantity* when the price of one is 33 $\frac{1}{3}\phi$. RULE.—*Multiply the dollars, considered as quantity, by 3.*
3. To find the *quantity* when the price of one is 25 ϕ . RULE.—*Multiply the dollars, considered as quantity, by 4.*
4. To find the *quantity* when the price of one is 20 ϕ . RULE.—*Multiply the dollars, considered as quantity, by 5.*
5. To find the *quantity* when the price of one is 16 $\frac{2}{3}\phi$. RULE.—*Multiply the dollars, considered as quantity, by 6.*
6. To find the *quantity* when the price of one is 12 $\frac{1}{2}\phi$. RULE.—*Multiply the dollars, considered as quantity, by 8.*
7. To find the *quantity* when the price of one is 10 ϕ . RULE.—*Annex a cipher to the dollars, and consider as quantity.*
8. To find the *quantity* when the price of one is 8 $\frac{1}{3}\phi$. RULE.—*Multiply the dollars, considered as quantity, by 12.*
9. To find the *quantity* when the price of one is 6 $\frac{1}{2}\phi$. RULE.—*Multiply the dollars, considered as quantity, by 16.*
10. To find the *quantity* when the price of one is 5 ϕ . RULE.—*Annex a cipher to the dollars, consider as quantity, and multiply by 2.*
11. To find the *quantity* when the price of one is 3 $\frac{1}{3}\phi$. RULE.—*Annex a cipher to the dollars, consider as quantity, and multiply by 3.*

12. To find the *quantity* when the price of one is $2\frac{1}{2}\phi$. RULE.—*Annex a cipher to the dollars, consider as quantity, and multiply by 4.*

13. To find the *quantity* when the price of one is $1\frac{3}{4}\phi$. RULE.—*Annex a cipher to the dollars, consider as quantity, and multiply by 6.*

14. To find the *quantity* when the price of one is $1\frac{1}{4}\phi$. RULE.—*Annex a cipher to the dollars, consider as quantity, and multiply by 8.*

MISCELLANEOUS CONTRACTIONS.

301. 1. To find the *quantity* when the price of one is \$1.25. RULE.—*Point off one place in the dollars, consider as quantity, and multiply by 8.*

2. To find the *quantity* when the price of one is \$1.66 $\frac{2}{3}$. RULE.—*Point off one place in the dollars, consider as quantity, and multiply by 6.*

3. To find the *quantity* when the price of one is \$2.50. RULE.—*Point off one place in the dollars, consider as quantity, and multiply by 4.*

4. To find the *quantity* when the price of one is \$6.66 $\frac{2}{3}$. RULE.—*Point off one place in the dollars, consider as quantity, and add $\frac{1}{2}$.*

5. To find the *quantity* when the price of one is \$7.50. RULE.—*Point off one place in the dollars, consider as quantity, and add $\frac{1}{4}$.*

6. To find the *quantity* when the price of one is \$12.50. RULE.—*Point off two places in the dollars, consider as quantity, and multiply by 8.*

MISCELLANEOUS EXAMPLES IN FINDING QUANTITY.

302. EXAMPLE 1.—How many pounds of tea, worth 66 $\frac{2}{3}\phi$ per lb., can be bought for \$147?

EXPLANATION.—Since the price of one pound is contained $1\frac{1}{2}$ times in \$1, the number of pounds bought will be $1\frac{1}{2}$ times the number of dollars invested; hence, add to the number of dollars (as pounds) $\frac{1}{2}$ of itself, and the result will be the number of pounds purchased.

EXAMPLE 2.—How many pounds of tea, at 75 ϕ per lb., will \$419.25 purchase?

EXPLANATION.—Since the price of one pound is contained $1\frac{1}{3}$ times in \$1, the number of pounds bought will be $1\frac{1}{3}$ times the number of dollars invested; hence, add to the number of dollars $\frac{1}{3}$ of itself, and the result will represent the number of pounds purchased.

EXAMPLE 3.—How many pounds of tea, at 87 $\frac{1}{2}\phi$ per lb., can be bought for \$316?

EXPLANATION.—Since the cost of one pound is contained $1\frac{1}{4}$ times in \$1, the number of pounds purchased will be $\frac{1}{4}$ greater than the dollars invested.

EXAMPLE 4.—At 83 $\frac{1}{3}\phi$ per yard, how many yards of cloth can be bought for \$1128.50?

EXPLANATION.—Since the price of one yard is contained $1\frac{1}{2}$ times in \$1, we can buy $\frac{1}{2}$ more yards than we have dollars to invest.

EXAMPLE 5.—At 80 ϕ per yard, how many yards can be bought for \$246.25?

EXPLANATION.—Since \$1 will buy $1\frac{1}{4}$ yards, \$246.25 will buy 246.25 times $1\frac{1}{4}$ yards; hence, to the number of dollars add $\frac{1}{4}$ of itself.

REMARK.—The teacher can profitably extend these exercises for mental and written drill for pupils.

303. To Find the Cost when the Price is an Aliquot Part of a Dollar.

EXAMPLE.—Required, the cost of 546 gallons of molasses, at $33\frac{1}{3}\phi$ per gallon?

EXPLANATION.—Since $33\frac{1}{3}\phi$ is $\frac{1}{3}$ of \$1, 3 gallons would cost \$1; and if \$1 will buy 3 gallons, it will require as many dollars to buy 546 gallons as 3 is contained times in 546, or \$182.

Rule.—*Divide the quantity, considered as dollars, by the number of units of the quantity that will cost \$1.*

EXAMPLES FOR PRACTICE.

304. Find the cost of

- | | |
|---|--|
| 1. 286 lb. of tea, at 50ϕ per lb. | 7. 1238 cans of salmon, at 20ϕ per can. |
| 2. 1152 yd. of linen, at $33\frac{1}{3}\phi$ per yd. | 8. 756 $\frac{3}{4}$ gal. of cider, at $8\frac{1}{2}\phi$ per gal. |
| 3. 527 lb. of lard, at $12\frac{1}{2}\phi$ per lb. | 9. 81 lb. of meal, at $1\frac{1}{2}\phi$ per lb. |
| 4. 455 gal. of molasses, at 25ϕ per gal. | 10. 111 qt. of berries, at $3\frac{1}{3}\phi$ per qt. |
| 5. 1751 doz. of eggs, at $16\frac{2}{3}\phi$ per doz. | 11. 1354 $\frac{1}{4}$ yd. of cotton, at 5ϕ per yd. |
| 6. 2133 lb. of pork, at $6\frac{1}{4}\phi$ per lb. | 12. 840 lb. of salt, at $1\frac{1}{4}\phi$ per lb. |

REMARK.—In the following examples treat the amount of each item as a separate result, and consider 5 or more mills as 1ϕ .

13. Find the total cost of the following :

- | | | |
|--|---|--|
| 86 yd., at $12\frac{1}{2}\phi$ per yd. | 591 yd., at 10ϕ per yd. | 1600 yd., at $16\frac{2}{3}\phi$ per yd. |
| 93 yd., at $6\frac{1}{4}\phi$ per yd. | 327 yd., at $33\frac{1}{3}\phi$ per yd. | 71 yd., at 50ϕ per yd. |
| 150 yd., at 25ϕ per yd. | 1141 yd., at 25ϕ per yd. | 947 yd., at $3\frac{1}{3}\phi$ per yd. |

REMARK.—Quarters are often written thus: $5^s = 5\frac{1}{4}$; $17^s = 17\frac{1}{4}$; $11^s = 11\frac{1}{4}$; this method is not used with other fractions.

14. Find the total cost of the following :

- | | | |
|--|--|--|
| 83 ² yd., at $6\frac{1}{4}\phi$ per yd. | 127 ² yd., at $16\frac{2}{3}\phi$ per yd. | 2855 ¹ yd., at 20ϕ per yd. |
| 71 ³ yd., at $8\frac{1}{3}\phi$ per yd. | 1000 yd., at $1\frac{2}{3}\phi$ per yd. | 872 ² yd., at 50ϕ per yd. |
| 230 ¹ yd., at 25ϕ per yd. | 547 ³ yd., at 5ϕ per yd. | 624 yd., at $2\frac{1}{2}\phi$ per yd. |

305. To Find the Cost when the Price is given and the Quantity is a Multiple, or an Aliquot part, of 100 or 1000.

REMARK.—When the quantity is in even hundreds or thousands, find the cost by multiplying the price, expressed as a decimal, by the number of hundreds or thousands. For parts of hundreds or thousands, add equivalent fractional parts.

EXAMPLE 1.—Find the cost of 100 yards of cloth, at $54\frac{1}{4}\phi$ per yard.

EXPLANATION.—Since 1 yard costs \$.5425, 100 yards will cost 100 times as much, or \$54.25.

EXAMPLE 2.—Find the cost of 300 yards, at $17\frac{1}{2}\phi$ per yard.

EXPLANATION.—Since 1 yard costs \$.175, 100 yards will cost \$17.50, and 300 yards, or 3 times 100 yards, will cost 3 times \$17.50, or \$52.50.

EXAMPLE 3.—Find the cost of 1000 yards, at $83\frac{3}{4}\phi$ per yard.

EXPLANATION.—Since 1 yd. costs \$.8375, 1000 yd. will cost 1000 times \$.8375, or \$837.50.

EXAMPLE 4.—Find the cost of 75 yards, at $\$1.37\frac{1}{2}$ per yard.

EXPLANATION.—Since 1 yard costs $\$1.37\frac{1}{2}$, 100 yards will cost $\$137.50$; 75 yards will cost $\frac{3}{4}$ less than $\$137.50$, or $\$103.125$, or $\$103.13$.

EXAMPLE 5.—Find the cost of 250 yards, at $\$1.75$ per yard.

EXPLANATION.—Since 1 yard costs $\$1.75$, 1000 yards will cost $\$1750$; 250 yards, or $\frac{1}{4}$ of 1000 yards, will cost $\frac{1}{4}$ of $\$1750$, or $\$437.50$.

EXAMPLE 6. PARTS OF PARTS.—What will be the cost of 1426 bushels of wheat, at $\$1.37\frac{1}{2}$ per bushel?

EXPLANATION.—At $\$1$ per bushel, 1426 bushels will cost.....	$\$1426.00$
At $\frac{1}{4} = 25\phi$ per bushel, 1426 bushels will cost.....	356.50
At $\frac{1}{8} = (\frac{1}{2} \text{ of } \frac{1}{4})$, or $12\frac{1}{2}\phi$ per bushel, 1426 bushels will cost	178.25
At $\$1.37\frac{1}{2}$ per bushel, 1426 bushels will cost	$\$1960.75$

EXAMPLE 7.—What is the cost of 824 yards of cloth, at $\$1.75$ per yard?

EXPLANATION.—At $\$1$ per yard, 824 yards will cost	$\$824.00$
At $\frac{1}{2} = 50\phi$ per yard, 824 yards will cost.....	412.00
At $\frac{1}{4} = 25\phi$ per yard, 824 yards will cost	206.00
At $\$1.75$ per yard, 824 yards will cost	$\$1442.00$

EXAMPLE 8.—At 55ϕ per lb., what will be the cost of 146 lb. of gunpowder?

EXPLANATION.—At $\$1$ per lb., 146 lb. will cost.....	$\$146.00$
At $\frac{1}{2} = 50\phi$ per lb., 146 lb. will cost.....	73.00
At $\frac{1}{20} = (\frac{1}{10} \text{ of } \frac{1}{2})$, or 5ϕ per lb., 146 lb. will cost	7.30
At 55ϕ per lb., 146 lb. will cost.....	$\$80.30$

EXAMPLES FOR WRITTEN PRACTICE.

306. Find the cost of

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|--------------------------------------|---------------------------------------|---------------------------------------|
| 1. 200 lb., @ $37\frac{1}{2}\phi$. | 8. 400 lb., @ $95\frac{1}{2}\phi$. | 15. 300 lb., @ $41\frac{3}{4}\phi$. |
| 2. 700 lb., @ $51\frac{1}{4}\phi$. | 9. 250 lb., @ $9\frac{5}{8}\phi$. | 16. 3000 lb., @ $12\frac{1}{2}\phi$. |
| 3. 150 lb., @ $14\frac{3}{4}\phi$. | 10. 75 lb., @ $60\frac{3}{4}\phi$. | 17. 2500 lb., @ $\$1.10$. |
| 4. 250 lb., @ $21\frac{3}{8}\phi$. | 11. 125 lb., @ $27\frac{1}{8}\phi$. | 18. 25 lb., @ $\$1.85$. |
| 5. 1000 lb., @ $\$1.12\frac{1}{2}$. | 12. 1100 lb., @ $\$1.42\frac{1}{2}$. | 19. 150 lb., @ $33\frac{1}{2}\phi$. |
| 6. 750 lb. @ $81\frac{3}{4}\phi$. | 13. 500 lb., @ $37\frac{1}{2}\phi$. | 20. 75 lb., @ $\$1.15$. |
| 7. 1250 lb., @ $\$2.62\frac{1}{2}$. | 14. 1500 lb., @ $18\frac{3}{4}\phi$. | 21. 125 lb., @ $\$1.25$. |

EXAMPLES FOR MENTAL PRACTICE.

REMARK.—All extensions in the following examples should be made mentally, the pupil writing only the cost of each item for *footing*.

307. 1. Find the total cost of

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|---|---|--|
| 516 lb., at 10ϕ per lb. | 216 lb., at $12\frac{1}{2}\phi$ per lb. | 1095 lb., at $33\frac{1}{3}\phi$ per lb. |
| 484 lb., at 5ϕ per lb. | 1120 lb., at 50ϕ per lb. | 125 lb., at 6ϕ per lb. |
| 1000 lb., at $7\frac{1}{2}\phi$ per lb. | 818 lb., at 25ϕ per lb. | 711 lb., at 30ϕ per lb. |
| 2500 lb., at 8ϕ per lb. | 1400 lb., at 20ϕ per lb. | 97 lb., at $8\frac{1}{3}\phi$ per lb. |
| 3000 lb., at 11ϕ per lb. | 381 lb., at 40ϕ per lb. | 150 lb., at $6\frac{1}{4}\phi$ per lb. |

2. Find the total cost of

686 yd., at 15¢ per yd.	297 yd., at 25¢ per yd.	800 yd., at 13¢ per yd.
2140 yd., at 5¢ per yd.	1100 yd., at 43¢ per yd.	959 yd., at 16 $\frac{2}{3}$ ¢ per yd.
853 yd., at 10¢ per yd.	1200 yd., at 28 $\frac{1}{3}$ ¢ per yd.	1000 yd., at 19 $\frac{2}{3}$ ¢ per yd.
246 yd., at 20¢ per yd.	298 yd., at 50¢ per yd.	2000 yd., at 21 $\frac{2}{3}$ ¢ per yd.
398 yd., at 30¢ per yd.	931 yd., at 25¢ per yd.	606 yd., at 12 $\frac{1}{2}$ ¢ per yd.
450 yd., at 33 $\frac{1}{3}$ ¢ per yd.	1315 yd., at 33 $\frac{1}{3}$ ¢ per yd.	150 yd., at 25¢ per yd.

3. Find the total cost of

1400 lb., at 4¢ per lb.	936 ² lb., at 12 $\frac{1}{2}$ ¢ per lb.	291 lb., at 50¢ per lb.
2168 lb., at 3 $\frac{1}{3}$ ¢ per lb.	2143 ¹ lb., at 15¢ per lb.	1437 ² lb., at 25¢ per lb.
7000 lb., at 5¢ per lb.	546 lb., at 11¢ per lb.	1978 ³ lb., at 33 $\frac{1}{3}$ ¢ per lb.
246 ² lb., at 6 $\frac{1}{4}$ ¢ per lb.	2980 lb., at 16 $\frac{2}{3}$ ¢ per lb.	844 ¹ lb., at 75¢ per lb.
596 ³ lb., at 8 $\frac{3}{4}$ ¢ per lb.	593 lb., at 13¢ per lb.	930 lb., at 66 $\frac{2}{3}$ ¢ per lb.
1252 ¹ lb., at 10¢ per lb.	1250 ² lb., at 6 $\frac{1}{4}$ ¢ per lb.	675 ² lb., at 12 $\frac{1}{2}$ ¢ per lb.

4. Find the total cost of

525 ² yd., at 8¢ per yd.	367 yd., at \$1.25 per yd.	2100 yd., at 75¢ per yd.
1178 ¹ yd., at 9¢ per yd.	28 ² yd., at \$2.50 per yd.	146 yd., at 25¢ per yd.
2853 ³ yd., at 10¢ per yd.	577 ¹ yd., at 55¢ per yd.	500 yd., at \$.37 $\frac{1}{2}$ per yd.
1400 yd., at 6 $\frac{1}{2}$ ¢ per yd.	315 yd., at 75¢ per yd.	1000 yd., at \$1.87 $\frac{1}{2}$ per yd.

5. Find the total cost of

83 ² yd., at 55¢ per yd.	127 ² yd., at 16 $\frac{2}{3}$ ¢ per yd.	2005 ³ yd., at 6 $\frac{1}{4}$ ¢ per yd.
71 ³ yd., at 75¢ per yd.	500 yd., at 18 $\frac{3}{4}$ ¢ per yd.	1000 yd., at 83 $\frac{3}{4}$ ¢ per yd.
107 ¹ yd., at 50¢ per yd.	269 ³ yd., at 12 $\frac{1}{2}$ ¢ per yd.	250 yd., at 27 $\frac{1}{4}$ ¢ per yd.
230 ³ yd., at 66 $\frac{2}{3}$ ¢ per yd.	2960 ¹ yd., at 90¢ per yd.	931 ² yd., at 33 $\frac{1}{3}$ ¢ per yd.
1753 ² yd., at 25¢ per yd.	183 ² yd., at 8 $\frac{1}{3}$ ¢ per yd.	768 ³ yd., at \$1.25 per yd.
46 yd., at 15¢ per yd.	2375 ³ yd., at 10¢ per yd.	1756 ¹ yd., at \$1.12 $\frac{1}{2}$ per yd.

6. Find the total cost of

629 ¹ yd., at 3 $\frac{1}{3}$ ¢ per yd.	250 yd., at 13 $\frac{5}{8}$ ¢ per yd.	5548 ² yd., at 14¢ per yd.
1140 ² yd., at 5¢ per yd.	400 yd., at 15 $\frac{1}{2}$ ¢ per yd.	1250 yd., at 7 $\frac{1}{2}$ ¢ per yd.
594 ³ yd., at 6 $\frac{1}{4}$ ¢ per yd.	756 ³ yd., at 16 $\frac{2}{3}$ ¢ per yd.	300 yd., at 23 $\frac{3}{4}$ ¢ per yd.
3469 yd., at 8 $\frac{1}{3}$ ¢ per yd.	1375 ¹ yd., at 20¢ per yd.	500 yd., at 41 $\frac{5}{8}$ ¢ per yd.
1291 ² yd., at 11¢ per yd.	1741 yd., at 25¢ per yd.	186 yd., at 16¢ per yd.
593 ³ yd., at 12 $\frac{1}{2}$ ¢ per yd.	906 ³ yd., at 66 $\frac{2}{3}$ ¢ per yd.	175 yd., at 15¢ per yd.

308. To Find the Quantity when the Price is an Aliquot Part of \$1.

EXAMPLE 1.—If oats cost 33 $\frac{1}{3}$ ¢ per bushel, how many bushels can be bought for \$54 ?

EXPLANATION.—Since 1 bushel costs 33 $\frac{1}{3}$ ¢, or $\frac{1}{3}$ of \$1, 3 bushels can be bought for \$1 ; and if \$1 will buy 3 bushels, \$54 will buy 54 times 3 bushels, or 162 bushels.

EXAMPLE 2.—If a yard of cloth costs 66 $\frac{2}{3}$ ¢, how many yards will \$84 buy ?

EXPLANATION.—Since the price is $\frac{1}{2}$ of itself less than \$1 per yard, the number of yards will be $\frac{1}{2}$ greater than the number of dollars expended; $\frac{1}{2}$ of 84 = 42; 84 + 42 = 126, or 126 yards.

EXAMPLE 3.—At $87\frac{1}{2}\phi$ per bushel, how many bushels of wheat can be bought for \$1267?

EXPLANATION.—Since the price is $\frac{1}{2}$ of itself less than \$1 per bushel, the number of bushels will be $\frac{1}{2}$ greater than the number of dollars expended; $\frac{1}{2}$ of 1267 = 181; $181 + 1267 = 1448$, or 1448 bushels.

REMARK.—Application of the principle of reciprocals can profitably be introduced at this point; the reasoning will be the same as in the examples given above.

EXAMPLE 4.—At $66\frac{2}{3}\phi$ per yard, how many yards of cloth can be bought for \$84?

EXPLANATION.— $66\frac{2}{3}\phi = \$\frac{2}{3}$; write its reciprocal, $\frac{3}{2}$, and multiply by \$84.

EXAMPLE 5.—At 75ϕ per yard, how many yards of cloth can be bought for \$84?

EXPLANATION.— $75\phi = \$\frac{3}{4}$; write its reciprocal, $\frac{4}{3}$, and multiply by \$84.

EXAMPLE 6.—At $87\frac{1}{2}\phi$ per yard, how many yards of cloth can be bought for \$84?

EXPLANATION.— $87\frac{1}{2}\phi = \$\frac{7}{8}$; write its reciprocal, $\frac{8}{7}$, and multiply by \$84.

Rules.—1. *Multiply the cost by the quantity that can be bought for \$1.* Or,

2. *Add to the cost (as quantity) such a part of itself as the price lacks of being \$1.*

EXAMPLES FOR PRACTICE.

309. 1. If 1 lb. of candy can be bought for 25ϕ , how many pounds can be bought for \$5.75?

2. At $33\frac{1}{3}\phi$ per yard, how many yards of cloth will \$1542.50 buy?

3. A boy expended \$1 for almonds, at $16\frac{2}{3}\phi$ per lb. How many pounds did he buy?

4. At 75ϕ per yard, how many yards of cloth can be bought for \$572.40?

5. If I invest \$175.30 in eggs, at 20ϕ per doz., how many dozens do I purchase?

6. A farmer sold $26\frac{1}{2}$ bu. buckwheat, at $87\frac{1}{2}\phi$ per bu., and took his pay in sugar at $6\frac{1}{4}\phi$ per lb. How many pounds should he have received?

7. A gardener exchanged 132 qt. of berries, at $8\frac{1}{3}\phi$ per qt., and 75 doz. corn, at $12\frac{1}{2}\phi$ per doz., for cloth at 25ϕ per yd. How many yards did he receive?

8. If I exchange 1920 acres of wild land, at \$7.50 per acre, for an improved farm at \$125 per acre, what should be the number of acres in my farm?

9. A farmer gave $8\frac{1}{2}$ cwt. of pork, at \$7.50 per cwt., 15 bu. of beans, at \$3.25 per bu., and $46\frac{1}{2}$ bu. of oats, at $33\frac{1}{3}\phi$ per bu., for 28 yd. of dress silk, at \$1.25 per yd., and $52\frac{1}{2}$ yd. of delaine, at $16\frac{2}{3}\phi$ per yd., receiving for the remainder, cotton goods at $12\frac{1}{2}\phi$ per yd. How many yards of cotton goods should be delivered to him?

10. When potatoes are worth $66\frac{2}{3}\phi$ per bu., and turnips 25ϕ per bu., how many pounds of coffee, at $16\frac{2}{3}\phi$ per lb., will pay for 24 bu. of potatoes and 18 bu. of turnips?

11. Having bought 1487 lb. A. sugar, at $6\frac{1}{4}\phi$ per lb.; 872 lb. C. sugar, at 5ϕ per lb.; 628 $\frac{1}{2}$ lb. Y. H. tea, at $33\frac{1}{3}\phi$ per lb.; 522 lb. J. tea, at 25ϕ per lb.; 650 lb. Rio coffee, at $12\frac{1}{2}\phi$ per lb.; and 81 sacks of flour, at \$1.25 per sack, I give in payment seven one-hundred dollar bills. How much should be returned to me?

310. To find the Cost of Articles Sold by the C.

C stands for 100. M stands for 1000.

EXAMPLE.—What is the cost of 416 lb. phosphate, at \$2.00 per hundred?

EXPLANATION.—416 lbs. = 4.16 hundred lbs. If 1 hundred pounds cost \$2.00, 4.16 hundred lb. will cost 4.16 times \$2, or \$8.32.

Rule.—*Reduce the quantity to hundreds and decimals of a hundred, by pointing off two places from the right, then multiply by the price per C.*

EXAMPLES FOR PRACTICE.

311. Find the cost of .

- | | |
|---|---|
| 1. 1753 lb. of salt, at \$1.25 per C. | 6. 905 lb. of lead, at \$3.50 per C. |
| 2. 8425 lb. of scrap iron, at \$1.10 per C. | 7. 1125 lb. of castings, at \$2.25 per C. |
| 3. 2156 lb. of fence wire, at \$3.25 per C. | 8. 1620 handles, at \$5.50 per C. |
| 4. 378 fence posts, at \$7.50 per C. | 9. 509 lb. of beef, at \$12.50 per C. |
| 5. 3295 lb. of guano, at \$4.50 per C. | 10. 23765 lb. of nails, at 15¢ per C. |

312. To Find the Cost of Articles Sold by the M.

EXAMPLE.—At \$7.00 per M, what will be the cost of 1544 bricks?

EXPLANATION.—1544 bricks = 1.544 thousand bricks; and if one thousand bricks cost \$7, 1.544 thousand bricks will cost 1.544 times \$7, or \$10.808 = \$10.81.

Rule.—*Reduce the quantity to thousands and decimals of a thousand, by pointing off three places from the right, then multiply by the cost per M.*

EXAMPLES FOR PRACTICE.

313. 1. What will be the cost of 1650 ft. pine lumber, at \$15 per M?

2. What will be the cost of 611 ft. oak lumber, at \$24 per M?
3. What will be the cost of 21168 ft. hemlock lumber, at \$7.50 per M?
4. What will be the cost of 9475 ft. elm lumber, at \$13 per M?
5. What will be the cost of 2120 ft. ash lumber, at \$25 per M?
6. What will be the cost of 2768 ft. maple lumber, at \$14 per M?
7. What will be the cost of 1100 ft. chestnut lumber, at \$18 per M?
8. Find the cost of 4560 ft. oak lumber, at \$22 per M.
9. Find the cost of 11265 ft. spruce lumber, at \$12.50 per M.
10. Find the cost of 6625 shingles, at \$5.25 per M.

11. A dealer bought the season's cut of a saw mill, which was as follows: 326475 ft. clear pine, at \$25 per M; 1467250 ft. seconds, at \$17.50 per M; 102500 ft. culls, at \$13 per M; 890000 ft. hemlock boards, at \$10.50 per M; 824650 ft. hemlock timber, at \$9 per M; 552720 ft. white oak plank, at \$21 per M; 75690 ft. red oak plank, at \$16 per M; 101145 ft. cherry, at \$35 per M. What was the amount of the purchase?

12. For constructing a house and barn I bought: 46210 ft. matched pine, at \$21 per M; 13516 ft. siding, at \$28.50 per M; 11260 ft. chestnut, at \$32 per M; 4680 ft. black walnut, at \$45 per M; 928 ft. cherry, at \$86 per M; 33725 ft. hemlock timber, at \$11 per M; 58660 shingles, at \$6.25 per M; 13700 brick, at 5.50 per M. What was the total cost?

314. To find the Cost of Articles Sold by the Short Ton, or Ton of 2000 lb.

EXAMPLE.—What will be the cost of 3108 lb. of coal, at \$6 per ton?

EXPLANATION.—3108 lb. = 3.108 thousand lb.; since 1 ton, or 2000 lb., cost \$6, $\frac{1}{2}$ ton, or 1000 lb., will cost $\frac{1}{2}$ of \$6, or \$3; and if 1000 lb. cost \$3, 3.108 thousand lb. will cost 3.108 times \$3, or \$9.324, or \$9.32.

Rule.—Divide the price of one ton by 2, and the result will be the price per 1000 lb. From the right of the quantity point off 3 places, thus reducing it to thousands and decimals of a thousand. Multiply by the price per 1000 lb.

EXAMPLES FOR PRACTICE.

- 315.** 1. At \$3 per ton, what will be the cost of 2680 lb. soft coal?
 2. At \$7 per ton, what will be the cost of 1345 lb. cannel coal?
 3. At \$36 per ton, what will be the cost of 4372 lb. phosphate?
 4. At \$2.50 per ton, what will be the cost of 11075 lb. salt?
 5. At \$34.50 per ton, what will be the cost of 116780 lb. pig iron?
 6. At \$47.60 per ton, what will be the cost of 84725 lb. steel rails?
 7. At \$125 per ton, what will be the cost of 15066 lb. sheet copper?
 8. At \$4.50 per ton, what will be the cost of 9362 lb. land plaster?
 9. At \$2.10 per ton, what will be the cost of 2640 lb. slack lime?
 10. At \$35 per ton, what will be the cost of 1115 lb. guano?
 11. What will be the freight, at \$5 per ton, on four cars of Mdse. of 21780, 23055, 41200, and 32460 lb. weight respectively?
 12. At \$16.50 per ton, what will be the express charges on five boxes weighing respectively 186, 610, 241, 519, and 356 lb?
 13. My furnace consumed, in one year, six loads of hard coal, weighing respectively 4125, 3960, 4305, 4440, 4055, and 3775 lb. If the coal was bought at \$4.60 per ton, what did it cost to run the furnace?
 14. A dealer stocked his yard with 17500 tons of coal, as follows: 850 tons cannel, at \$7.40 per ton; 52600 lb. soft, at \$2.50 per ton; 193410 lb. of egg, at \$3.20 per ton, and the remainder chestnut, at \$3.60 per ton. What was the value of the dealer's stock?

316. To Find the Cost of Products of Varying Weights per Bushel.

EXAMPLE 1.—Required, the cost of 104 lb. of clover seed, at \$6.35 per bushel of 60 lb.

EXPLANATION.—At \$6.35 per lb., the cost would be 104 times \$6.35, or \$660.40; but since the price was not \$6.35 per lb., but \$6.35 per bu. of 60 lb., the cost will be $\frac{1}{6}$ of \$660.40, or \$11.006, or \$11.01.

EXAMPLE 2.—Required, the cost of 100 lb. of blue grass seed, at \$1.25 per bushel of 14 lb.

EXPLANATION.—At \$1.25 per lb. the cost would be \$125; but since the price was not \$1.25 per lb., but \$1.25 per bu. of 14 lb., the cost would be $\frac{1}{14}$ of \$125, or \$8.93.

Rule.—Multiply the number of pounds weight by the price per bushel, and divide the product by the number of pounds in 1 bushel.

REMARK.—Parts of bushels are often written in smaller figures at the right and above as pounds. Thus 1^{44} bu. clover seed = $1\frac{4}{5}$ bu. = 1 bu. 44 lb. = 104 lb. 21^{12} bu. oats = $21\frac{1}{2}$ bu. = 21 bu. 12 lb. = 682 lb. 119^{25} bu. corn = $119\frac{5}{8}$ bu. = 119 bu. 25 lb. = 7689 lb.

EXAMPLES FOR PRACTICE.

317. How much should be paid for a load of

1. Wheat, weighing 2142 lb., at \$.80 per bushel of 60 lb.
2. Corn, weighing 2506 lb., at \$.65 per bushel of 58 lb.
3. Barley, weighing 3381 lb., at \$.75 per bushel of 48 lb.
4. Millet, weighing 1768 lb., at \$1 per bushel of 45 lb.
5. Oats, weighing 2255 lb., at \$.35 per bushel of 32 lb.
6. Buckwheat, weighing 2172 lb., at \$.60 per bushel of 48 lb.
7. Beans, weighing 2761 lb., at \$1.25 per bushel of 62 lb.
8. Peas, weighing 2500 lb., at \$1.40 per bushel of 60 lb.
9. Hungarian grass seed, weighing 3146 lb., at \$2.50 per bushel of 45 lb.
10. Red top grass seed, weighing 2059 lb., at \$.90 per bushel of 14 lb.
11. Timothy seed, weighing 2677 lb., at \$2 per bushel of 44 lb.
12. Kentucky blue grass seed, weighing 2266 lb., at \$1.50 per bushel of 14 lb.
13. Clover seed, weighing 2941 lb., at \$5.10 per bushel of 45 lb.
14. Flax seed, weighing 2727 lb., at \$2.25 per bushel of 56 lb.
15. Castor beans, weighing 3050 lb., at \$3 per bushel of 46 lb.
16. Potatoes, weighing 2599 lb., at \$.65 per bushel of 60 lb.
17. Turnips, weighing 2160 lb., at \$.30 per bushel of 56 lb.
18. Apples, weighing 2701 lb., at \$.25 per bushel of 56 lb.
19. Sweet potatoes, weighing 3349 lb., at \$1 per bushel of 55 lb.
20. Onions, weighing 2021 lb., at \$.85 per bushel of 57 lb.
21. Rye, weighing 1367 lb., at \$.64 per bushel of 56 lb.

22. The products of a farm were ten loads each of wheat, barley, corn, oats, and potatoes. The wheat sold at \$1.12 per bushel of 60 lb., the barley at 85¢ per bushel of 48 lb., corn at 70¢ per bushel of 58 lb., oats at 32¢ per bushel of 32 lb., and potatoes at 62¢ per bushel of 60 lb. The loads of wheat weighed respectively 2585, 2640, 2721, 2594, 3063, 3354, 3145, 2720, 2938, and 2890 lb.; the barley 2163, 2487, 2225, 3004, 3121, 2742, 2907, 2525, 3140, and 3082 lb.; the corn 3100, 3126, 3097, 3040, 2872, 2950, 2777, 2981, 2547, and 2939 lb.; the oats 1973, 2946, 2172, 3148, 2500, 1951, 2631, 2997, 3005, and 2775 lb.; the potatoes 2846, 2891, 2805, 2863, 2984, 2901, 3046, 3280, 3395, and 2584 lb. How much was received from the five products?

REMARK.—Add each ten loads, and compute bushels but once for each product.

BILLS, STATEMENTS, AND INVENTORIES.

319. A **Bill** is a written statement in detail of articles sold or services rendered.

REMARK.—A Bill should state the names of both parties, the terms of credit, the name, quantity, and price of each item, and the entire amount. The Bill is said to be receipted when the words "Received Payment," or "Paid" and the creditor's signature, have been written at the bottom.

320. An **Invoice** is a written description of merchandise sold, or shipped to be sold on account of the shipper.

REMARK 1.—The terms Invoice and Bill are now used interchangeably; formerly the term Invoice was applied only to written statements of merchandise shipped to be sold for the owner.

2. An Invoice should bear the date of the sale or shipment, the special distinguishing marks, if any, upon the goods, the names of seller and buyer, or consignor and consignee, the items, prices, footing, discounts, if any, terms of sale, and manner of shipment.

321. A **Statement** is based upon *itemized* bills previously rendered, and is a written exhibit of the sum of the items charged in each of the bills, including also the dates on which the several bills were rendered.

322. An **Inventory** is an itemized *schedule* of the property possessed by an individual, firm, or corporation, and not shown by the regular books of account; or it may include all of the property possessed by an individual, firm, or corporation, such as book accounts, notes, cash, merchandise, etc., and also the debts due by the individual, firm, or corporation. This, however, is generally called a statement of the business.

REMARK.—An inventory is usually made upon the event of taking off a balance sheet, of a change in the business, of the admission of a partner, of the issue of stock, or, in case of embarrassment or insolvency, for examination by creditors, together with the other resources and liabilities of the business.

323. Contractions and Abbreviations used in Business.

<i>A1</i> First Quality.	<i>Ct.</i> Cent.	<i>E. & O. E.</i> Errors and
<i>Acct.</i> Account.	<i>Chgd.</i> Charged.	Omissions Excepted.
<i>Agt.</i> Agent.	<i>Co.</i> Company.	<i>Exch.</i> Exchange.
<i>Amt.</i> Amount.	<i>C. O. D.</i> Collect on	<i>Fol.</i> Folio or page.
<i>Bal.</i> Balance.	Delivery.	<i>Fr't.</i> Freight.
<i>Bbl. or Bar.</i> Barrel.	<i>Com.</i> Commission.	<i>Ft.</i> Foot.
<i>Bdl.</i> Bundle.	<i>Con.</i> Consignment.	<i>Gal.</i> Gallon.
<i>Blk.</i> Black.	<i>Cr.</i> Creditor.	<i>Gr.</i> Gross.
$\frac{b}{L}$ Bill of Lading.	<i>Cwt.</i> Hundred weight.	<i>Guar.</i> Guaranteed.
<i>Bot.</i> Bought.	<i>Dft.</i> Draft.	<i>Hhd.</i> Hogshead.
<i>Bro.</i> Brother.	<i>Dis.</i> Discount.	<i>i. e.</i> That is.
<i>Bu.</i> Bushel.	<i>Do. or ditto.</i> The same.	<i>In.</i> Inch.
<i>Bx.</i> Box.	<i>Doz.</i> Dozen.	<i>Ins.</i> Insurance.
<i>Cd.</i> Cord.	<i>Dr.</i> Debtor.	<i>Jr.</i> Junior.
ϕ Cent.	<i>Ea.</i> Each.	<i>Lb.</i> Pound.

<i>Mdse.</i> Merchandise.	<i>P. or p.</i> Page.	<i>Rec'd.</i> Received.
<i>Mem.</i> Memorandum.	<i>Pp. or pp.</i> Pages.	<i>Rec't.</i> Receipt.
<i>Messrs.</i> Gentlemen or Sirs.	<i>Pay't.</i> Payment.	<i>R. R.</i> Railroad.
<i>Mr.</i> Mister.	<i>Pd.</i> Paid.	<i>Schr.</i> Schooner.
<i>Mrs.</i> Mistress.	<i>Per.</i> By, or by the.	<i>Ship't.</i> Shipment.
<i>N. B.</i> Take notice.	<i>Pkg.</i> Package.	<i>Str.</i> Steamer.
<i>Net.</i> Without discount.	<i>P. O.</i> Post Office.	<i>Sunds.</i> Sundries.
<i>No.</i> Number.	<i>Pr.</i> Pair.	<i>Super.</i> Superfine.
<i>Oz.</i> Ounce.	<i>Pc.</i> Piece.	<i>Wt.</i> Weight.
	<i>Qr.</i> Quarter.	<i>Yd.</i> Yard.

REMARK.—In abbreviating measures of capacity, weight, distance, or time, it is unnecessary to add an *s* for the plural.

324. Time Abbreviations and Contractions used in Business.

<i>Jan.</i> or <i>Jan'y.</i> January.	<i>Nov.</i> November.	<i>Cent.</i> Century.
<i>Feb.</i> or <i>Feb'y.</i> February.	<i>Dec.</i> December.	<i>d.</i> Day.
<i>Mar.</i> March.	<i>Mo.</i> Month.	<i>h.</i> Hour.
<i>Apr.</i> April.	<i>Yr.</i> Year.	<i>m.</i> Minute.
<i>Aug.</i> August.	<i>Inst.</i> Present month.	<i>sec.</i> Second.
<i>Sept.</i> September.	<i>Prox.</i> Next month.	<i>wk.</i> Week.
<i>Oct.</i> October.	<i>Ult.</i> Last month.	

325. Signs and Symbols in Common Use.

@ At; as, at a price.	% Care of.	$\frac{n}{\%}$ New account.
# Number.	✓ Check mark.	$\frac{\%}{\%}$ Old account.
Ⓕ By, or by the.	% Per cent. or Hun-	X By, in surface
‰ Account.	dredths.	measures.

BILLS.

326. Find the footing of each of the following bills:

(1.)

KNOXVILLE, TENN., Dec. 31, 1888.

JOHN R. KNOX,
153 Pearl St., City,

Bought of CULVER & CASS.

3	sacks Cream Flour	95¢	2	85
2	bu. Potatoes	80¢	1	60
$\frac{1}{2}$	bu. Sweet Potatoes	90¢		45
2	lb. Ginger	22¢		44
2	lb. Jap. Tea	55¢	1	10
1	lb. O. H. Tea	75¢		75
2	gal. Syrup	45¢		90
$\frac{1}{2}$	bu. Onions	\$1		50
4	lb. Crackers	11¢		44

Paid,

CULVER & CASS,
Per CASS.

(2.)

Folio 246.

SAGINAW, MICH., Sept. 1, 1888.

MCGRAW & SAGE,

Tonawanda, N. Y.,

To WALLACE W. WESTON, Dr.

Terms, Sight Draft without notice after ninety days; 5% if paid within 60 days.

26416 ft. Clear Pine	28.00 per M.
146250 ft. Pine Plank	23.50 per M.
81275 ft. Clapboards	25.00 per M.
11670 Cedar Posts	7.00 per C.
71300 Shingles "A"	4.10 per M.
56200 ft. Pine Timber	21.00 per M.
111224 Cedar R. R. Ties	34.50 per C.
91050 ft. Flooring	27.50 per M.
25508 Shingles "B,"	3.60 per M.
31000 Barn Boards	15.75 per M.

*

(3.)

WORCESTER, MASS., May 15, 1888.

OLE PAULSEN & BRO.,

Detroit, Mich.,

Folio 41.

Sales Bk. 219.

To FRANK DRAKE & SON, Dr.

Terms cash.

Case.	No.	Yd	Price.	Items.	Amount.
# 19	15	Pieces Bleached Cotton, 41 ² 46 ³ 41 ¹ 45 ² 44 44 ¹ 47 ¹ 45 ³ 42 42 ³ 43 ³ 43 ¹ 47 44 44 ²	6 $\frac{3}{4}$ ¢		
# 5	12	Pieces Muslin, 37 ¹ 32 ³ 33 35 ³ 34 ¹ 32 35 ² 33 ³ 37 38 ¹ 38 ¹ 36	7¢		
# 31	9	Pieces Delainé, 39 40 ² 41 ¹ 39 ³ 38 ² 40 42 ³ 44 ¹ 42	16¢		
# 7	24	Pieces Windsor Prints, 21 ³ 27 ³ 25 ³ 28 26 22 ² 24 25 32 31 ² 28 24 ¹ 25 27 ² 22 28 ¹ 24 ¹ 22 21 ² 26 24 31 ² 32 22	5 $\frac{1}{4}$ ¢		
# 21	21	Pieces Merrimac Prints, 28 ¹ 32 34 ³ 28 ² 26 24 ¹ 22 ² 24 ² 26 ² 24 26 ¹ 33 28 ² 34 27 ¹ 30 32 ³ 24 30 ² 31 30 ²	4 $\frac{1}{4}$ ¢		

REMARK.—Any conditions as to time of credit, manner of payment, interest on balance, or discount for prepayment, are properly placed on a bill or statement.

* An M of shingles is equivalent to one thousand shingles averaging 4 inches in width.

(4.)

Book 3, Page 308.

CHICAGO, ILL., Aug. 1, 1888.

H. H. BARNES & Co.

Boston, Mass.,

Bought of PEASE & SONS.

Terms, Interest after sixty days.

							Pkg.				
6	baskets	Pork	Loins, net								
	312	301	297	315	302	313	8½¢	\$			
3	tubs	Lard,	71-14	70-15	69-14						
		pkg.	10¢	each			11¢				
4	casks	Shoulders,	428-68	419-70							
		423-65	432-72	pkg.	90¢	each	9½¢				
25	bar.	Mess	Pork				\$22.50				
20	casks	Hams,	395-67	412-71	402-71						
		411-67	408-68	425-71	400-69	399-70					
		398-71	426-68	419-69	423-69	407-67					
		415-75	418-68	409-71	403-71	421-71					
		428-68	400-78	pkg.	75¢	each	13½¢				

STATEMENTS.

327. Find the amount of each of the following statements :

(1.)

Folio 421.

BIRMINGHAM, ALA., Jan. 1, 1889.

RICHMOND & NEW ORLEANS RAILWAY Co.,

To CLIMAX FOUNDRY CO., Dr.

1888.		To Bill rendered							
Nov.	4	"	"	"	-----	590	25		
"	7	"	"	"	-----	375	13		
"	13	"	"	"	-----	1150			
"	18	"	"	"	-----	1560	25		
"	21	"	"	"	-----	2506	50		
"	25	"	"	"	-----	763	28		
"	29	"	"	"	-----	846	20		
"	30	"	"	"	-----	1000			
Dec.	3	"	"	"	-----	12750			
"	6	"	"	"	-----	2634	19		
"	7	"	"	"	-----	9374	75		
"	10	"	"	"	-----	871	03		
"	15	"	"	"	-----	767	20		
"	20	"	"	"	-----	8500			
"	21	"	"	"	-----	76	50		
"	22	"	"	"	-----	1438	10		
"	25	"	"	"	-----	119	93		
"	30	"	"	"	-----	1408	27		

Please remit.

INVENTORIES.

328. Find the amount of each of the following inventories :

(1.)

MERCHANDISE INVENTORY, JAN. 1, 1888.

8	pc. F. A. Cambric 56 52 45 50 52 54 46 50—405	22¢		
5	gr. Jet Buttons,	1.12½		
15	pc. P. D. Goods 55 45 ³ 55 ² 50 ³ 51 52 46 ¹ 50 52 ¹ 54 48 ² 50 ³ 52 55 ¹ 50	50¢		
4	pc. G. Flannel 35 ³ 40 40 ² 40 ³	25¢		
6	pc. E. Lining 40 52 ² 54 55 ¹ 45 ² 50 ²	3½¢		
10	pc. V. Barege 20 ¹ 25 23 ² 27 26 ³ 22 24 ² 22 26 ³ 28	16½¢		
10	pc. B. H. Checks 45 52 55 41 40 ² 51 ³ 51 ¹ 53 50 ² 46	24¢		
5	pc. W. Prints 25 ² 31 ³ 30 28 ² 27	5½¢		
15	pc. A. F. Cashmere 62 ¹ 65 ³ 60 ¹ 63 58 ³ 60 ² 56 ² 58 ² 60 62 ² 55 ³ 58 ¹ 60 ³ 58 55 ¹	19¢		
20	pc. L. Gingham, 45 48 ¹ 46 ¹ 44 ² 45 ³ 44 ³ 46 44 48 46 42 50 ² 51 ³ 46 ² 47 ¹ 46 ¹ 48 49 45 ¹ 48	8½¢		

(2.)

STARBUCK & MARTIN'S INVENTORY, JAN. 1, 1889.

Schedule A. (*Personal Property.*)

3	Delivery Horses, \$110, \$95, \$165,			
4	“ Express Wagons,	\$80		
3	“ Sleighs,	\$35		
4	sets Single Harness,	\$12.50		
	Robes, Blankets, and Whips,		15	
	Grocery stock, as by Schedule “G,”		13246	09
	Bills receivable, as by “ “ “H,”		7246	25
	Accts. “ “ “ “ “I,”		6242	10
	Fixtures in store, movable,		975	50

Schedule B. (*Real Estate.*)

7	Vacant Lots on Bank St.,	\$1250		
3	Houses on Clayton Pk.,			
	No. 12, 18, and 20,	\$2150		
	Warehouse on Canal,		13500	

MISCELLANEOUS EXAMPLES.

329. 1. Maurice H. Decker, bought of Silas Kingsbury & Co., Elmira, N. Y., July 5, 1888, 1760 ft. pine, at \$29 per M; 40 cedar posts, at \$12.50 per C; nails and hardware, \$6.21; 11248 ft. stringers, at \$4.75 per M. What was the amount of the bill?

2. Geo. W. Banning, bought of E. B. Henry & Co., Syracuse, N. Y., June 13, 1888, on account, 2 doz. carpet stretchers, at \$3; 10 grindstones, at \$2.25; 5 doz. steelyards, at \$9; 15 blacksmith drills, at \$7; 12 clothes wringers, at \$4.50; 6 doz. wrought wrenches, at \$12.25; 3 copying presses, at \$5; 7 doz. cow bells, at \$8.50; 15 doz. cast steel axes, at \$12. Find the amount of the bill.

3. Wm. J. Howard, bought for cash of Howe & Collins, carpet dealers, Rochester, N. Y., July 1, 1888, 100 yd. Moquette, at \$1.75; 250 yd. body Brussels, at \$1.50; 325 yd. tapestry Brussels, at \$1.00; 500 yd. 3-ply ingrain, at 75¢; 275 yd. 2-ply ingrain, at 65¢; 300 yd. matting, at 25¢; 200 yd. lining, at 12½¢. How much money was required to pay the bill?

4. Henry R. Smith, bought of O. L. Warren, Waverly, N. Y., Dec. 15, 1888, terms, 60 da.; 2% off, in 10 da.; 3 doz. Eagle wash boards, at \$1.75; 5 doz. Novelty wash boards, at \$2.25; 5 M. No. 4 paper bags, at \$1.75; 3 doz. butter bowls, at \$2; 5 doz. O. C. trays, at \$4; 1½ doz. feather dusters, at \$18; 10 gro. Gates' matches, at \$2.75; 15 broom racks, at \$2.25; 5 doz. wood shovels, at \$7.50; ⅝ doz. oil tanks, at \$16. What was the amount of the bill?

5. Jeffrey & Co., bought of Perry & Co., Buffalo, N. Y., Sept. 1, 1888:

10 pc. F. of L. cotton, 50 60 ¹ 65 ³ 51 60	10 pc. M. shirting, 40 41 46 34 51 45 50
55 52 62 61 56, at 8¢.	43 52 42, at 5¢.
5 doz. C. silk, at 80¢	15 pc. crash, 600 yd., at 5¢.
4 pc. A. F. cashmere, 62 ² 51 ³ 55 60, at 19¢.	6 pc. C. jeans, 50 ³ 45 ¹ 50 55 51 ² 46, at 5¢.
5 pc. A. L. L. cotton, 40 46 ² 51 ² 55 42 ¹ , at 4¢.	10 doz. M. L. thread, at 59¢.
500 lb. W. S. warp, at 15¢.	10 pc. R. print, 41 55 45 51 46 50 40 56 42 52, at 4½¢.

Find the amount of the bill.

6. W. C. Blanchard, bought of M. C. Wood, Utica, N. Y., July 15, 1888:

10 pc. R. gingham, 60 61 ² 50 ¹ 60 ³ 51 61 ³ 61 50 55 51 ³ , at 8¢.	4 pc. N. sateen, 55 ³ 55 50 60 ³ , at 5½¢.
10 doz. F. E. braid, at 23¢.	5 gross T. Braid, at \$7.62½.
10 pc. B. checks, 45 41 55 ¹ 42 52 40 ² 50 55 51 ³ 45 ² , at 24¢.	3 doz. L. shirts, at \$7.20.
15 gro. G. buttons, at \$1.12½.	6 pc. T. R. print, 25 35 30 ³ 31 21 25 ¹ , at 4¾¢.
2 pc. T. A. flannel, 65 60, at 30¢.	10 cases E. Batts, at \$6.00.
6 pc. E. lining, 40 55 ¹ 45 ² 52 41 50 ¹ , at 5¢.	20 gro. S. P. buttons, at 49¢.
5 doz. L. L. gloves, at \$3.05.	4 pc. V. barege, 20, 23 25 25, at 16¾¢.
	7 pc. W. Print, 45 ³ 51 45 50 46 ² 55 50 ² , at 5¼¢.

What was the amount of the bill?

7. I. F. Hoyt, bought of Mann & Moore, Sept. 4, 1888, terms 30 da.:

10 pc. N. sateen, 55 ² 51 50 ³ 54 ¹ 56 55 ¹ 52 ² 53 51 ³ 50, at 5½¢.	20 pc. R. Gingham, 50 52 ¹ 51 51 ² 55 60 ³ 62 ¹ 61 ² 58 55 ² 56 ¹ 53 ³ 51 55 ³ 61 ² 61 58 ¹ 56 54 ² 51 ¹ , at 6¼¢.
15 pc. T. A. flannel, 62 ³ 65 ¹ 61 58 ² 55 63 ¹ 65 ³ 62 60 ² 63 56 ³ 60 ¹ 58 62 ² 65 ¹ at 33½¢.	10 pc. B. checks, 45 52 ¹ 41 ² 40 55 ³ 50 ³ 45 51 ¹ 42 50 ³ , at 25¢.

What was the footing of the bill?

8. H. B. Smith, bought of Jones Bros. & Co., Dec. 3, 1888:

19 pc. M. gingham, 47 ² 36 41 ³ 49 ¹ 39 ³ 41 32 ³ 34 36 ¹ 42 ³ 46 35 ³ 38 ¹ 45 50 ¹ 48 ³ 33 ² 39 ¹ 36, at 11¢.	44 ¹ 44 ³ 51 40 ² 39 ² 37 ³ 35 38 ² 35 41 ³ 46 ³ 49 ² 38 ¹ 41 ³ 38 ² 36 ¹ , at 6⅔¢.
20 pc. P. B. sheeting, 32 ³ 32 ¹ 37 ² 40 ¹ Find the footing of the bill.	10 pc. B. D. velvet, 21 ² 27 ³ 25 26 ² 29 ³ 22 ² 24 ³ 21 20 ³ 23 ² , at \$6.50.

9. Drown Bros. & Co., bought of W. B. Adams & Co., for cash, June 18, 1888:

20 pc. L. gingham, 58 ² 46 ¹ 41 ³ 38 ¹ 46 ² 45 ³ 51 ² 55 38 ² 35 37 ³ 49 ³ 40 ² 51 ³ 44 ¹ 44 ² 40 37 ¹ 33 ³ 46 ² , at 8½¢.	37 ² 34 36 ² 42 ³ 48 43 ² 53 ¹ 38 ¹ 42, at 5½¢.
24 pc. W. print, 44 ¹ 46 ³ 51 ² 39 ³ 41 ² 45 48 ³ 51 34 ³ 37 ² 35 36 ² 41 ³ 34 ³ 49 ¹	20 pc. E. lining, 45 54 ¹ 39 ² 48 ³ 46 ² 38 ² 47 ¹ 37 ² 45 ³ 46 ³ 42 ⁴ 44 ³ 45 ³ 43 ¹ 35 ² 54 ² 34 ³ 42 ² 53 ² 44 ¹ , at 4¼¢.

What sum of money was required to pay the bill?

10. Find the amount of the following inventory:

25 pc. M. gingham, 46 ² 48 ² 39 ¹ 47 ² 41 ¹ 50 39 ³ 50 ³ 42 44 ² 36 ² 34, 36 ¹ 49 ² 40 ³ 41 ³ 39 ² 40 ¹ 49 ² 45 38 ³ 33 38 ² 46 ² 32 ¹ , at 10¼¢.	39 43 ² 47 ² 42 36 ² , at 8¢.
40 pc. L. gingham, 35 36 ² 32 ³ 41 ¹ 38 ¹ 40 ¹ 35 ³ 38 ² 46 48 ² 43 34 ³ 46 39 33 ¹ 37 ³ 34 ² 48 36 ² 32 38 ³ 47 ¹ 50 48 ² 41 ¹ 35 ¹ 39 42 ³ 44 41 ² 45 ¹ 48 43 ³ 36 33 ¹	15 pc. E. lining, 47 41 ³ 49 50 ² 46 45 ¹ 38 ³ 36 41 ² 38 ¹ 45 ³ 33 40 ² 39 ¹ 45, at 3½¢.
	10 pc. L. plaid, 46 ² 48 ¹ 38 ¹ 35 ³ 40 ¹ 38 ³ 41 32 ² 36 ³ 35, at 10¢.
	4 pc. C. denims, 39 ² 61 ¹ 48 ³ 36 ² at 12½¢.

DENOMINATE NUMBERS.

330. Denominate numbers may be either simple or compound.

331. A **Simple Denominate Number** is a unit or a collection of units of but one denomination.

332. A **Compound Denominate Number** is a *concrete number* expressed in two or more different denominations; as 5 lb. 4 oz. 12 dr.; 4 yr. 7 mo. 12 da.

REMARK.—*Compound denominate numbers* are sometimes called *compound numbers*.

333. **Compound Numbers** express divisions of time, and of the money, weights, and measures of the different countries.

REMARK.—Most denominate scales are varying, but the uniform decimal scale is used throughout the *metric system*, and, except in Great Britain, in the money of most civilized countries. The *units* of all denominate numbers are treated by the decimal scale.

334. A **Denominate Fraction** is a fraction expressing one or more of the equal parts of a *denominate* or *concrete* unit; as $\frac{3}{4}$ of a ton, $\frac{3}{4}$ of a yd., $\frac{4}{5}$ of a gal.

335. **Reduction of Denominate Numbers** is the process of changing them from one denomination to another, without altering their value. It is of two kinds, *Reduction Descending* and *Reduction Ascending*.

336. **Reduction Descending** is the process of changing a denominate number to an equivalent number of a *lower* denomination; as the change of barrels to an equivalent in gallons, quarts, pints, or gills.

337. **Reduction Ascending** is the process of changing a denominate number to an equivalent of a higher denomination; as the change of gills to an equivalent in pints, quarts, gallons, or barrels.

MEASURES OF TIME.

338. **Time** is the measure of *duration*; its computations, being based upon planetary movements, are the same in all lands and among all peoples.

339. The **Solar Day** is the *unit of time*; it includes one revolution of the earth on its axis, and is divided into 24 hours, counting from midnight to midnight again.

340. **Noon**, marked **M.** for *Meridian*, is that moment of time at which a line, called a *Meridian*, projected from the centre of the earth to the sun, would pass through the point of observation.

341. **A. M.** (*Ante-Meridian*) denotes the 12 hours before noon.

342. P. M. (*Post-Meridian*) denotes the time between noon and the following midnight.

REMARKS.—1. For astronomical calculations, the day begins at 12 o'clock noon, but for civil affairs, it begins at 12 o'clock midnight.

2. In banking business, the law fixes the end of the day at the hour appointed for closing the bank.

343. The Solar Year is the exact time required by the earth to make one complete revolution around the sun. It is equal to 365 days, 5 hours, 48 minutes, 49.7 seconds, nearly $365\frac{1}{4}$ days.

344. The Common Year consists of 365 days for 3 successive years; and every *fourth* year, except it be a centennial year, contains 366 days, one day being added for the excess of the solar year over 365 days; this day is added to the month of February, which then has 29 days, and the year is called *Leap Year*. The slight error still existing after this addition, is again corrected by excluding from the leap years the centennial years which are not divisible by 400. Thus 1900, 2100, 2200, while divisible by 4, are not divisible by 400, hence will not be leap years; while 2000, 2400, 2800, being divisible by 400, will be leap years.

REMARKS.—1. The correction last named was made by a decree of Pope Gregory XIII., in 1685, and is known as the Gregorian calendar. It is used in all civilized countries except Russia, and is so nearly correct that an error of one day will not be shown for 4000 years, hence it is practically correct.

2. The calendar in general use previous to 1685 was known as the Julian calendar, having been established by Julius Cæsar, 46 B. C. This calendar is still in use in Russia, and as the difference in the two calendars is now 12 days, the current date in Russia is 12 days behind that of the other civilized countries of the world; thus when it is Jan. 1 in Russia, it is Jan. 13 in all other countries.

3. The Julian and the Gregorian calendars are sometimes designated by the terms Old Style (O. S.), and New Style (N. S.)

345. Rule for Leap Years.—I. *All years divisible by 4, except centennial years, are leap years.*

II. *All centennial years divisible by 400 are leap years.*

Table.

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
4 weeks	= 1 lunar month	mo.
30 days	= 1 commercial month	mo.
365 days	= 1 common year	yr.
366 days	= 1 leap year	yr.
12 calendar months	= 1 civil year	yr.
10 years	= 1 decade	
100 years	= 1 century	C.

Scale, descending, 12, 30, 24, 60, 60; ascending, 60, 60, 24, 30, 12.

REMARK.—In most business transactions 30 days are considered a month, and twelve such months a year.

346. The Calendar Months are as follows:

1st. January (Jan.)	having 31 days.	7th. July (July)	having 31 days.
2nd. February (Feb.)	“ 28-29 “	8th. August (Aug.)	“ 31 “
3rd. March (Mar.)	“ 31 “	9th. September (Sept.)	“ 30 “
4th. April (Apr.)	“ 30 “	10th. October (Oct.)	“ 31 “
5th. May (May)	“ 31 “	11th. November (Nov.)	“ 30 “
6th. June (June)	“ 30 “	12th. December (Dec.)	“ 31 “

347. The year begins with the first day, or First, of January, and is divided into four *seasons* of three months each.

348. The **Seasons** are *Winter, Spring, Summer, and Autumn*, or *Fall*.

The Winter months are *December, January, and February*.

The Spring months are *March, April, and May*.

The Summer months are *June, July, and August*.

The Autumn months are *September, October, and November*.

REMARK.—The ancient Roman year began with March 1, and thus September, October, November, and December ranked, as their Latin derivation indicates, as the 7th, 8th, 9th, and 10th months respectively of the Roman year.

REDUCTION OF TIME.

349. The reduction of expressions of time from higher to lower denominations, or the reverse, may be accomplished in the same manner as the reduction of United States money heretofore explained, the only difference being that the scale in the latter is uniform, while that in the former is varying.

350. To Reduce Time from Higher to Lower Denominations.

EXAMPLE.—Reduce 3 yr. 7 mo. 11 da. 7 hr. 25 m. 38 sec. to seconds.

OPERATION.		EXPLANATION.—	
3 yr. 7 mo. 11 d. 7 hr. 25m. 38 sec.		Since one year equals 12 months, 3 years equal 36 months, and 7 months added gives 43 months; since one month equals 30 days, 43 months equal 1290 days, and 11 days added gives 1301 days; since one day equals 24 hours, 1301 days equal 31224 hours, and 7 hours added gives 31231 hours; since one hour equals 60 minutes, 31231 hours equal 1873860 minutes, and 25 minutes added gives 1873885 minutes; since one minute equals 60 seconds, 1873885 minutes equal 112433100 seconds, and 38 seconds added gives 112433138 seconds.	
12			
36 mo.			
7 mo.	OPERATION CONTINUED.		
	31231 hr.		
43 mo.	60		
30	1873860 m.		
1290 da.	25 m.		
11 da.	1873885 m.		
1301 da.	60		
24	112433100 sec.		
31224 hr.	38 sec.		
7 hr.	112433138 sec.		
31231 hr.			

REMARK.—The reduction descending of *any* compound denominate number can be accomplished as above, by observing the scale of the table to which it belongs.

Rule.—*Beginning with the highest, multiply the units of each denomination by the number in the scale required to reduce it to the denomination next lower; add the units, if any, of such lower denomination, and so continue from the given to the required denomination.*

351. To Reduce Time from Lower to Higher Denominations.

EXAMPLE.—Reduce 112433138 seconds to years.

OPERATION.

$$\begin{array}{r}
 60 \overline{) 112433138} \text{ sec.} \\
 60 \overline{) 1873885} \text{ min.} + 38 \text{ sec.} \\
 24 \overline{) 31231} \text{ hr.} + 25 \text{ min.} \\
 30 \overline{) 1301} \text{ da.} + 7 \text{ hr.} \\
 12 \overline{) 43} \text{ mo.} + 11 \text{ da.} \\
 3 \text{ yr.} + 7 \text{ mo.}
 \end{array}$$

$$112433138 \text{ sec.} = 3 \text{ yr. } 7 \text{ mo. } 11 \text{ da. } 7 \text{ hr. } 25 \text{ min. } 38 \text{ sec.}$$

EXPLANATION.—Divide the given seconds by 60, to reduce to minutes; the minutes thus obtained, by 60, to reduce to hours; the hours by 24, to reduce to days; the days by 30, to reduce to months, and the months by 12, to reduce to years.

Rule.—*Divide the given units by that number in the scale which will reduce them to units of the next higher denomination, and so continue from the given to the required denomination. Any remainder obtained will be of the same denomination as the dividend from which it arises.*

ADDITION OF TIME.

352. To Add Time.

Time expressions may be added as simple numbers, if only it be observed that the scale from the lowest to the highest order is 60, 60, 24, 30, and 12. The highest denomination in common use is the year.

EXAMPLE.—Add 41 yr. 8 mo. 22 da. 19 hr. 27 min. 14 sec., and 5 yr. 6 mo. 11 da. 10 hr. 50 min. 56 sec.

OPERATION.

$$\begin{array}{r}
 41 \text{ yr. } 8 \text{ mo. } 22 \text{ da. } 19 \text{ hr. } 27 \text{ min. } 14 \text{ sec.} \\
 5 \text{ yr. } 6 \text{ mo. } 11 \text{ da. } 10 \text{ hr. } 50 \text{ min. } 56 \text{ sec.} \\
 \hline
 47 \text{ yr. } 3 \text{ mo. } 4 \text{ da. } 6 \text{ hr. } 18 \text{ min. } 10 \text{ sec.}
 \end{array}$$

EXPLANATION.—Arrange the numbers so that those of the same denomination stand in the same vertical line. Then begin with the lowest denomination, which is seconds, and add: 14 seconds plus 56 seconds equals 70 seconds, equals 1 minute plus 10 seconds; write the 10 underneath the column of seconds, and carry the 1 to the next column; 27 minutes plus 50 minutes equals 77 minutes, and 77 minutes plus 1 minute (to carry) equals 78 minutes, equals 1 hour plus 18 minutes; write and carry as before; 19 hours plus 10 hours equals 29 hours, and 29 hours plus 1 hour (to carry) equals 30 hours, equals 1 day plus 6 hours; 22 days plus 11 days equals 33 days, and 33 days plus 1 day (to carry) equals 34 days, equals 1 month plus 4 days; 8 months plus 6 months equals 14 months, and 14 months plus 1 month (to carry) equals 15 months, equals 1 year plus 3 months; 41 years plus 5 years equals 46 years, and 46 years plus 1 year (to carry) equals 47 years.

Rule.—*Add as in abstract numbers, and reduce according to the table of Time.*

SUBTRACTION OF TIME.

353. Difference in time is found in two ways:

1st. By counting the actual number of days from the given to the required date. Thus, the number of days between May 13 and September 7 is 117, counting 18 days left in May, 30 for June, 31 for July, 31 for August, and the 7 of September.

2d. By Compound Subtraction. Subtraction in either simple or compound numbers is really the same, except that in the latter a varying scale is employed. That is, it may, and usually does, involve a transformation in either case. This will always be required unless the several minuend terms, or orders are each equal to or greater than the corresponding subtrahend term.

354. To Find the Difference in Time by Compound Subtraction.

EXAMPLE.—Subtract 5 yr. 4 mo. 21 da. from 8 yr. 1 mo. 18 da.

OPERATION.			EXPLANATION.—Write the numbers so that those of the same denomination stand in the same column. Then begin with the lowest denomination to subtract. Since 21 days cannot be subtracted from 18 days, transform, or borrow one from the next denomination; 1 month = 30 days, and 18 days added = 48 days; 48 days — 21 days = 27 days, which write underneath the column of days; the 1 month having been borrowed from the minuend, there are no months remaining from which to subtract the 4 months in the subtrahend, hence, borrow one from the next denomination; 12 months — 4 months = 8 months, which write underneath the column of months; there now remains 7 years from which to subtract; 7 years — 5 years = 2 years, which write underneath the column of years. This completes the operation, giving a remainder of 2 years, 8 months, and 27 days.
8 yr.	1 mo.	18 da.	
5 yr.	4 mo.	21 da.	
2 yr.	8 mo.	27 da.	

Rule.—*Subtract as in abstract numbers, observing the varying scale.*

EXAMPLES FOR PRACTICE.

REMARK.—In the following examples, the difference in time should be found by compound subtraction, unless it be otherwise stated.

- 355.** 1. Reduce 27051 seconds to minutes.
 2. Reduce 83129 seconds to hours and minutes.
 3. Reduce 610251 seconds to higher denominations.
 4. How many years, months, days, hours, and minutes, in 749520360 seconds?
 5. How many hours from half-past three o'clock P. M. Oct. 13, 1888, to noon on the fourth day of July, 1889?
 6. A note entitled to 93 days' time was dated Oct. 13, 1888. Counting actual time, on what day should it be paid?
 7. How many days between Nov. 3, 1890, and Mar. 1, 1900?
 8. A mortgage dated July 2, 1888, was paid Sept. 14, 1891. How many days did it run?
 9. How long does a note run if dated Sept. 22, 1887, and paid Aug. 31, 1888?
 10. How much time will a man gain for labor in 60 years, by rising 45 minutes earlier each day, beginning Jan. 1, 1888.

11. How many more minutes in the eleven years before Jan. 1, 1890, than in the eleven years after that date?
12. How many seconds of difference in the time of one solar year and 12 lunar months of 29 da. 12 hr. 44 min. and 3 sec. each?

CIRCULAR MEASURE.

356. Circular Measure is used in surveying, navigation, astronomy, and geography; for reckoning latitude and longitude, determining location of places and vessels, and in computing differences of time.

357. Every circle, great or small, is divisible into four equal parts; these parts are called *quadrants*, and are divisible into ninety equal parts, each of which is called a *degree*; every circle, therefore, may be divided into 360 equal parts, called degrees.

REMARK.—The divisions into twelfths called *signs*, and into sixths called *sexants*, are in occasional use.

Table.

60 seconds (") = 1 minute (').	30 degrees = 1 sign (S.)
60 minutes = 1 degree (°).	12 signs or 360° = 1 circle (C.)
Scale, { descending, 12, 30, 60, 60; or, 360, 60, 60.	
{ ascending, 60, 60, 30, 12; or, 60, 60, 360.	

REMARK.—Minutes of the earth's circumference are called nautical or geographic miles.

EXAMPLES FOR PRACTICE.

358. 1. Reduce 2154' to degrees.
2. Reduce 87406" to degrees, minutes, and seconds.
3. Reduce 330581" to higher denominations.
4. How many seconds in a circle?
5. How many minutes in 2 S. 21° 47'?
6. How many seconds in 1 S. 27° 8' 57"?
7. Reduce 8162 geographic miles to degrees.
8. How many geographic miles in the circumference of the earth?
9. By two different observations the position of a ship was shown to have changed 519 geographic miles. How much was her change in degrees and minutes?

LATITUDE, LONGITUDE, AND TIME.

359. Latitude is distance north or south from the equator. A place is said to be in *north latitude* if *north* of the equator; and to be in *south latitude* if *south* of the equator.

360. Longitude is distance east or west from any given starting point or meridian. A place is said to be in *west longitude* if *west* of the given meridian; and to be in *east longitude* if *east* of the given meridian.

361. Since every circle may be divided into 360 equal parts, or degrees, and the sun appears to pass from east to west around the earth, or through 360° of longitude, once in every 24 hours, it will pass through $\frac{1}{24}$ of 360° , or 15° of longitude, in 1 hour; through 1° of longitude in $\frac{1}{15}$ of 1 hour, or 4 minutes; and through $1'$ of longitude in $\frac{1}{60}$ of 4 minutes, or 4 seconds.

Table.

360° of longitude	=	24 hours or 1 day of time,	----	da.
15° “	=	1 hour of time,	-----	hr.
1° “	=	4 minutes “	-----	min.
1’ “	=	4 seconds “	-----	sec.

REMARK.—Standard Time.—Previous to 1883 there were fifty-three different time-standards in use by the railroads of the United States, and as these standards were based on the local time of the principal cities which served as the center of operations of the different roads, they were a constant source of annoyance and trouble, both to the railroads and to the traveling public. To obviate this difficulty the principal railroads of the United States and Canada adopted, in 1883, what is known as the “Standard Time System.” This system divides the United States and Canada into four sections or time-belts, each covering 15° of longitude, $7\frac{1}{2}^\circ$ of which are east and $7\frac{1}{2}^\circ$ west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt. The governing meridians are the 75th, the 90th, the 105th and the 120th west of Greenwich, and as these meridians are just 15° apart, there is a difference in time of *exactly* one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. The time of the 75th meridian, which is about 4 minutes slower than New York time and about 1 minute faster than Philadelphia time, is called “Eastern Time,” and when it is astronomical noon on this meridian it is noon on every railroad clock from Portland, Me., to Buffalo and Pittsburg, and from Quebec to Charleston. The time of the 90th meridian, one hour slower than “Eastern Time,” and 9 minutes slower than Chicago time, is known as “Central Time,” and all roads operated in the second belt are run by “Central Time.” The time of the 105th meridian, one hour slower than “Central Time,” is distinguished as “Mountain Time.” Time in the fourth belt, which is governed by the 120th meridian, and extends to the Pacific coast, is called “Pacific Time;” it is one hour behind “Mountain Time,” two behind “Central Time,” and three behind “Eastern Time.” The changes from one time-standard to another are made at the termini of roads, or at well-known points of departure, and where they are attended with the least inconvenience and danger. As this system has produced satisfactory results and has been adopted by most of the principal cities for local use, it is probable that the business of the whole country will, before many years, be regulated by standard railroad time.

362. To Find the Difference in Time, when the Difference in Longitude is given.

EXAMPLE.—If the difference in longitude of two places be $9^\circ 15'$, what must be their difference in time?

OPERATION.	EXPLANATION.
$9^\circ + 15'$	Since each minute of distance equals 4 seconds of time, 15 minutes of distance will equal 15 times 4 seconds, or 60 seconds, which equals one minute of time. And since each degree of distance equals 4 minutes of time, 9 degrees will equal 9 times 4 minutes, or 36 minutes; adding the one minute obtained above, gives 37 minutes as the required result.
<u>4</u>	
37 min. 0 sec.	

Rule.—*Multiply the units of distance by 4, and reduce according to the table of Time.*

EXAMPLES FOR PRACTICE.

REMARK.—Examples under this topic will be restricted to variations of solar time.

363. 1. Cincinnati is $84^{\circ} 24'$, and San Francisco 122° , west longitude. What is their difference in time?

2. New York is $74^{\circ} 1'$, and Halifax $63^{\circ} 36'$, west longitude. Find their difference in time.

3. St. Petersburg is $30^{\circ} 19'$ east, and St. Louis $90^{\circ} 15'$ west longitude. When it is noon at St. Petersburg, what is the time at St. Louis.

REMARK.—If one place be east and the other west of the given meridian, to find their difference in longitude, add their respective distances from the meridian taken.

4. The longitude of the City of Mexico is $99^{\circ} 5'$, and that of Boston $71^{\circ} 3'$, west longitude. Find their difference in time.

5. If on leaving London, $0^{\circ} 0'$ of longitude, my watch, keeping correct time, indicates 46 minutes, 15 seconds, after 3 P. M., what time should it indicate on my arrival at Astoria, Oregon, 124° west, where it is then noon?

364. To Find the Difference of Longitude, when the Difference in Time is Given.

EXAMPLE.—The difference in time between two places is 2 hours, 19 minutes, and 48 seconds. What is their difference of longitude?

OPERATION.

2 hr. 19 min. 48 sec. = 139 min. 48 sec.

4) 139 min. 48 sec.

$34^{\circ} + (3 \text{ min. } 48 \text{ sec.})$

3 min. 48 sec. = 228 sec.

4) 228 sec.

57'

2 hr. 19 min. 48 sec. = $34^{\circ} 57'$.

EXPLANATION.—2 hours, 19 minutes, and 48 seconds equal 139 minutes and 48 seconds; since each 4 minutes of time equal 1 degree of distance, 139 minutes and 48 seconds equal 34 degrees, with 3 minutes and 48 seconds, or 228 seconds, remainder; and since each 4 seconds of time equal $1'$ of distance, 228 seconds equal $57'$ of distance. Therefore, if the difference in time between two points be 2 hours, 19 minutes, and 48 seconds, their difference in longitude will be $34^{\circ} 57'$.

Rule.—*Reduce the difference in time to minutes and seconds, and divide by 4; the quotient will be the difference of longitude, in degrees, minutes, and seconds.*

EXAMPLES FOR PRACTICE.

365. 1. What is the difference in the longitude of New York and San Francisco, their difference of time being 3 hr. 11 min. 56 sec.

2. The longitude of Sitka is $135^{\circ} 18'$ west. What is the longitude of the city of Jerusalem if, when it is 9 o'clock and 5 minutes A. M. at Sitka, it is 27 minutes and 4 seconds after 8 P. M. in Jerusalem?

3. Find the difference in latitude of Chicago, situated $41^{\circ} 54'$ north, and Valparaiso, $33^{\circ} 4'$ south.

4. What is the latitude of Washington, if it be $61^{\circ} 46' 20''$ north of Rio Janeiro, and the latter place be $24^{\circ} 54'$ south latitude?

5. When it is $20\frac{1}{2}$ minutes after noon at Washington, it is 21 min. 26 sec before noon at Chicago, $87^{\circ} 30'$ west. What is the longitude of Washington?

MISCELLANEOUS EXAMPLES.

366. 1. A messenger leaves the Greenwich Observatory, westward bound, at noon, Dec. 31, and by a uniform rate of speed encircles the globe in 24 hours. Where is he at the end of the old year?

2. Suppose the messenger be eastward bound, at what point will he meet the new year?

3. When it is 20 minutes past 10 A. M. at Cape Horn, 68° west, what is the time at Cape of Good Hope, $18^{\circ} 19'$ east?

4. When it is noon at London, what is the time at St. Augustine, $81^{\circ} 35'$ west? At Berlin, $13^{\circ} 30'$ east? At New Orleans, 90° west? At Sidney, $152^{\circ} 20'$ east? At Paris, $20^{\circ} 20' 22\frac{1}{2}''$ east? At New York, $74^{\circ} 3'$ west?

ENGLISH MONEY.

367. English or Sterling Money is the legal currency of Great Britain.

Table.

4 farthings (far.)	=	1 penny d.
12 pence	=	1 shilling ... s.
20 shillings	=	{ 1 sovereign.. sov.
		{ 1 pound £.

Scale, ascending, 4, 12, 20; descending, 20, 12, 4.

368. The standard unit is the pound sterling, the value of which, in United States money, is shown, together with the other coins, in the following

Comparative Table.

The farthing = $\frac{1}{4}$ cent.		The shilling = $24\frac{1}{4}$ cents.
The penny = $2\frac{3}{4}$ cents.		The pound = \$4.8665.

REMARK.—The farthing is but little used, except as a fractional part of the penny.

COINS OF GREAT BRITAIN.

369. The gold coins are the sovereign and the half-sovereign.

The silver coins are the crown (equal to 5 shillings), the half-crown, the florin (equal to 2 shillings), the shilling, the six-penny and three-penny pieces.

The copper coins are the penny, the half-penny, and the farthing.

The guinea (equal to 21 shillings) and the half-guinea are in use, but are no longer coined.

REDUCTION OF ENGLISH MONEY.

370. To Reduce English Money from Lower to Higher Denominations.

EXAMPLE.—Reduce 13206 farthings to units of higher denominations.

OPERATION.	EXPLANATION.—
4) 13206 far.	Since 4 farthings equal one penny, 13206 farthings equal as many pence as 4 is contained times in 13206, or
12) 3301 d. + 2 far.	3301, plus 2 remainder, equal 3301 pence, 2 farthings; since 12 pence equal 1 shilling, 3301 pence equal 275 shillings, plus 1
20) 275 s. + 1 d.	penny; since 20 shillings equal 1 pound, 275 shillings equal 13
£13 + 15 s.	pounds, plus 15 shillings. Therefore, 13206 farthings equal £13,
13206 far. = £13, 15 s. 1 d. 2 far.	15 s. 1 d. 2 far.

Rule.—*Divide by the units in the scale from the given to the required denomination.*

EXAMPLES FOR PRACTICE.

371. Reduce

- | | |
|-----------------------------|---|
| 1. 5124 s. to pounds. | 3. 13042 d. to pounds. |
| 2. 11916 far. to shillings. | 4. 187409 far. to higher denominations. |

372. To Reduce English Money from Higher to Lower Denominations.

EXAMPLE.—How many farthings in £9, 4 s. 3 d. 2 far.?

OPERATION.

£9, 4 s. 3 d. 2 far.

20	OPERATION
180 s.	CONTINUED.
4 s.	2211 d.
184 s.	4
12	8844 far.
2208 d.	2 far.
3 d.	8846.
2211 d.	

EXPLANATION.—Since 1 pound equals 20 shillings, 9 pounds equal 180 shillings, and 180 shillings, plus 4 shillings, equal 184 shillings; since 1 shilling equals 12 pence, 184 shillings equal 2208 pence, and 2208 pence, plus 3 pence, equal 2211 pence; since 1 penny equals 4 farthings, 2211 pence equal 8844 farthings, and 8844 farthings, plus 2 farthings, equal 8846 farthings. Therefore, £9, 4 s. 3 d. 2 far. = 8846 far.

Rule.—*Multiply by the units in the scale from the given to the required denomination.*

EXAMPLES FOR PRACTICE.

373. 1. How many pence in £27?

2. How many farthings in 19 s. 11 d.?
3. How many pence in £161, 17 s. 9 d.?
4. Reduce £41, 1 s. 10 d. 2 far. to farthings.
5. How many farthings in £13, 15 s. 1 d. 2 far.?

374. To Reduce English Money to Equivalents in United States Currency.

EXAMPLE.—Reduce £15, 3 s. 7 d. 2 far. to dollars and cents.

FIRST EXPLANATION.—Since £1 equals \$4.8665, £15 equal \$72.9975; since 1 shilling equals 24½ cents, 3 shillings equal \$.73; since 1 penny equals 2½ cents, 7 pence equal \$.1414; since 1 farthing equals ⅙ cent, 2 farthings equal \$.0101. Therefore, £15, 3 s. 7 d. 2 far. = \$73.8789, or \$73.88.

SECOND EXPLANATION.—Call each 2 shillings $\frac{1}{10}$ of a pound, then 3 shillings equal £.15; call the pence and farthings, reduced to farthings, so many $\frac{1}{1000}$ of a pound, then 7 pence, plus 2 farthings, equal 30 farthings, equal £.030; to these add the £15, and the result is £15.18. And, since £1 equals \$4.8665, £15.18 equal 15.18 times \$4.8665, or \$73.88, as before found.

Rules.—1. *Multiply each of the orders of Sterling money by its equivalent in United States currency, and add the results.* Or,

2. *Reduce the Sterling expression to pounds and decimals of a pound by calling each 2 shillings $\frac{1}{10}$ of a pound, and the pence and farthings, reduced to farthings, so many $\frac{1}{1000}$ of a pound; multiply the pounds and decimals of a pound thus obtained by 4.8665, and the product will be the answer in dollars and cents.*

REMARK.—This is exact to within $\frac{1}{240}$ of the part represented by the pence and farthings.

EXAMPLES FOR PRACTICE.

375. Reduce to equivalents in United States money

1. £71, 19 s. 5 d. and 3 far.
2. £108, 11 d. and 1 far.
3. £13057, 10 s. and 4 d.
4. £3, 1 s. 9 d. and 2 far.
5. £11, 3 s. 1 d. 1 far.

376. To Reduce United States Money to Sterling equivalents.

EXAMPLE.—Reduce \$5164.28 to equivalents in English money.

OPERATION.

$$\begin{array}{r}
 4.8665 \) \ 5164.28 \\
 \hline
 £1061 + £.189 \text{ rem.} \\
 \quad \quad \quad 20 \\
 \quad \quad \quad \hline
 \quad 3 \text{ s.} + .78 \text{ s. rem.} \\
 \quad \quad \quad 12 \\
 \quad \quad \quad \hline
 \quad \quad 9 \text{ d.} + 36 \text{ d. rem.} \\
 \quad \quad \quad 4 \\
 \quad \quad \quad \hline
 \quad \quad 1 \text{ far.} + .44 \text{ far.}
 \end{array}$$

EXPLANATION.—Since \$4.8665 equal £1, \$5164.28 equal £1061.189; multiply the decimal by the units in the scale, 20, 12, 4, in order, pointing off as in decimals, and obtain 3 s. 9 d. 1 far., which, added to the £1061, equals £1061, 3 s. 9 d. 4 far.

Rule.—*Divide the expression of decimal currency by 4.8665, and the integers of the quotient will be pounds Sterling; reduce the decimal of the quotient, if any, by multiplying by the lower units in the scale.*

EXAMPLES FOR PRACTICE.

- 377.** 1. Reduce \$185 to equivalents in English money.
2. Reduce \$308.50 to equivalents in English money.
3. Reduce \$2500 to equivalents in English money.
4. Reduce \$3658.21 to equivalents in English money.
5. Reduce \$110085.75 to equivalents in English money.

MEASURES OF WEIGHT.

378. **Weight** is the measure of gravity, and is of three kinds, distinguished from each other by their uses, viz :

Troy weight, with units of pounds, ounces, pennyweights, and grains, used for weighing precious metals.

Avoirdupois weight, with units of tons, hundred weights, pounds, ounces, and drams, used for weighing products and general merchandise.

Apothecaries' weight, with units of pounds, ounces, drams, scruples, and grains, used by druggists.

TROY WEIGHT.

379. The **Troy pound** is the standard of weight, and is equal to 22.7944 cubic inches of pure water, at its greatest density. The grains of the other weights are the same as the Troy grains

Table.

24 grains (gr.) = 1 pennyweight.... pwt.

20 pennyweights = 1 ounce oz.

12 ounces = 1 pound..... lb.

Scale { descending, 12, 20, 24.
 { ascending, 24, 20, 12.

1 lb. = 5760 grains.

1 oz. = 480 grains.

REDUCTION OF DENOMINATE NUMBERS.

380. To Reduce Denominate Numbers from Higher to Lower Denominations.

EXAMPLE—Reduce 6 lb. 11 oz. 15 pwt. 21 gr., Troy, to grains.

OPERATION.

6 lb. 11 oz. 15 pwt. 21 gr.
 12
 72 oz.
 11 oz.
 83 oz.
 20
 1660 pwt.
 15 pwt.
 1675 pwt.
 24
 40200 gr.
 21 gr.
 40221 gr.

FIRST EXPLANATION.—Since 1 pound equals 12 ounces, 6 pounds equal 72 ounces, and 72 ounces plus 11 ounces equal 83 ounces; since 1 ounce equals 20 pennyweights, 83 ounces equal 1660 pennyweights, and 1660 pennyweights plus 15 pennyweights equal 1675 pennyweights; since 1 pennyweight equals 24 grains, 1675 pennyweights equal 40200 grains, plus 21 grains equal 40221 grains. Therefore, 6 lb. 11 oz. 15 pwt. 21 gr. Troy, = 40221 gr.

SECOND EXPLANATION.—Since 1 pound equals 5760 grains, 6 pounds equal 34560 grains; since 1 ounce equals 480 grains, 11 ounces equal 5280 grains; since 1 pennyweight equals 24 grains, 15 pennyweights equal 360 grains; to these add the 21 grains, and the entire sum is 40221 grains.

REMARK.—A thorough knowledge of the unit equivalents, together with readiness in the use of the multiplication table, renders the second form much the shorter of the two methods.

Rules.—1. *Multiply the units of the highest denomination given by that number in the scale which will reduce it to the denomination next lower, and add the units of that lower denomination; continue in this manner until the required denomination is reached. Or,*

2. *Multiply the units of each denomination by the number of units of the desired equivalent which it takes to make one of that denomination, and add the products thus obtained.*

381. To Reduce Denominate Numbers from Lower to Higher Denominations.

EXAMPLE.—Reduce 40221 gr., Troy, to higher denominations.

FIRST OPERATION.

$$\begin{array}{r} 24 \) \ 40221 \text{ gr.} \\ \underline{20 \) \ 1675 \text{ pwt.} + 21 \text{ gr.}} \\ 12 \) \ 83 \text{ oz.} + 15 \text{ pwt.} \\ \underline{\hspace{1.5cm}} \\ 6 \text{ lb.} + 11 \text{ oz.} \end{array}$$

40221 gr., Troy, = 6 lb. 11 oz. 15 pwt. 21 gr.

SECOND OPERATION.

$$\begin{array}{r} 5760 \) \ 40221 \text{ gr. (6 lb.} \\ \underline{34560} \\ 480 \) \ 5661 \text{ gr. (11 oz.} \\ \underline{5280} \\ 24 \) \ 381 \text{ gr. (15 pwt.} \\ \underline{360} \\ 21 \text{ gr.} \end{array}$$

40221 gr., Troy, = 6 lb. 11 oz. 15 pwt. 21 gr.

EXPLANATION.—Since 24 grains equal 1 pennyweight, 40221 grains equal 1675 pennyweights, plus 21 grains; since 20 pennyweights equal 1 ounce, 1675 pennyweights equal 83 ounces, plus 15 pennyweights; since 12 ounces equal 1 pound, 83 ounces equal 6 pounds, plus 11 ounces. Therefore, 40221 gr., Troy, = 6 lb. 11 oz. 15 pwt. 21 gr.

EXPLANATION.—Since 5760 grains equal 1 pound, 40221 grains equal 6 pounds, plus 5661 grains; since 480 grains equal 1 ounce, 5661 grains equal 11 ounces, plus 381 grains; since 24 grains equal 1 pennyweight 381 grains equal 15 pennyweights, plus 21 grains. Therefore, 40221 gr. = 6 lb. 11 oz. 15 pwt. 21 gr., as before found.

REMARK.—The first form is advised for practice, as the operations may usually be performed by short division.

EXAMPLE 2.—Reduce 11426 gr., Troy, to higher denominations.

EXPLANATION.—Divide the given number by 24, the integers of the quotient by 20, the integers of the new quotient by 12.

Rules.—1. *Divide by the successive units in the scale. Or,*

2. *Divide by the unit equivalents of each of the higher denominations.*

382. To Reduce Denominate Fractions from a Higher to a Lower Denomination.

EXAMPLE.—Reduce $\frac{7}{1880}$ lb., Troy, to the fraction of a pennyweight.

FIRST OPERATION.

$$\frac{7}{1880} \times \frac{1^2}{1} \times \frac{2^0}{1} = \frac{1680}{1880} = \frac{28}{31}.$$

SECOND OPERATION.

$$\frac{7}{1880} \times \frac{4^2}{1} \times \frac{3^3}{1} = \frac{28}{31} \text{ pwt.}$$

31

EXPLANATION.— $\frac{7}{1880}$ of a pound equals $\frac{7}{1880}$ of the 12 ounces in 1 pound, or $\frac{84}{1880}$ ounces; $\frac{84}{1880}$ of an ounce equals $\frac{84}{1880}$ of the 20 pennyweights in 1 ounce, or $\frac{1680}{1880}$, which equals $\frac{28}{31}$ pennyweights. Therefore, $\frac{7}{1880}$ lb., Troy, = $\frac{28}{31}$ pwt.

Rule.—*Multiply the fraction by the units in the scale, from the given to the required denomination.*

383. To Reduce a Denominate Fraction from a Lower to a Higher Denomination.

EXAMPLE.—Reduce $\frac{3}{8}$ of a grain to the fraction of a pound, Troy.

OPERATION.

$$\frac{3}{8} \times \frac{1}{24} \times \frac{1}{20} \times \frac{1}{12} = \frac{1}{14400}.$$

$$\frac{3}{8} \text{ gr.} = \frac{1}{14400} \text{ lb., Troy.}$$

EXPLANATION.— $\frac{3}{8}$ of a grain equals $\frac{3}{8}$ of $\frac{1}{24}$ of a pennyweight; $\frac{3}{8}$ of $\frac{1}{24}$ of a pennyweight equals $\frac{3}{8}$ of $\frac{1}{24}$ of $\frac{1}{20}$ of an ounce; $\frac{3}{8}$ of $\frac{1}{24}$ of $\frac{1}{20}$ of an ounce equals $\frac{3}{8}$ of $\frac{1}{24}$ of $\frac{1}{12}$ of a pound, or $\frac{1}{14400}$ of a pound.

Rule.—*Divide by the units in the scale, from the given to the required denomination.*

384. To Reduce Denominate Fractions to Integers of Lower Denominations.

EXAMPLE.—Reduce $\frac{3}{16}$ of a pound, Troy, to integers of lower denominations.

OPERATION.

$$\frac{3}{16} \times \frac{1}{2} = \frac{3}{8} \text{ oz.} = 2\frac{1}{4} \text{ oz.}$$

$$\frac{1}{4} \times \frac{5}{1} = 5 \text{ pwt.}$$

$$\frac{3}{16} \text{ lb., Troy,} = 2 \text{ oz. } 5 \text{ pwt.}$$

EXPLANATION.— $\frac{3}{16}$ of a pound equals $\frac{3}{16}$ of the 12 ounces in a pound, or $\frac{9}{8}$ ounces, which reduced gives $2\frac{1}{4}$ ounces; $\frac{1}{4}$ of an ounce equals $\frac{1}{4}$ of the 20 pennyweights in an ounce, or 5 pennyweights. Therefore, $\frac{3}{16}$ of a pound, Troy, equals 2 ounces, 5 pennyweights.

Rule.—*Multiply the denominate fraction by the unit next lower in the scale, and if the product be an improper fraction reduce it to a whole or mixed number.*

385. To Reduce a Compound Denominate Number to a Fraction of a Higher Denomination.

EXAMPLE.—Reduce 7 oz. 5 pwt. 9 gr. to the fraction of a pound, Troy.

OPERATION.

$$\begin{array}{r} 7 \text{ oz. } 5 \text{ pwt. } 9 \text{ gr.} \\ 20 \\ \hline 140 \text{ pwt.} \\ 5 \text{ pwt.} \\ \hline 145 \text{ pwt.} \\ 24 \\ \hline 3480 \text{ gr.} \\ 9 \text{ gr.} \\ \hline 3489 \text{ gr.} = \frac{3489}{34560} \text{ lb.} \end{array}$$

FIRST EXPLANATION.—Since 1 ounce equals 20 pennyweights, 7 ounces equal 140 pennyweights; 140 pennyweights plus 5 pennyweights equals 145 pennyweights; since 1 pennyweight equals 24 grains 145 pennyweights equal 3480 grains; 3480 grains plus 9 grains equals 3489 grains; since 1 pound equals 5760 grains, 3489 grains equal $\frac{3489}{5760}$ of a pound.

SECOND EXPLANATION.—Since 1 ounce equals 480 grains, 7 ounces equal 3360 grains; since 1 pennyweight equals 24 grains, 5 pennyweights equal 120 grains; 3360 grains, plus 120 grains, plus 9 grains equal 3489 grains; since 1 pound, Troy, equals 5760 grains, 3489 grains equal $\frac{3489}{5760}$ of a pound. Therefore, 7 ounces, 5 pennyweights, 9 grains, equal $\frac{3489}{5760}$ of a pound, Troy.

Rule.—*Reduce the compound denominate number to its lowest denomination for a numerator, and a unit to the same denomination for a denominator; the fraction thus formed is the answer sought.*

386. To Reduce a Denominate Decimal to Units of Lower Denominations.

EXAMPLE.—Reduce .865 of a pound, Troy, to integers of lower denominations.

OPERATION.

$$\begin{array}{r}
 .865 \text{ lb.} \\
 \underline{12} \\
 10.380 \text{ oz.} \\
 \underline{20} \\
 7.60 \text{ pwt} \\
 \underline{24} \\
 14.4 \text{ gr.} \\
 .865 \text{ lb., Troy,} = 10 \text{ oz. } 7 \text{ pwt. } 14.4 \text{ gr.}
 \end{array}$$

EXPLANATION.—.865 of a pound equals .865 of the 12 ounces in 1 pound, or 10.38 ounces; .38 of an ounce equals .38 of the 20 pennyweights in 1 ounce, or 7.6 pennyweights; .6 of a pennyweight equals .6 of the 24 grains in 1 pennyweight, or 14.4 grains. Therefore, .865 of a pound, Troy, equals 10 ounces, 7 pennyweights, 14.4 grains.

Rule.—*Multiply the decimal by that unit in the scale which will reduce it to units of the next lower denomination, and in the product point off as in decimals. Proceed in like manner with all decimal remainders.*

387 To Reduce Denominate Numbers to Decimals of a Higher Denomination.

EXAMPLE.—Reduce 8 oz. 3 pwt. 15 gr. to the decimal of a pound, Troy.

OPERATION.

$$\begin{array}{r}
 24 \overline{) 15} \text{ gr.} \\
 \underline{.625} \\
 3. \text{ pwt.} \\
 20 \overline{) 3.625} \text{ pwt.} \\
 \underline{.18125} \text{ oz.} \\
 8. \text{ oz.} \\
 12 \overline{) 8.18125} \text{ oz.} \\
 \underline{.68177\frac{1}{2}} \text{ lb.}
 \end{array}$$

EXPLANATION.—Since 24 grains equal 1 pennyweight, 15 grains equal $\frac{15}{24}$ or .625 of a pennyweight; 3 pennyweights plus .625 pennyweights equal 3.625 pennyweights; since 20 pennyweights equal 1 ounce, 3.625 pennyweights equal .18125 of an ounce, and 8 ounces plus .18125 of an ounce equal 8.18125 ounces; since 12 ounces equal 1 pound, 8.18125 ounces equal $.68177\frac{1}{2}$ of a pound. Therefore, 8 oz. 3 pwt. 15 gr. = $.68177\frac{1}{2}$ lb., Troy.

Rule.—*Divide the lowest denomination given by the number in the scale next higher, and to the quotient add the integers of the next higher denomination. So continue to divide by all the successive orders of units in the scale.*

ADDITION OF DENOMINATE NUMBERS.

388. EXAMPLE.—Find the sum of 2 lb. 5 oz. 13 pwt. 4 gr., 17 lb. 11 oz. 18 pwt. 20 gr., and 9 lb. 9 oz. 6 pwt. 15 gr.

OPERATION.

lb.	oz.	pwt.	gr.
2	5	13	4
17	11	18	20
9	9	6	15
<hr/>			
30 lb.	2 oz.	18 pwt.	15 gr.

EXPLANATION.—Since each of the given expressions is a compound number of the same class, and they all have the same varying scale, their addition may be performed the same as in simple numbers; in reducing the sum of each column from a lower to a higher order, observe the units in the ascending scale.

Rule.—I. *Write the numbers of the same unit value in the same column.*

II. *Beginning with the lowest denomination, add as in simple numbers, and reduce to higher denominations according to the scale.*

SUBTRACTION OF DENOMINATE NUMBERS.

389. EXAMPLE.—Subtract 11 lb. 7 oz. 13 pwt. 9 gr. from 23 lb. 4 oz. 17 pwt. 6 gr.

OPERATION.			
lb.	oz.	pwt.	gr.
23	4	17	6
11	7	13	9
<hr/>			
11 lb.	9 oz.	3 pwt.	21 gr.

EXPLANATION.—Subtract as in simple numbers. If a subtrahend term be numerically greater than the corresponding minuend term, *borrow* 1 from the next higher minuend term, reduce it to equivalent units in the denomination next lower, add them to the minuend units, and from their sum take the subtrahend units.

Rule.—Write the numbers as for simple subtraction; take each subtrahend term from its corresponding minuend term for a remainder. In case any subtrahend term be greater than the minuend term, borrow 1 as in simple subtraction, and reduce it to the denomination required.

MULTIPLICATION OF DENOMINATE NUMBERS.

390. EXAMPLE.—Each of five bars of silver weighed 16 lb. 3 oz. 10 pwt. 21 gr. What was the total weight?

OPERATION.			
lb.	oz.	pwt.	gr.
16	3	10	21
<hr/>			
81 lb.	5 oz.	14 pwt.	9 gr.

EXPLANATION.—Multiply 21 grains by 5 and obtain 105 grains, which reduce to pennyweights by dividing by 24, and obtain 4 pennyweights, with a remainder of 9 grains; multiply 10 pennyweights by 5, add the 4 pennyweights, and reduce to ounces by dividing by 20, obtaining 2 ounces, 14 pennyweights; multiply 3 ounces by 5, add the 2 ounces and divide by 12, obtaining 1 pound, 5 ounces; multiply 16 pounds by 5, add the 1 pound and obtain 81 pounds.

Rule.—Beginning with the lowest denomination, multiply each in succession, and reduce the product to higher denominations by the scale.

REMARKS.—1. In order that the pupil may have all problems under each denominate subject given together, and so make an exhaustive study separately of each, it has seemed proper to include all of the reductions under a typical subject, that of TROY WEIGHT, and hereafter, as may be needed, reference will be made to such reductions.

2. The teacher will appreciate the above change, as each subject will thus be made to include enough work for a lesson, and the confusion often arising from giving in the same lesson several tables, with varying scales, may be avoided.

DIVISION OF DENOMINATE NUMBERS.

391. EXAMPLE.—If 7 lb. 7 oz. 12 pwt. 18 gr. of silver be made into 6 plates of equal weight, what will be the weight of each?

OPERATION.			
lb.	oz.	pwt.	gr.
6) 7	7	12	18
<hr/>			
1 lb.	3 oz.	5 pwt.	11 gr.

EXPLANATION.—One plate will weigh $\frac{1}{6}$ as much as 6 plates. Write the dividend and divisor as in short division. Divide 7 pounds by 6, obtaining a quotient of 1 pound and an undivided remainder of 1 pound; reduce this remainder to ounces (12) and add to the 7 ounces of

the dividend, obtaining 19 ounces, which divide by 6, obtaining 3 ounces and an undivided

remainder of 1 ounce; reduce this remainder to pennyweights (20) and add to the 12 pennyweights of the dividend, obtaining 32 pennyweights, which divide by 6, obtaining 5 pennyweights and an undivided remainder of 2 pennyweights; reduce this remainder to grains (48) and add to the 18 grains of the dividend, obtaining 66 grains, which divide by 6, obtaining 11 grains, and thus completing the division. Therefore, the weight of each plate will be 1 pound, 3 ounces, 5 pennyweights, 11 grains.

Rule.—*Write the terms as in short division; divide as in integers and reduce remainders, if any, to next lower orders by the scale.*

REMARKS.—1. Should the highest dividend order not contain the divisor, reduce its units to the order next lower, and so proceed to the end.

2. The above and like divisions may be accomplished by the reduction of the denominate expressions to the lowest order in its scale, then effecting the division and afterwards reducing the quotient to higher denominations.

COMPOUND DENOMINATE DIVISION.

392. EXAMPLE.—How many plates, each weighing 1 lb. 3 oz. 5 pwt. 11 gr., can be made from 7 lb. 7 oz. 12 pwt. 18 gr. of silver?

OPERATION.

1 lb. 3 oz. 5 pwt. 11 gr. = 7331 gr.
 7 lb. 7 oz. 12 pwt. 18 gr. = 43986 gr.
 $43986 \text{ gr.} \div 7331 \text{ gr.} = 6$

EXPLANATION.—Reduce each of the given expressions to its equivalent in grains. Since one plate weighs 7331 grains, and the weight of the silver to be used is 43986 grains, as many plates can be made as the weight of one plate, 7331 grains, is contained times in the 43986 grains to be so used, or 6 plates.

Rule.—*Reduce the dividend and divisor to the same denomination, and divide as in simple numbers.*

EXAMPLES FOR PRACTICE.

- 393.** 1. Reduce 31 lb. 10 oz. 13 pwt. to pennyweights.
 2. How many grains in 27 lb. 17 pwt. 20 gr.?
 3. How many pounds, ounces, and pennyweights in 23051 gr.?
 4. Reduce 30297 grains to higher denominations.
 5. Reduce $\frac{3}{40}$ of a pound to grains.
 6. $\frac{1}{2100}$ of a pound is what part of a pennyweight?
 7. $\frac{3}{80}$ of a grain is what fraction of an ounce?
 8. Reduce $\frac{7}{8}$ of a pennyweight to the fraction of a pound.
 9. Reduce $\frac{9}{4}$ of a pound to lower denominations.
 10. Reduce $\frac{5}{8}$ of an ounce to lower denominations.
 11. Reduce 9 oz. 1 pwt. 21 gr. to the fraction of a pound.
 12. What fraction of a pound equals 11 oz. 11 pwt. 18 gr.?
 13. What is the value in lower denominations of .6425 lb.?
 14. Find the equivalents in lower denominations of .905 oz.?
 15. 3 oz. 11 pwt. 12 gr. is what decimal of a pound?
 16. Reduce 17 pwt. 12 gr. to the decimal of an ounce.
 17. Add 236 lb. 4 oz. 15 pwt., 83 lb. 11 oz. 21 gr., 46 lb. 16 pwt., 105 lb. 9 oz. 11 gr.

18. What is the sum of 16 lb. 16 pwt. 16 gr., 100 lb. 1 oz. 5 pwt. 20 gr., 76 lb. 7 oz. 6 pwt. 13 gr., 19 lb. 2 oz. 10 pwt. 20 gr.?
19. Find the equivalents in lower denominations of .1425 oz.
20. 1 pwt. 15 gr. is what decimal of a pound?
21. Subtract 41 lb. 11 oz. 6 pwt. 18 gr. from 50 lb. 2 oz.
22. What is the difference between 19 lb. 9 oz. 11 pwt. and 11 oz. 16 pwt. 22 gr.?
23. What will be the cost of 15 gold chains, each weighing 1 lb. 3 oz. 18 pwt. 18 gr., at 7¢ per grain?
24. I bought 7 lb. 7 oz. 12 pwt. 18 gr. of old gold, at \$1.05 per pwt. What was the sum paid?
25. A manufacturer made 18 vases from 7 lb. 8 oz. 8 pwt. 18 gr. of silver. What was their average weight?
26. If 12 rings be made from 1 lb. 8 oz. of gold, what will be the weight of each?
27. A miner having 77 lb. 10 oz. 5 pwt. of gold dust, divided $\frac{1}{4}$ of it among his laborers, and had the remainder made into chains averaging 3 oz. 3 pwt. 3 gr. of pure gold each. If he sold the chains for \$52.50 each, how much did he receive for them?
28. What is the aggregate weight of five purchases of old silver, weighing respectively 4 lb. 9 oz. 20 gr., 13 lb. 17 pwt. 22 gr., 20 lb. 1 oz. 17 pwt. 4 gr., 8 lb. 2 oz., and 27 lb. 12 pwt. 21 gr.?
29. I bought 27 lb. 11 oz. 1 gr. of old silver, and after having used 15 lb. 15 pwt. 15 gr., sold the remainder at 5¢ per pwt. What quantity was sold, and how much was received for it?
30. A goldsmith bought 3 lbs. 9 oz. 1 pwt. 16 gr. of old gold, at 80¢ per pwt., and made it into pins of 40 grains weight each, which he sold at \$2 apiece. How much did he gain or lose?

AVOIRDUPOIS WEIGHT.

394. Avoirdupois Weight is used for all ordinary purposes of weighing.

Table.

16 ounces	= 1 pound	lb.
100 pounds	= 1 hundred-weight..	cwt.
20 hundred-weight., or 2000 pounds	= 1 ton	T.

Scale, descending, 20, 100, 16; ascending, 16, 100, 20.

REMARK.—At the United States Custom Houses, in weighing goods on which duties are paid and to a limited extent in coal and iron mines, the *long ton* of 2240 pounds is still used.

Long Ton Table.

16 ounces	= 1 pound	lb.
28 pounds	= 1 quarter	qr.
4 quarters, or 112 pounds	= 1 hundred-weight ...	cwt.
20 hundred-weight, or 2240 pounds	= 1 ton	T.

Additional Table of Weights of Products,

As usually given, but varied by the laws of different States :

Apples, green,-----	56 lb. per bushel.	Mineral Coal, -----	80 lb. per bushel.
Charcoal, -----	22 lb. per bushel.	Peas, -----	60 lb. per bushel.
Hungarian Grass Seed,	45 lb. per bushel.	Potatoes, sweet, -----	55 lb. per bushel.
Malt, -----	38 lb. per bushel.	Red Top Grass Seed,--	14 lb. per bushel.
Millet, -----	45 lb. per bushel.	Turnips, -----	56 lb. per bushel.

Table of Gross Weights for Freighting.

Ale and Beer,-----	330 lb. per barrel.	Highwines,-----	400 lb. per barrel.
Apples, -----	150 lb. per barrel.	Lime, -----	230 lb. per barrel.
Beef (200 lb. net),	330 lb. per barrel.	Oil, -----	400 lb. per barrel.
Cider, -----	400 lb. per barrel.	Pork (200 lb. net),	330 lb. per barrel.
Corn Meal,-----	200 lb. per barrel.	Potatoes, -----	180 lb. per barrel.
Eggs,-----	180 lb. per barrel.	Salt, -----	300 lb. per barrel.
Fish,-----	300 lb. per barrel.	Vinegar,-----	400 lb. per barrel.
Flour (196 lb. net),	200 lb. per barrel.	Whiskey, -----	400 lb. per barrel.

Estimates on Lumber, Wood, Etc., for Freighting.

Pine, Hemlock, and Poplar, seasoned, per M,-----	3000 lb.
Black Walnut, Ash, Maple, and Cherry, per M,---	4000 lb.
Oak and Hickory, per M,-----	5000 lb.
Soft wood, dry, per cord, -----	3000 lb.
Hard wood, dry, per cord, -----	3500 lb.

REMARK.—For unseasoned lumber, add one-third.

Brick, common, each,-----	4 lb.	Sand, cubic yard,-----	3000 lb.
Brick, fire, each,-----	6 lb.	Gravel, cubic yard,-----	3200 lb.
Stone, cubic yard,----		4000 lb.	

REMARK.—For assistance in the solution of the following examples, the pupil is referred to the explanations and rules under TROY WEIGHT.

EXAMPLES FOR PRACTICE.

395. 1. How many pounds Avoirdupois in 17 T. 6 cwt. 69 lb.?
2. Reduce 31275 lb. Avoirdupois to higher denominations.
3. $\frac{3}{8}$ of a ton Avoirdupois equals how many pounds?
4. Reduce $\frac{1}{16}$ of a cwt. Avoirdupois to ounces.
5. Reduce .3842 of a ton Avoirdupois to lower denominations.
6. How many Avoirdupois pounds in .625 of a ton?
7. 17 cwt. 72 lb. 4 oz. Avoirdupois is what fraction of a ton?
8. Reduce 51 lb. 12 oz. Avoirdupois to the fraction of a hundred-weight.
9. What decimal part of a hundred-weight is 24 lb. 2 oz. Avoirdupois?
10. Reduce 19 cwt. 99 lb. 15 oz. Avoirdupois to the decimal of a ton.

11. What is the sum of 7 T. 4 cwt. 78 lb. 5 oz., 3 T. 17 cwt. 19 lb. 11 oz., 5 T. 18 cwt. 96 lb., 13 T. 1 cwt. 11 oz.?

12. A farmer sold 4 loads of hay, weighing respectively 1 T. 2 cwt. 14 lb., 19 cwt. 90 lb., 1 T. 3 cwt. 97 lb., 1 T. 5 cwt., and received for it \$16 per ton. How much did he receive?

13. Six loads of lime weighed 13 T. 15 cwt. 4 lb. What was their average weight?

APOTHECARIES' WEIGHT.

396. Apothecaries' Weight is used by druggists in retailing, and by apothecaries in mixing medicines.

Table.

20 grains	= 1 scruple	sc.
3 scruples	= 1 dram	dr.
8 drams	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

Scale, descending, 12, 8, 3, 20; ascending, 20, 3, 8, 12.

REMARKS.—1. The pound, ounce, and grain are the same as in Troy weight. The only difference between these weights is in the subdivisions of the ounce.

2. Drugs and medicines are sold at wholesale by Avoirdupois weight.

EXAMPLES FOR PRACTICE.

REMARK.—For assistance, refer to rules and explanations under TROY WEIGHT.

397. 1. Reduce 5128 sc. to higher denominations.

2. How many drams in 61 lb. 5 oz.?

3. 10 oz. 1 dr. 1 sc. 15 gr. equal what fraction of a pound?

4. Reduce .955 of a pound to lower denominations.

5. How many scruples in $\frac{7}{32}$ of a pound?

6. Add $6\frac{3}{4}$ lb., $7\frac{1}{2}$ oz., $3\frac{5}{8}$ dr. and $2\frac{1}{2}$ sc.

7. Find the sum of $\frac{1}{2}$ lb., 7 oz., 7 dr., 1 sc. and 16 gr.

8. From 21 lb. 5 oz. 3 dr. 1 sc. 11 gr., take 14 lb. 1 oz. 7 dr. 19 gr.

9. What is the difference between 16 lb. 1 oz. 4 dr. 2 sc. 12 gr., and $9\frac{9}{16}$ lb.?

10. In compounding six cases of medicine, an apothecary used for each 2 lb. 7 oz. 6 dr. 18 gr. What was the aggregate weight?

11. If 19 lb. 4 oz. 7 dr. 1 sc. 5 gr. be divided into 21 packages of equal weight, what will be the weight of each?

Comparative Table of Weights.

	Troy.	Apothecaries'.	Avoirdupois.
1 pound	= 5760 grains	= 5760 grains	= 7000 grains.
1 ounce	= 480 grains	= 480 grains	= 437.5 grains.
	175 pounds	= 175 pounds	= 144 pounds.

QUESTIONS.—1. Which is heavier, a pound Troy or a pound Avoirdupois?

2. Which is heavier, an ounce Troy or an ounce Avoirdupois?

REMARKS.—1. A cubic foot of water weighs $62\frac{1}{2}$ lb. or 1000 oz. Avoirdupois.

2. In weighing diamonds and gems, the *unit* generally employed is the *carat*, which is about 3.2 Troy grains.

3. The term *carat* is also used to express the fineness of gold, 24 *carats* fine being *pure*; thus 18 *carat* gold = $\frac{3}{4}$ pure

EXAMPLES FOR PRACTICE.

398. 1. A dealer bought 131 lb. 5 oz. of drugs by Avoirdupois weight, at \$6.25 per pound, and retailed them at 5¢ per scruple. What was his gain?

2. How much is gained or lost by buying 23 lb. 4 oz. of medicine by Avoirdupois weight, at 50¢ per oz., and selling by Apothecaries weight, at $1\frac{1}{2}$ ¢ per grain?

3. Reduce $5\frac{5}{8}$ lb., Avoirdupois, to Troy units.

4. What is the remainder after subtracting $51\frac{5}{16}$ lb. Troy from 60 lb. 10 oz. Avoirdupois?

5. I bought by Avoirdupois weight 45 lb. 6 oz. of drugs, and from the stock sold by Apothecaries weight 29 lb. 4 oz. 3 dr. 1 sc. 10 gr. What is the remainder worth, at 75¢ per Avoirdupois ounce?

6. Having bought $1\frac{3}{8}$ of a pound of roots by Avoirdupois weight, I sold $1\frac{3}{8}$ of a pound by Apothecaries weight. What was the remainder worth, at 10¢ per scruple?

MEASURES OF CAPACITY.

399. **Dry Measure** is used for measuring grains, seeds, fruit, vegetables, etc.—all articles not liquid.

The units are pints, quarts, pecks, and bushels.

Table.

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.

Scale, descending, 4, 8, 2; ascending, 2, 8, 4.

REMARKS.—1. The United States Standard Unit of Dry Measure is the *bushel*, which, as a circular measure, is $18\frac{1}{2}$ inches in diameter and 8 inches deep, contains 2150.42 cubic inches, and is in uniform use for measuring shelled grains; while the *heaped bushel* of 2747.71 cubic inches is used for measuring apples, roots, and corn unshelled.

2. The British Imperial bushel contains 2218.19 cubic inches. The English Quarter mentioned in prices current = 8 bu. of 70 lb. each, or 560 lb. avoirdupois = $\frac{1}{4}$ long ton.

3. For weights of different commodities, refer to *Table*, page 125.

EXAMPLES FOR PRACTICE.

REMARK.—For assistance, refer to rules under TROY WEIGHT.

400. 1. How many pints in $14\frac{1}{2}$ bu.?

2. Reduce 9 bu. 1 pk. 3 qt. 1 pt. to pints.

3. Add 51 bu. 3 pk. 1 pt.; 46 bu. 2 pk.; 37 bu. 2 qt. 1 pt.; 51 bu. 1 pk. 7 qt.

4. From $\frac{2}{3}$ of a bushel, take $\frac{5}{8}$ of a peck.

5. What is the difference between $7\frac{1}{2}$ bu. and 2 bu. 2 pk. 2 qt. 1 pt.?
6. A teamster's 12 loads of wheat measured 1000 bu. 1 pk. 6 qt. 1 pt. How much was the average of each load?
7. What will be the cost, at 45¢ per bushel, of 5 loads of oats, weighing respectively 2619 lb., 2554 lb., 2124 lb., 3051 lb., and 2745 lb.?

LIQUID MEASURE.

401. Liquid Measure is used for measuring water, oil, milk, cider, molasses, etc.

The units are gills, pints, quarts, gallons, and barrels.

Table.

4 gills (gi.)	= 1 pint	pt.
2 pints	= 1 quart	qt.
4 quarts	= 1 gallon	gal.
$31\frac{1}{2}$ gallons	= 1 barrel	bar. or bbl.

Scale, descending, $31\frac{1}{2}$, 4, 2, 4; ascending, 4, 2, 4, $31\frac{1}{2}$.

REMARKS.—1. The standard unit of Liquid Measure is the gallon, which contains **231** cubic inches.

2. Casks, called hogsheads, pipes, butts, tierces, tuns, etc., are indefinite measures; their capacity, being determined by gauging, is usually marked upon them.

3. In sales of oils and liquors, and in certain other cases, the *barrel* is also of indefinite capacity.

EXAMPLES FOR PRACTICE.

REMARK.—For assistance refer to rules under TROY WEIGHT.

- 402.** 1. How many gills in 5 bar. 27 gal. 3 qt. 1 pt. of cider?
2. Reduce 31479 gi. to higher denominations.
3. .646 of a barrel equals how many gills?
4. From .895 of a barrel take 21 gal. 2 qt. 1 pt. 1 gi.
5. From a cask containing 68 gal. $4\frac{1}{2}$ qt. of wine, 1.625 bar. were sold. What was the remainder worth, at 50¢ per pint?
6. A reservoir contained 896 gal. 2 qt. of water, and its contents were put into 116 kegs. What was the quantity put into each?
7. From $\frac{5}{8}$ of a barrel take 4 gal. 1 qt. 3 gi.
8. If 2 qt. 1 pt. 1 gi. of oil be consumed per day for the year 1890, what will be its cost for the year at 8¢ per gallon?
9. From a cask of brandy containing 69 gal. 1 pt. and costing \$3.75 per gallon, one-fourth leaked out, and the remainder was sold at 20¢ per gill. What was the amount of gain or loss?

Comparative Table of Dry and Liquid Measures.

	Cu. in. in one gallon.	Cu. in. in one quart.	Cu. in. in one pint.
Dry Measure.....	($\frac{1}{2}$ pk.) 268 $\frac{1}{2}$	67 $\frac{1}{2}$	33 $\frac{3}{4}$.
Liquid Measure.....	231	57 $\frac{3}{4}$	28 $\frac{1}{2}$.

- REMARKS.—1. A pint of water weighs about 1 pound, *Avoirdupois*.
 2. Potatoes and grains are usually sold to dealers and shippers by **weight**.
 3. The beer gallon of 282 cubic inches is nearly obsolete.

EXAMPLES FOR PRACTICE.

- 403.** 1. Reduce 21 bu. 6 qt. 1 pt., dry measure, to pints, liquid measure.
 2. A grocer bought 12 bu. 3 pk. 3 qt. of chestnuts by dry measure, and when selling used a liquid pint measure. How many pints did he gain by the change?
 3. A bushel of cherries, bought at 10¢ per quart, dry measure, was sold at the same price per quart, wine measure. How much was thereby gained?
 4. A cask of cranberries, containing $5\frac{7}{8}$ bu., was bought for \$15, and retailed at 10¢ per quart by wine measure. What was the gain?
 5. A blundering clerk bought of a gardener 375 quarts of currants, measuring them by a liquid quart measure, and when selling used a dry quart measure. If he bought at 6¢ per quart and sold at 7¢, how much less did he receive than if he had measured by dry measure when buying and by liquid measure when selling?

MEASURES OF EXTENSION.

404. **Extension** is that which has one or more of the dimensions, *length*, *breadth*, and *thickness*; it may therefore be a line, a surface, or a solid.

405. A **Line** has only one dimension—*length*.

REMARKS.—1. The United States Standard of linear, surface, and solid measure, is the **yard** of 3 feet, or 36 inches.

2. The standard, prescribed at Washington, has been fixed with the greatest precision. It was determined by a brass rod, or *pendulum*, which vibrates *seconds* in a vacuum at the sea level, in the latitude of London, Eng., and in a temperature of 62° Fahrenheit. This pendulum is divided into 391393 equal parts, and 360000 of these parts constitute a yard.

406. A **Surface** or **Area** has two dimensions—*length* and *breadth*.

407. A **Solid** has three dimensions—*length*, *breadth*, and *thickness*.

LINEAR MEASURE.

408. **Linear** or **Long Measure** is used in measuring lengths and distances.

Table.

12 inches (in.).....	= 1 foot	ft.
3 feet	= 1 yard	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod	rd.
320 rods	= 1 statute mile	mi.

Scale, descending, 320, $5\frac{1}{2}$ 3, 12; ascending, 12, 3, $5\frac{1}{2}$, 320.

1 Mile = 320 rods, or 5280 feet, or 63360 inches.

Special Table.

$\frac{1}{8}$ of an inch = 1 Size, applied to shoes.	3 geographic miles = 1 League, used for measuring distances at sea.
18 inches = 1 Cubit.	
3.3 feet = 1 Pace.	60 geographic miles or 69.16 statute miles = 1 Degree of Latitude on a meridian, or Longitude on the equator.
5 paces = 1 Rod,	
4 inches = 1 Hand, used to measure the height of animals.	360 degrees = Equatorial circumference of the earth.
6 feet = 1 Fathom, used to measure depths at sea.	
1.152 $\frac{2}{3}$ statute miles = 1 Geographic or Nautical mile, used for measuring distances at sea.	1 geographic mile = 1 Knot, used to determine the speed of vessels.

REMARKS.—1. In civil engineering, and at the Custom Houses, the foot and inch are divided into tenths, hundredths, and thousandths.

2. The yard is divided into halves, quarters, eighths, and sixteenths, for measuring goods sold by the yard.

3. The furlong of 40 rods is little used.

4. Degrees are of unequal length; those of latitude varying from 68.72 miles at the Equator to 69.34 miles in the polar regions. The average length, 69.16 miles, is the standard adopted by the United States Coast Survey.

5. A degree of longitude is 69.16 statute miles at the equator, but decreases gradually toward the poles, where it is 0.

EXAMPLES FOR PRACTICE.

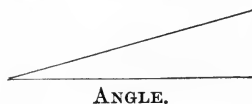
REMARK.—For assistance refer to Rules under TROY WEIGHT.

409. 1. Reduce 2 mi. 1 rd. 7 ft. to inches.
2. Reduce 2501877 inches to higher denominations.
3. What part of a mile is $\frac{5}{12}$ of a foot?
4. Reduce $\frac{2}{3}$ of a mile to integers of lower denominations.
5. What fraction of a rod is 11 ft. 2 in.?
6. Reduce .542 of a mile to integers of lower denominations.
7. Reduce 285 rd. 7 ft. 4 in. to the decimal of a mile.
8. A wheelman ran 71 mi. 246 rd. 1 yd. 2 ft. 6 in. in the forenoon, and 20 mi. 10 rd. 8 in. less in the afternoon. What distance did he run in the entire day?
9. If a yacht makes an average of 227 mi. 227 rd. 2 yd. 2 ft. 2 in. per day, for the seven days of a week, what distance will be passed?
10. If the Servia steams 2905 mi. in six days, what is her average rate per day?

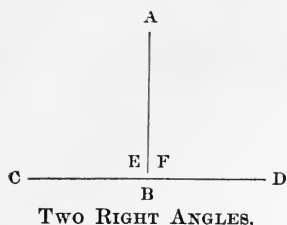
SQUARE MEASURE.

410. **Square Measure** is used for computing the surface of land, floors, boards, walls, roofs, etc.

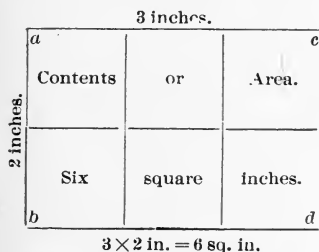
411. The **Area** of a figure is the quantity of surface it contains.



412. An **Angle** is the difference in the direction of two lines proceeding from a common point called the *vertex*.



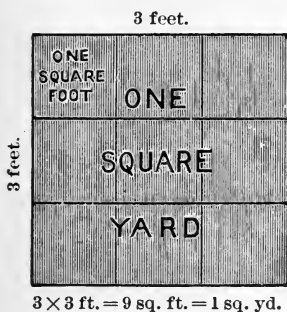
RECTANGLE.



413. A Right Angle is the angle formed when one straight line meets another so as to make the adjacent angles equal. The lines forming the angles are said to be *perpendicular* to each other. E and F are right angles, and the lines A B and C D are perpendicular to each other.

414. A Rectangle is a plane or flat surface, having four straight sides and four square corners, or four right angles.

415. The Contents or Area of any surface having a uniform length and a uniform breadth is found by multiplying the length by the breadth. In the accompanying diagram, in which the angles (*a, b, c, d*), are all right angles, and the corners all square corners, the area is 6 square inches, and is found by multiplying 2 inches by 3 inches.



416. A Square is a figure bounded by four equal lines, and having four right angles.

REMARK.—A *square inch* is a square, each side of which is 1 inch. A *square foot* is a square, each side of which is 1 foot. A *square yard* is a square, each side of which is 1 yard.

Table of Square Measure.

144 square inches (sq. in.)	-----	= 1 square foot	sq. ft.
9 square feet	-----	= 1 square yard	sq. yd.
30 $\frac{1}{4}$ square yards, or }	-----	= 1 square rod	sq. rd.
272 $\frac{1}{4}$ square feet	-----			
160 square rods	-----	= 1 acre	A.
640 acres	-----	= 1 square mile	sq. mi.
36 square miles (6 miles square),	-----	= 1 township	Tp.

Scale, descending, 36, 640, 160, 30 $\frac{1}{4}$, 9, 144; ascending, 144, 9, 30 $\frac{1}{4}$, 160, 640, 36.

REMARK.—All the units of *square* measure, except the acre, are derived by *squaring* the corresponding units of *linear* measure; as, a square foot is a surface one foot square; a square rod is a surface 1 rod or 16 $\frac{1}{4}$ feet square; a square mile is a surface 1 mile or 320 rods square.

417. The Unit of Land Measure is the acre, equal to 208.71 ft. \times 208.71 ft.

REMARKS.—1. In sections of the United States where the original grants were from France, the *arpent*, a French unit of surface, equal to about $\frac{5}{8}$ of an acre, is still sometimes used.

2. The Rood, equal to 40 square rods, is but little used

418. Dimension stuff is sold by board measure.

419. The Unit of Board Measure is a square foot surface, one inch thick, called a *board foot*.

420. To Find the Number of Board Feet in a Board.

Rule.—Multiply the length in feet by the width in inches, and divide by 12; the quotient will be the number of square feet.

REMARK.—If the board tapers evenly, find the *mean* or average width, by adding the width of the two ends, and dividing by 2.

421. To Find the Number of Board Feet in Timbers or Planks.

Rule.—Multiply the length in feet by the product of the width and thickness in inches, and divide by 12.

422. To Find the Number of Squares in a Floor or Roof.

REMARK.—In flooring, roofing, slating, etc., the *square*, or 100 square feet, is used as a unit of measure.

Rule.—Point off two decimal places from the right of the number of surface feet.

423. To Find the Number of Yards of Carpeting that Would be Required to Cover a Floor.

Rule.—I. Divide one of the dimensions of the floor by 3, add the wastage, if any, and the result will be the length, in yards, of 1 strip of the carpet.

II. Divide the other dimension by the width of the carpet, and the quotient will be the number of strips it will take to cover the floor.

III. Multiply the length of each strip by the number of strips, and the product will be the number of yards required.

REMARK.—In carpeting and papering, it is usually necessary to allow for certain waste in matching the figures of patterns, and often carpets may be laid with less waste one way of the room than the other. Dealers charge for all goods furnished, regardless of the waste.

EXAMPLES FOR PRACTICE.

REMARK.—For assistance refer to rules under TROY WEIGHT.

424. 1. Reduce 5 A. 110 sq. rd. 7 sq. ft. to square inches.

2. Reduce 4 sq. mi. 527 A. 105 $\frac{1}{2}$ sq. rd. to square feet.

3. Reduce .1754 of a square mile to lower denominations.

4. Reduce $\frac{1}{16}$ of an acre to lower denominations.
5. What fraction of a square mile is $\frac{7}{16}$ of a square foot?
6. What decimal part of an acre is 150 sq. rd. 3 sq. yd. 7 sq. ft. 100 sq. in.?
7. From .6375 of an acre take $\frac{1}{4}$ of a square rod.
8. To the sum of $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{4}{5}$ of an acre, add .0055 of a square mile.
9. How many squares in a roof, each side of which is 26×60 feet?
10. How many yards of carpet, 1 yard wide, will be required to cover a floor 10.5 yd. long by 6 yd. wide, if no allowance be made for matching?
11. How many feet in 8 boards, each 15 ft. long, 9 in. wide, and 1 in. thick?
12. How many feet in 15 boards, each 16 ft. long and 1 in. thick, the boards being 13 in. wide at one end and 10 in. at the other?
13. How many acres in a square field, each side of which is 64 rods in length?
14. What will be the cost of a tract of land 508 rd. long and 1350 rd. wide, at \$25 per acre?
15. A field $87\frac{1}{2}$ rd. wide and 240 rd. long, produced $27\frac{3}{4}$ bu. of wheat to the acre. What was the crop worth, at 90¢ per bushel?
16. A farm in the form of a rectangle is 75 rd. wide; if the area is 167.5 A., how long is the farm?
17. I wish to build a shed which will cover $\frac{2}{5}$ of an acre of land. If the width of the shed is 42 ft., what must be its length?
18. 17.75 bu. of timothy seed is sown on land, at the rate of 6 lb. per acre. What is the area thus seeded?
19. What is the difference between a square rod and a rod square?
20. What is the difference between two square rods and two rods square?
21. A square yard will make how many surfaces 5 in. by 9 in.?
22. How many acres of flooring in a six-story block 160 ft. by 220 ft.?
23. A rectangular field containing $16\frac{1}{2}$ A. is 45 rd. wide. What is its length?
24. How many fields, each of 10 A. 56 sq. rd. 21 sq. yd. 5 sq. ft. and 28 sq. in., can be formed from a farm containing 124 A. 40 sq. rd. 16 sq. yd. 8 sq. ft. 48 sq. in.?
25. How many acres in a road 17206 ft. long and 66 ft. wide?
26. What will be the cost, at \$3.50 per M, of the shingles for a roof 26 ft. wide and 110 ft. long, if the shingles are 6 in. wide and 4 inches of their length be exposed to the weather?
27. A hall $7\frac{1}{2}$ ft. wide and $19\frac{1}{2}$ ft. long is covered with oil cloth, at 65¢ per sq. yd. How much did it cost?
28. If a farm of 106 A. $94\frac{1}{2}$ sq. rd. is divided equally into 11 fields, what will be the area of each field?
29. Reduce 240689740 sq. in. to higher denominations.
30. How many rods of fence will enclose 160 A. of land lying in the form of a square?
31. How many additional rods will divide the farm into four fields of equal area?

32. How many yards of brussels carpeting, $\frac{3}{4}$ of a yard wide, laid lengthwise of the room, will be required to cover a room 22 ft. by 17 ft. 4 in., if the waste in matching be 6 in. on each strip?

REMARK.—When the width of the room is not exactly divisible by the width of the carpet, drop the fraction in the quotient and add 1 to the whole number. The waste in such cases is either cut off or turned under in laying.

33. What will it cost, at 21¢ per sq. yd., to plaster the sides and ceiling of a room 24 ft. by $31\frac{1}{2}$ ft. and $10\frac{1}{2}$ ft. high, if one-sixth of the surface of the sides is taken up by doors and windows?

34. A street 4975 ft. long and 40 ft. wide was paved with Trinidad asphaltum, at \$2.65 per square yard. What was the cost?

35. A skating rink, 204 ft. by $196\frac{1}{2}$ ft., was floored with 2 in. plank, at \$23.50 per M. What was the cost of the lumber?

36. What will be the cost of the carpet border for a room $16\frac{1}{2}$ ft. by 21 ft., if the price be $62\frac{1}{2}$ ¢ per yard?

37. How many single rolls of paper, 8 yd. long and 18 in. wide, will it take to cover the ceiling of a room 56 ft. long and 27 ft. 4 in. wide, if there be no waste in matching?

REMARK.—When no allowance is made for waste in matching, divide the surface to be papered by the number of square feet in one roll of the paper.

38. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will be required to carpet a room 32 ft. long and 25 ft. wide, if the lengths of carpet are laid crosswise of the room, and 8 inches is lost on each length in matching the pattern? How many yards if the lengths are laid lengthwise and 6 in. is lost in matching? If the carpet is laid in the most economical way, what will be the cost, at \$2.55 per yard?

39. How many sheets of tin, 20 in. by 14 in., will be required to cover a roof 60.5 ft. wide and 156.25 ft. long?

40. What is the difference between four square feet and four feet square?

41. What will it cost, at \$1.15 per yard, to carpet a flight of stairs 11 ft. 4 in. high, the tread of each stair being 10 in. and the riser 8 in.?

42. How many shingles, averaging 4 in. wide and laid 5 in. to the weather, will cover the roof of a barn, one side of the roof being 24 ft. wide and the other 42 ft. wide, the length of the barn being 60 ft.?

43. Divide an acre of land into 8 equal sized lots, the length of each of which shall be twice its frontage. What will be the dimensions of each lot?

44. How many granite blocks, 12 in. by 18 in., will be required to pave a mile of roadway 42 ft. in width?

45. What will be the cost, at 20¢ per sq. yd., for plastering the ceiling and walls of a room 22 ft. wide, 65 ft. long, and 15 ft. high, allowance being made for 8 doors 4 ft. 6 in. wide by 11 ft. 6 in. high, and 10 windows each 42 in. wide by 8 ft. high?

46. I wish to floor and ceil a room $27\frac{1}{2}$ yd. long and 15 yd. 2 ft. wide, with matched pine. What will be the cost of the material, at \$26.40 per M?

INVOLUTION.

425. A **Power** of a number is the product arising from multiplying a number by itself, or repeating it several times as a factor.

426. A **Perfect Power** is a number that can be exactly produced by the involution of some number as a root; thus, 64 and 16 are perfect powers, because $8 \times 8 = 64$, and $2 \times 2 \times 2 \times 2 = 16$.

427. The **Square** of a number is its *second power*.

428. The **Cube** of a number is its *third power*.

429. **Involution** is the process of finding any power of a number; and a number is said to be *involved* or *raised* to a power, when any power of it is found.

EXAMPLES FOR PRACTICE.

- | | |
|--|--------------------------------|
| 430. 1. What is the square of 1 ? | 5. What is the square of 9 ? |
| 2. What is the square of 3 ? | 6. What is the square of 10 ? |
| 3. What is the square of 4 ? | 7. What is the square of 99 ? |
| 4. What is the square of 5 ? | 8. What is the square of 256 ? |

REMARK.—From the solution of the above examples the pupil will observe:

1st. That the square of any number expressed by one figure cannot contain less than 1 nor more than 2 places.

2d. That the addition of 1 place to any number will add 2 places to its square.

EVOLUTION.

431. **Evolution** is the process of extracting the root of a number considered as a power. It is the reverse of Involution, and each may be proved by the other.

432. A **Root** of a number is one of the equal factors which, multiplied together, will produce the given number; as, $4 \times 4 \times 4 = 64$; 4 is the root from which the number 64 is produced.

SQUARE ROOT.

433. The **Square Root of a Number** is such a number as, multiplied by itself, will produce the required number.

434. The operation of finding one of the two equal factors of a square, or product, is called *extracting the square root*.

REMARK.—The square root of any number, then, is one of its two equal factors, the given number being considered a product.

435. In practical operations, a surface and one of its dimensions being given, the wanting dimension is found by dividing the surface by the given dimension.

from the 6 hundreds or 6, the square of 2 (tens) = 400 or 4, gives 225 as a remainder. Having now taken away the square of the *tens*, the remainder, 225, must be equal to 2 times the *tens* multiplied by the square of the units, plus the square of the units. Since the *tens* are 2 or 20, twice the *tens* = 40. Observe, therefore, that 225 must equal 40 times the *units* of the root, together with the square of such units. If, then, 225 be divided by 40, the quotient, 5, will nearly, if not exactly represent the units of the root sought. Using 40, then, as a trial divisor, the second, or unit figure of the root is found to be 5. The term, *twice the tens multiplied by the units*, is equal to 2 (20 \times 5), or 200, and the units, or 5, squared = 25; the sum of these wanting *terms*, or 225, is the remainder, or what is left after taking from the power the square of the first figure of the root. Therefore, the square root of 625 is 25.

Rule.—I. *Beginning at the right, separate the given number into periods of two places each.*

II. *Take the square root of the greatest perfect square contained in the left hand period for the first root figure; subtract its square from the left hand period, and to the remainder bring down the next period.*

III. *Divide the number thus obtained, exclusive of its units, by twice the root figure already found for a second quotient, or root figure; place this figure at the right of the root figure before found, and also at the right of the divisor; multiply the divisor thus formed by the new root figure, subtract the result from the dividend, and to the remainder bring down the next period, and so proceed till the last period has been brought down, considering the entire root already found as so many tens, in determining subsequent root figures.*

REMARKS.—1. Whenever a divisor is greater than the dividend, place a cipher in the root and also at the right of the divisor; bring down another period and proceed as before.

2. When the root of a mixed decimal is required, form the periods from the decimal point right and left, and if necessary supply a decimal cipher to make the decimal periods of two places each.

3. A root may be carried to any number of decimal places by the use of decimal periods.

4. Any root of a common fraction may be obtained by extracting the root of the numerator for a numerator of the root, and the root of the denominator for the denominator of the root.

5. To find a root, decimally expressed, of any common fraction, reduce such common fraction to a decimal, and extract the root to any number of places.

EXAMPLES FOR PRACTICE,

439. 1. Find the square root of 196.
2. Find the square root of 225.
3. Find the square root of 144.
4. Find the square root of 576.
5. Find the square root of 1225.
6. Find the square root of 5625.
7. Find the square root of 42436.
8. Find the square root of 125.44.
9. Find the square root of 50.2681.
10. Find the square root of 482, carried to three decimal places.
11. Find the square root of 25.8, carried to two decimal places.

12. Find the square root of 106.413, carried to four decimal places.
13. What is the square root of $\frac{9}{16}$?
14. What is the square root of $\frac{25}{4}$?
15. What is the square root, decimally expressed, of $1\frac{1}{16}$, carried to three decimal places?
16. What is the square root, decimally expressed, of $\frac{501}{693}$, carried to two decimal places?
17. What is the square root of 30368921, carried to one decimal place.
18. What is the square root of 4698920043, carried to two decimal places.

FIG. T.

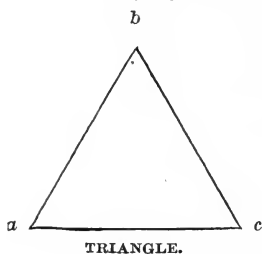
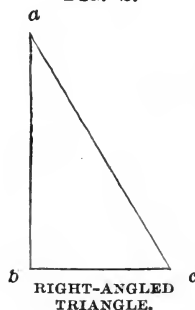


Fig. T. is a *triangle*, having angles at a , b , c .

FIG. S.



444. A **Right-angled Triangle** is a triangle having a *right* angle.

Fig. S is a right-angled triangle, the angle at b being a right angle. The line ab is the Perpendicular; the line bc is the Base; the line ac is the Hypotenuse.

REMARK.—It is a geometrical conclusion that the square formed on the hypotenuse is equal to the sum of the squares formed on the base and the perpendicular

445. To find the hypotenuse, when the base and perpendicular are given.

RULE.—To the square of the base add the square of the perpendicular, and extract the square root of their sum.

To find the base, when the hypotenuse and perpendicular are given.

RULE.—From the square of the hypotenuse take the square of the perpendicular, and extract the square root of the remainder.

To find the perpendicular, when the hypotenuse and base are given.

RULE.—Take the square of the base from the square of the hypotenuse, and extract the square root of the remainder.

EXAMPLES FOR PRACTICE.

446. 1. The base of a figure is 6 ft. and the perpendicular 8 ft. Find the hypotenuse.

2. The perpendicular is 17.5 ft. and the base is 46.6 ft. Find the hypotenuse to three decimal places.

3. The hypotenuse is 110 ft. and the base is 19.5 ft. Find the perpendicular to two decimal places.
4. The hypotenuse is 86 ft. and the base is equal to the perpendicular. Find both of the wanting terms to two decimal places.
5. The hypotenuse is 127 ft. and the base is equal to $\frac{1}{2}$ of the perpendicular. Find wanting terms to three decimal places.

REMARKS.—1. Observe, in example 4, that the square root of $\frac{1}{4}$ the square of the hypotenuse is equal to the base; and in example 5, that the square root of $\frac{1}{4}$ of the square of the hypotenuse is equal to the base.

2. Carry all roots to two decimal places.

6. What is the length of one side of a square field, the area of which is one acre?

7. How many feet of fence will enclose a square field containing five acres?

8. I wish to lay out ten acres in the form of a square. What must be its frontage in feet and inches?

9. What is the distance from the top of a perpendicular flag-staff 105 ft. high to a point 4 rods from the base and on a level with it?

10. What is the width of a street in which a ladder 60 ft. long can be so placed that it will reach the eaves of a building 40 ft. high on one side of the street, and of another building 50 ft. high on the opposite side of the street?

11. What length of line will reach from the lower corner to the opposite upper corner of a room 64 ft. long, 27 ft. wide, and 21 ft. high?

12. If a farm be one mile square, how far is it diagonally across from corner to corner? Find the answer in rods, feet, and inches.

13. How many rods of fence will enclose a square field containing 20 acres?

14. A farm of 180 acres is in the form of a rectangle, the length of which is twice its width. How many rods of fence will enclose it?

15. What will be the base line of a farm of 136 A. 40 sq. rd. if it is in the form of a right-angled triangle, with the base equal to the perpendicular?

SURVEYOR'S LONG MEASURE.

447. The **Unit** of measure used by land surveyors is Gunter's Chain, 4 rods, or 66 feet, in length, and consisting of 100 links.

REMARK.—Rods are seldom used in Surveyor's Measure, it being customary to give distances in chains and links or hundredths.

Table.

7.92 inches.....	= 1 link ... l.
25 links	= 1 rod rd.
4 rods, or 66 feet...	= 1 chain .. ch.
80 chains, or 320 rods	= 1 mile ... mi.

Scale, descending, 80, 4, 25, 7.92; ascending, 7.92, 25, 4, 80.

EXAMPLES FOR PRACTICE.

448. 1. Reduce 3 mi. 27 ch. 19 l. 4 in. to inches.
 2. Reduce 14841 l. to higher denominations.
 3. Reduce $\frac{1}{4}$ of a chain to lower denominations.
 4. Reduce .953 of a mile to links.
 5. A lot having a frontage of 4 rods contains $\frac{1}{4}$ of an acre. What is its depth in chains, links, and inches?
 6. A field 37 ch. 42 l. long, and 30 ch. 21 l. wide, will require how many feet of fence to enclose it?
 7. How many rods of fence wire will enclose a farm 21 ch. 50 l. long and 18 ch. 60 l. wide, if the fence be made 6 wires high?
 8. A garden is $307\frac{3}{4}$ feet long and $250\frac{1}{2}$ feet wide. What is the girt, in chains, links, and inches, of a wall surrounding it?
 9. An errand boy goes from his starting point east 33 ch. 50 l. 3 in., thence north 14 ch. 90 l. 2 in., and returns. How many full steps of 2 feet 4 inches did he take, and what was the remaining distance in inches?

SURVEYOR'S SQUARE MEASURE.

449. The Unit of land measure is the acre.

Table.

625 square links (sq. l.)	= 1 square rod	sq. rd.
16 square rods	= 1 square chain	sq. ch.
10 square chains, or 160 square rods	} = 1 acre	A.
640 acres		
	= 1 square mile	sq. mi.

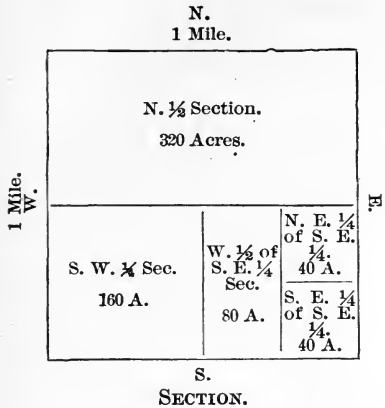
REMARK.—In surveying United States lands, a selected *North* and *South* line is surveyed as a *Principal Meridian*, and an *East* and *West* line, intersecting this, is surveyed as a *Base Line*. From these, other lines are run at right angles, six miles apart, which divide the territory into *Townships* six miles square.

W. 6 miles.		N. 6 miles.						E.	
		6	5	4	3	2	1		
		7	8	9	10	11	12		
		18	17	16	15	14	13		
		19	20	21	22	23	24		
		30	29	28	27	26	25		
		31	32	33	34	35	36		
		S. TOWNSHIP.							

The surface of the earth being convex, these *meridians* converge slightly. The townships and sections are, therefore, not perfectly rectangular; thus is created the necessity for occasional *offsets* called *Correction Lines*.

Each township (Tp.) is divided into 36 equal squares of 1 square mile each, as shown in the first diagram. These squares are called sections (Sec.), and are divided into halves and quarters; each quarter-section, 160 acres, is in turn divided into halves, or lots of 80 acres, and quarter or half lots of 40 acres each, as shown in the second diagram.

The row of townships running north and south is called a *Range*; the townships in each range are numbered north and south from the base line, and the ranges numbered east and west from the principal



meridian. The numbering of the sections in every township is as in the township diagram given, and the corners of all quarter-sections are permanently marked by monuments of stone or wood, and a description of each monument and its location (surroundings) made in the field notes of the surveyor.

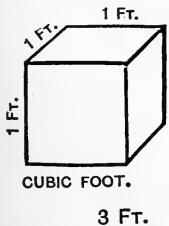
The advantages of the United States survey over all others are: 1st, its official character and uniformity; and 2d, its simplicity. Any one having a sectional map of the United States may place a pencil point upon any described land, thus knowing absolutely its exact location.

For example, Sec. 26, Tp. 24, N. of Range 8, E. of the 5th Principal Meridian, describes a section in the 24th tier of townships north of the base line, and 8th range east of the fifth principal meridian.

EXAMPLES FOR PRACTICE.

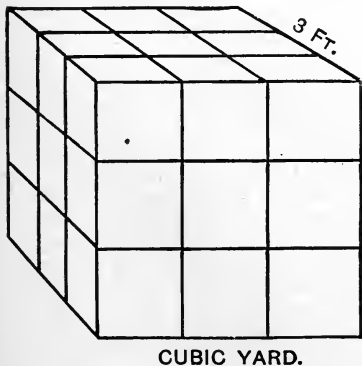
450. 1. Make a diagram of a township, and locate S. $\frac{1}{2}$ of Sec. 21, and mark its acreage.
2. Make a diagram of a township, and locate S. E. $\frac{1}{4}$ of Sec. 16, and mark its acreage.
3. Make a diagram of a township, and locate N. W. $\frac{1}{4}$ of S. W. $\frac{1}{4}$ of Sec. 12, and mark its acreage.
4. Make a diagram of a township, and locate Secs. 35, 26, and E. $\frac{1}{2}$ of 27, and mark their acreage.

CUBIC MEASURE.



451. Cubic Measure is used in measuring *solids* or volume.

452. A **Solid** is that which has *length*, *breadth*, and *thickness*; as the walls of buildings, bins of grain, timber wood, stone, etc.



453. A **Cubé** is a regular solid bounded by *six equal square sides*, or *faces*; hence its length, breadth, and thickness are equal.

454. The **Measuring Unit of solids** is a cube, the edge of which is a linear unit. Thus a cubic foot is a cube, each edge of which is 1 foot; a cubic yard is a cube, each edge of which is 1 yard. See the accompanying diagrams.

455. To Find the Volume of a Solid.

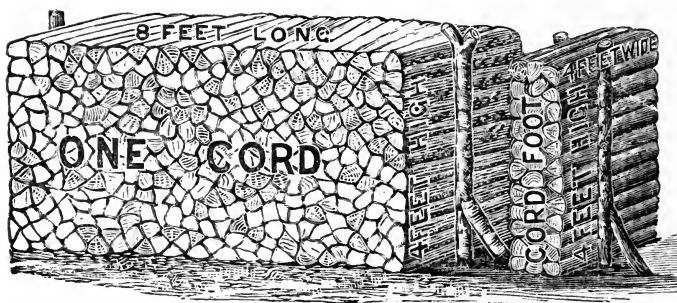
Rule.—*Multiply together its length, breadth, and thickness.*

Table.

1728 cubic inches (cu. in.)	= 1 cubic foot	...cu. ft.
27 cubic feet	= 1 cubic yard	...cu. yd.
128 cubic feet	= 1 cord of wood	...cd.

Special Cubic Measures.

100 cubic feet	= 1 register ton (shipping).
40 cubic feet	= 1 freight ton (shipping).
$16\frac{1}{2}$ cubic feet	= 1 perch of masonry.



456. A Cord of wood is a pile 8 feet long, 4 feet wide, and 4 feet high.

457. A Cord Foot is one foot in length of such a pile.

458. To Find the Cubical Contents of Square Timber.

Rule.—*Multiply together the feet measurements of length, width, and depth.*

459. To Carry Timbers, one person supporting an end and two others with bar.

DIRECTIONS.—Let the *two* with the bar lift at a point $\frac{1}{4}$ the length from the end.

REMARK.—1. Formerly a perch of masonry was $24\frac{1}{4}$ cu. ft.; but the perch of $16\frac{1}{2}$ cu. ft., which is $16\frac{1}{2}$ ft. long, 1 ft. high, and 1 ft. wide, is now in general use.

2. A cubic yard of earth is called a load.

3. Mechanics estimate their work on walls by the girt, and no allowance is made for windows or doors. In estimating the amount of material required, such allowances are made.

Formulas for Rectangular Solids.

$$\text{Length} \times \text{Breadth} \times \text{Height} = \text{Volume.}$$

$$\text{Volume} \div (\text{Length} \times \text{Breadth}) = \text{Height.}$$

$$\text{Volume} \div (\text{Length} \times \text{Height}) = \text{Breadth.}$$

$$\text{Volume} \div (\text{Breadth} \times \text{Height}) = \text{Length.}$$

REMARK.—The three given dimensions must be expressed in units of the same denomination

460. To Find the Number of Bricks for a Wall

Rule.—*Multiply the cubic feet by $23\frac{1}{4}$, and add $\frac{1}{10}$.*

REMARK.—For guide in purchasing material the above will be found correct for bricks 8 in. \times 4 in. \times 2 in., after allowing for mortar.

461. To Find the Number of Perches in a Wall.

Rule.—*Divide the contents of the wall, in feet, by $16\frac{1}{2}$.*

EXAMPLES FOR PRACTICE.

462. 1. Reduce 468093 cu. in. to higher denominations.
2. Reduce 132 cu. yd. 11 cu. ft. 981 cu. in. to cubic inches.
3. What is the volume of a solid 8 ft. 3 in. long, 5 ft. 10 in. high, and 4 ft. 6 in. wide?
4. How many cubic feet of air in a room 26 ft. 8 in. long, 22 ft. 6 in. wide, and 12 ft. high?
5. How many cubic yards of earth must be removed in digging a cellar 60 ft. long, $30\frac{1}{2}$ ft. wide, and $7\frac{1}{2}$ ft. deep?
6. How many perches of masonry, of $16\frac{1}{2}$ feet each, in a wall 85 ft. long, 32 ft. high, and $1\frac{1}{2}$ ft. thick?
7. Reduce $\frac{5}{8}$ of a cubic inch to the fraction of a cubic yard.
8. What decimal part of a cubic yard is 7 cu. ft. 108 cu. in.?
9. What fraction of a cubic foot is 220 cu. in.?
10. Reduce .525 of a cubic yard to lower denominations.
11. What will be the cost, at 21¢ per cubic yard, of excavating for a reservoir 180 ft. long, 105 ft. 3 in. wide, and 15 ft. 9 in. deep?
12. What will be the cost of building the walls of a block 140 ft. long, 66 ft. wide, and 57 ft. high, at \$1.40 per perch of $16\frac{1}{2}$ cu. ft., if the wall is 16 in. thick, and no allowance be made for openings?
13. How many common bricks will be required for the above wall, allowance being made for 28 windows each $3\frac{1}{2}$ ft. wide and 8 ft. high, 48 windows each 3 ft. 9 in. wide and 8 ft. high, and 4 doors each 8 ft. wide and 11 ft. high?
14. A room 28 ft. long, 18 ft. wide, and 12 ft. high, will store how many cords of wood?
15. How many cords of wood in a pile 108 ft. long, 7 ft. 9 in. high, and 6 ft. wide?
16. From a pile of wood 71 ft. 6 in. long, 9 ft. 4 in. wide, and 6 ft. 8 in. high, $21\frac{3}{4}$ cords were sold. What was the length of the pile remaining?
17. At \$4.75 per cord, what will it cost to fill with wood a shed 34 ft. long, 18 ft. wide, and 10 ft. high?
18. What is the weight of a block of granite 11 ft. 3 in. long, 3 ft. 6 in. thick, and 8 ft. 4 in. wide, if it weighs 166 lb. per cubic foot?
19. What is the weight of a white oak timber 15 in. square and 40 ft. long, if the weight per cubic foot be 72.5 lb.?
20. How many cubes 1 in. on each edge can be cut from a cubic yard of wood, if no allowance be made for waste by sawing?
21. Find the contents of a cube, each edge of which is 2 yd. $7\frac{1}{2}$ in.
22. How many perches of masonry in a wall $7\frac{1}{2}$ ft. high and 2 ft. thick, enclosing a yard $12\frac{1}{4}$ rods long and $9\frac{1}{2}$ rods wide? How many bricks will be required, and if bricks cost \$6.50 per M and laying them cost \$1.60 per M, what will be the cost of the wall?

23. What is the volume of a rectangular solid 11 ft. long, $4\frac{1}{2}$ ft. wide, and 4 ft. high?

24. A cask holding 256 $\frac{1}{4}$ gal. of water will hold how many bushels of wheat?

PRODUCERS' AND DEALERS' APPROXIMATE RULES.

463. To find the contents of a bin or elevator in bushels, stricken measure.

RULE.—*Multiply the cubic feet by .8, and add 1 bushel for each 300, or in that proportion.*

To find the contents of a bin or crib in bushels, by heaped measure.

RULE.—*Multiply the cubic feet by .63.*

REMARK.—If the crib flare, take the mean width.

To find the number of shelled bushels in a space occupied by unshelled corn.

RULE.—*Divide the cubic inches by 3840, or multiply the cubic feet by .45.*

To find the dimensions of a bin to hold a certain number of bushels.

RULE.—*To the number of bushels add one-fourth of itself, and the sum will be the cubic feet required, to within one three-hundredth part.*

To find the exact number of stricken bushels in a bin. RULE.—*Divide the cubic inches by 2150.42.*

To find the exact number of heaped bushels in a bin. RULE.—*Divide the cubic inches by 2747.71.*

To find the capacity of circular tanks, cisterns, etc. RULE.—*The square of the diameter, multiplied by the depth in feet, will give the number of cylindrical feet. Multiply by $5\frac{1}{2}$ for gallons, or multiply by .1865 for barrels.*

REMARK.—In tanks or casks having bilge, find the mean diameter by taking one-half of the sum of the diameters at the head and bilge.

To find the number of perches of masonry in a wall, of $24\frac{3}{4}$ cubic feet in a perch. RULE.—*Multiply the cubic feet by .0404.*

To find the number of perches of masonry in a wall, of $16\frac{1}{2}$ cubic feet in a perch. RULE.—*Multiply the cubic feet by .0606.*

REMARK.—The above is correct within $\frac{1}{1000}$ part. In large contracts add $\frac{1}{10}$ of 1%.

EXAMPLE.—How many perches, of $24\frac{3}{4}$ cu. ft. each, in a wall 150 ft. long, 50 ft. high, and 2 ft. thick?

EXPLANATION.—*Short Method.*— $150 \times 50 \times 2 = 15000$; $15000 \times .0404 = 606$; add $\frac{1}{1000}$, or .606 = 606.606.

Extended Method.— $150 \times 50 \times 2 = 15000$; $15000 \div 24.75 = 606.6$, same as before.

Same example, perch of $16\frac{1}{2}$ cu. ft.

EXPLANATION.—*Short Method.*— $150 \times 50 \times 2 = 15000$; $15000 \times .0606 = 909$; add $\frac{1}{1000}$ = .9; $909 + .9 = 909.9$.

To find the number of cubic feet in a log. RULE.—*Divide the average diameter in inches by 3, square the quotient, multiply by the length of the log in feet, and divide by 36.*

To find the number of feet, board measure, in a log. **RULE.**—*Multiply the cubic feet, as above obtained, by 9.*

HAY MEASUREMENTS.

464. Few products are so difficult of accurate measurement as hay, owing to the pressure, or the want of it, in packing, time of settling, volume in bulk, and freedom from obstruction in packing. Plainly, the larger (higher) the stack, or mow, and the greater the foreign weight in compress, the more compact it will be.

465. The accepted measurements are of three kinds:

1st. To find the weight of hay in a load or shed loft, unpressed. **RULE.**—*Allow 540 cubic feet for a ton.*

2d. To find the weight in common hay barn, or small (low) stack. **RULE.**—*Allow 405 cubic feet for a ton.*

3d. To find the weight in mow bases in barns, compressed with grain, and in butts of large stacks of timothy hay. **RULE.**—*Allow 324 cubic feet for a ton.*

CUBE ROOT.

466. The **Cube** or **Third Power** of a number, is the *product* of three equal factors.

467. The **Cube Root** of a number is one of the three equal factors the *product* of which represents the *cube*. Thus, a cubic foot = $12 \times 12 \times 12$, or 1728 cubic inches, the product of its length, breadth, and thickness; and since 12 is one of the three equal factors of 1728, it must be its *cube root*.

468. The operation of finding one of the equal factors of a cube is called *extracting the cube root*.

469. As shown in the explanation of extracting the square root, the first point to be settled in extracting any root is the relative number of unit orders or places in the number and its root.

470. Units and Cubes Compared.

UNITS.	CUBES.
$1^3 = 1$	
$2^3 = 8$	
$3^3 = 27$	
$4^3 = 64$	
$5^3 = 125$	
$6^3 = 216$	
$7^3 = 343$	
$8^3 = 512$	
$9^3 = 729$	
$10^3 = 1000$	

REMARK.—From this comparison may be inferred the following:

1st. The cube of any number expressed by a single figure cannot have less than one nor more than three places or unit orders.

2d. Each place added to a number will add three places to its cube.

3d. If a number be separated into periods of three figures each, beginning at the right hand, the number of places in the root will equal the number of periods and partial periods if there are any.

471. To help in understanding the cube root, first form a *cube* and thus ascertain its *component parts* or *elements*. Take 57 as the number to be cubed.

OPERATION.

$$\begin{array}{r}
 50 + 7 \\
 \hline
 50 + 7 \\
 (50 \times 7) + 7^2 \\
 \hline
 50^2 + (50 \times 7) \\
 50^2 + 2 (50 \times 7) + 7^2 \\
 \hline
 50 + 7 \\
 (50^2 \times 7) + 2 (50 \times 7^2) + 7^3 \\
 \hline
 50^3 + 2 (50^2 \times 7) + (50 \times 7^2) \\
 \hline
 50^3 + 3 (50^2 \times 7) + 3 (50 \times 7^2) + 7^3 = 185193
 \end{array}$$

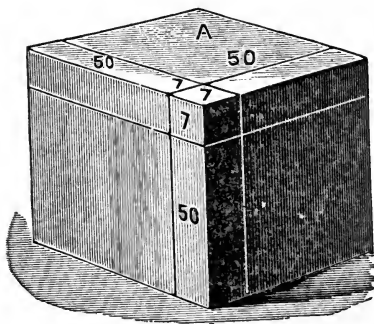
EXPLANATION.—Cubing 57, we have $57 \times 57 \times 57 = 185193$; or, separating 57 into its tens and units gives 5 tens or $50 + 7$ units; or, $50 + 7$. Cube the given number, by using it in this form three times as a factor, and the result is 185193.

472. From this result observe that $57^3 =$ the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens, multiplied by the square of the units, plus the cube of the units; or that the cube of any number made up of *tens* and *units* $= t^3 + 3t^2u + 3tu^2 + u^3$, which for the purpose of reference we will call *Formula (a)*. And if all orders above simple units are considered tens, *Formula (a)* will apply to the cube of any number.

473. To assist in understanding the operation of extracting the cube root, observe the forms and dimensions of the illustrative blocks, and the relation of each to the other in the formation of the complete cube.

OPERATION

$$\begin{array}{r}
 t \quad u \\
 t^3 + 3t^2u + 3tu^2 + u^3 = 185.193 \quad | \quad 5 \quad 7 \\
 t^3 = 125 \quad \text{--- or } 125000 \\
 \hline
 3t^2u + 3tu^2 + u^3 = 60193 = \text{rem.}
 \end{array}$$



$$\begin{array}{r}
 t = 50 \\
 t^2 = 2500 \\
 3t^2 = 7500 \\
 3t = 150 \\
 \hline
 3t^2 + 3t = 7650 \text{ trial divisor.} \\
 3t^2u = 52500 \\
 3tu^2 = 7350 \\
 u^3 = 343 \\
 \hline
 3t^2u + 3tu^2 + u^3 = 60193.
 \end{array}$$

EXPLANATION.—Since the block (A) is a cube, the number representing the length of its side will be its cube-root.

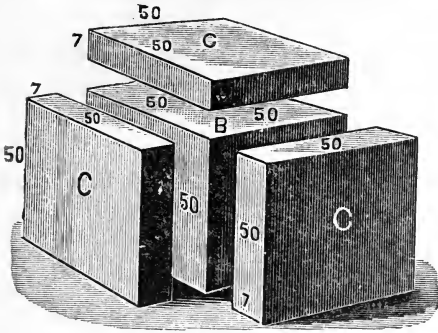
The given number consists of two periods of three figures each, therefore its cube root will contain two places, tens and units.

Since the given number is a product of its root taken three times as a factor, the first figure, or highest order of the root, must be obtained from the first left hand period, or highest order of the power; therefore, find first the greatest cube in 185; since 185 comes between 125 (the cube of 5) and 216 (the cube of 6) the tens of the root must be 5 plus a certain remainder; therefore, write 5 in the root as its *tens* figure.

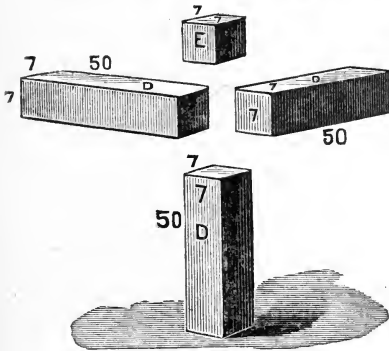
Subtracting the cube of the root figure thus found (5 tens, or 50)³ = 125000, by taking 125 from the left hand period, 185, and so obviate the necessity of writing the ciphers; to this remainder bring down the next, or right hand period, 193, thus obtaining as the entire remainder 60193.

Referring to *Formula (a)*, observe that, having subtracted from the given number the cube of its *tens* (t^3), the remainder, 60193, must be equal to $3t^2u + 3tu^2 + u^3$

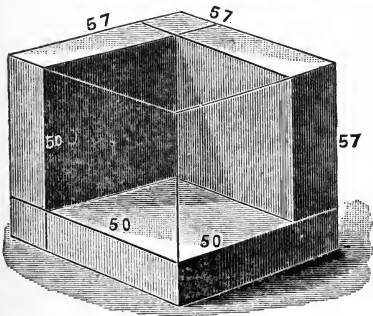
If a cube (B), 50 inches in length on each side, is formed, its contents will equal 125000



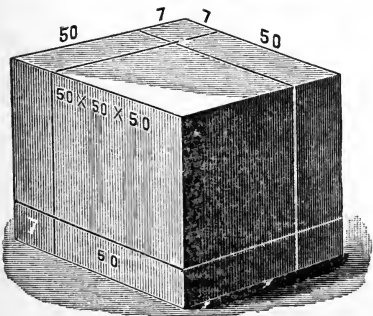
cubic inches, and it will be shown that the remaining 60193 cubic inches are to be so added to cube (B) that it will retain its cubical form. In order to do this, equal additions must be made to three adjacent sides, and these three sides, being each 50 inches in length and 50 inches in width, the addition to each of them in surface, or area, is 50×50 , or 50^2 , and on the three sides, $3(50^2)$, or $3t^2$, as in the squares (C). It will also be observed that three oblong blocks (D) will be required to fill out the vacancies in the edges, and also the small cube (E), to fill out the corner.



Since each of the oblong blocks has a length of 5 tens, or 50, inches, the three will have a length of 3×50 inches, or $3t$. Observe, now, the surface to be added to cube (B), in order to include in its contents the 60193 remaining cubic inches, has been nearly, but not exactly obtained; and since cubic contents divided by surface measurements must give units of length, the thickness of the three squares (C), and of the three oblong pieces (D), will be determined by dividing 60193 by the surface of the three squares, plus the surface of the three oblong blocks, or by $3t^2 + 3t$; this division may give a quotient too large owing to the omission in the divisor of the small square in the corner; hence such surface measure taken as a divisor, may with propriety, be called a *trial divisor*. So using it, 7 is obtained as the second, or unit figure of the root.



Assuming this 7 to be the thickness of the three square blocks (C), and both the height and thickness of the three oblong blocks (D), gives for the solid contents of the three square blocks (C), 52500, and for the solid contents of the three oblong blocks (D), 7350, or $3t^2u + 3tu^2 = 59850$; and by reference to the *Formula (a)*, observe that the only term or element required to complete the cube of $(t + u)$ is the cube of the units (u^3).



Now, by reference to the illustrative blocks, observe that by placing the small cube (E) in its place in the corner, the cube is complete. And since (E) has been found to contain $7 \times 7 \times 7$, or 343 cubic inches, add this to the sum of $3t^2u + 3tu^2 + u^3$, and obtain $3t^2u + 3tu^2 + u^3 = 60193$; and if to this t^3 , or 125000 is added, the result is $t^3 + 3t^2u + 3tu^2 + u^3 = 185193$, *Formula (a)*; then subtracting 60193 from the remainder, 60193, nothing remains.

This proves that the cube root of 185193 is 57. By the operation is also proved the correctness of *Formula (a)*: The cube of any number equals the cube of its tens, plus three times the square of its tens mul-

tiplied by its units, plus three times its tens multiplied by the square of its units, plus the cube of its units.

Rule.—I. *Beginning at the right, separate the given number into periods of three figures each.*

II. *Take for the first root figure the cube root of the greatest perfect cube in the left hand period; subtract its cube from this left hand period, and to the remainder bring down the next period.*

III. *Divide this remainder, using as a trial divisor three times the square of the root figure already found, so obtaining the second or units figure of the root; next, subtract from the remainder three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

REMARKS.—1. In examples of more periods than two, proceed as above, and after two root figures are found, treat both as tens for finding the third root figure. For finding subsequent root figures, treat all those found as so many tens.

2. In case the remainder, at any time after bringing down the next period, be less than the *trial divisor*, place a cipher in the root and proceed as before.

3. Should the cube root of a *mixed* decimal be required, form *periods* from the decimal point right and left. If the decimal be *pure*, point off from the decimal point to the right, and if need be annex decimal ciphers to make periods full.

4. To obtain approximate roots of imperfect cubes, to any desired degree of exactness, annex and use decimal periods.

5. The cube root of a common fraction is the cube root of its numerator divided by the cube root of its denominator.

6. The cube root of any common fraction may be found to any desired degree of exactness, either by extracting the root of its terms separately (adding decimal periods if need be) or by first reducing the common fraction to a decimal and then extracting the root.

7. The 4th root can be obtained by extracting the square root of the square root.

8. The 6th root is obtained by taking the cube root of the square root, or the square root of the cube root.

EXAMPLES FOR PRACTICE.

474. 1. What is the cube root of 1728 ?

2. What is the cube root of 15625 ?

3. What is the cube root of 110592 ?

4. What is the cube root of 65939264 ?

5. Find the cube root of 2146, to three decimal places.

6. Find the cube root of 119204, to two decimal places.

7. Find the cube root of 46982, to one decimal place.

8. Find the cube root of $\frac{64}{125}$.

9. Find the cube root of $\frac{17728}{15625}$.

10. Find the cube root of $\frac{110592}{65939264}$.

11. Find the cube root of $\frac{501}{716}$, to one decimal place.

12. Find the cube root of $\frac{1000}{125000}$, to two decimal places.

13. Find the cube root of 25.416237, to two decimal places.

14. Find the cube root of 3496.25, to three decimal places.

15. Find the cube root of .4106, to three decimal places.

16. Find the decimal equivalent of the cube root of $\frac{1}{16}$, to two decimal places, by reducing the fraction to a decimal of six places and extracting the root of the decimal.

17. What must be the hight of a cubical bin that will hold 1000 bu. of wheat?
18. The width and hight of a crib of unshelled corn are equal, and each is one-third of its length. If the contents of the crib are 7465 bushels, what is its length?
19. If the hight of an oat bin is twice its width, and its length is three and one-half times its hight, what must be its dimensions, if the bin holds 1750 bushels?
20. A cubical cistern contains 630 barrels. How deep is it?
21. A square cistern, the capacity of which is 420 barrels, has a depth equal to only one-half its width. Find its dimensions.

DUODECIMALS.

475. Duodecimals are denominate fractions of either linear, square, or cubic measure. They are found by successive divisions of the unit by 12, and are added, subtracted, multiplied, and divided in the same manner as compound numbers, though they may be treated as fractions, 12 being the uniform denominator. The scale is uniformly 12.

476. The Unit of measure in Duodecimals is the foot. Its first division by 12 gives *primes* ('); primes divided by 12 give *seconds* ("), seconds divided by 12 give *thirds* (""), and so on.

REMARK.—Duodecimals are but little used.

MISCELLANEOUS MEASUREMENTS.

477. A Triangle is a plane figure bounded by three straight lines.

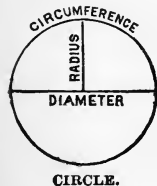
478 To find the area of a triangle, the base and hight being given.

RULE.—*Multiply the base by one-half the hight.*

To find the area of a triangle, when the three sides are given. RULE.—*Find one-half of the sum of the three sides; from this subtract each side separately; multiply together the four results thus obtained, and extract the square root of the product.*

To find the area of any plane figure, the opposite sides of which are equal and parallel. RULE.—*Multiply the base by the perpendicular hight.*

To find the area of a plane figure, whose opposite sides are parallel but of unequal length. RULE.—*Obtain the average length, and multiply by the perpendicular hight.*



479. A Circle is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the center.

480. The Circumference of a circle is the curved line bounding it.

481. The **Diameter** of a circle is a straight line passing through the center and terminating in the circumference.

482. The **Radius** of a circle is a straight line passing from the center to any point of the circumference.

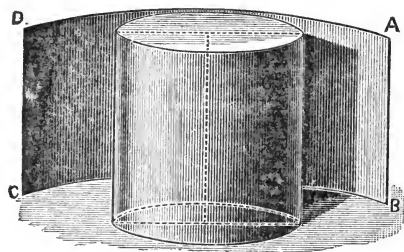
483. To find the circumference of a circle, the diameter being given. **RULE.**—*Multiply the diameter by 3.1416.*

To find the diameter of a circle, the circumference being given. **RULE.**—*Divide the circumference by 3.1416.*

To find the area of a circle, the circumference and diameter being given. **RULE.**—*Multiply the circumference by the diameter, and divide the product by 4.*

To find the side of a square equal in area to a given circle. **RULE.**—*Multiply the circumference by .2821.*

To find the area of a square that can be inscribed within a given circle. **RULE.**—*Multiply the square of the radius by 2, and extract the square root of the result.*



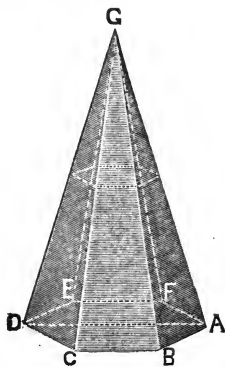
CYLINDER AND RECTANGLE.

484. A **Cylinder** is a circular body of uniform diameter, the ends of which are parallel circles.

REMARK.—The convex surface of a cylinder is equal to the surface of a rectangular body, the length and height of which are equal to the circumference and height of the cylinder. See the figure, A, B, C, D, back of the cylinder in the accompanying diagram.

485. To find the surface or area of a cylinder. **RULE.**—*Multiply the circumference by the height.*

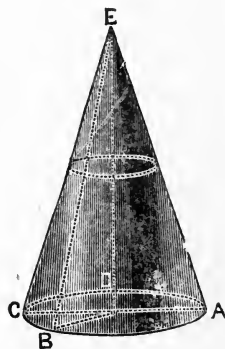
To find the contents of a cylinder. **RULE.**—*Multiply the area of the base by the height.*



PYRAMID.

486. A **Pyramid** is a solid, the base of which has three or more equal sides, terminating in a point called a vertex.

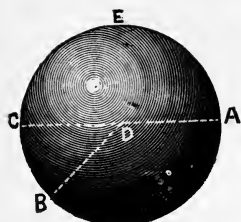
487. A **Cone** is a solid which has a circular base, its convex surface terminating in a point called a vertex.



CONE.

488. To find the surface of a regular pyramid or cone. **RULE.**—*Multiply the perimeter or circumference of the base, by one-half the slant height.*

To find the contents of a pyramid or cone. **RULE.**—*Multiply the area of the base by one-third the perpendicular height.*



SPHERE.

489. A **Sphere** is a solid bounded by a curved surface, all points of which are equally distant from a point within called the center.

490. The **Diameter** of a sphere is a line drawn through its center, terminating each way at the surface.

491. To find the surface of a sphere. **RULE.**—*Multiply the square of its diameter by 3.1416.*

To find the volume of a sphere. **RULE.**—*Multiply the cube of the diameter by .5236.*

To find how large a cube may be cut from any given sphere, or may be inscribed within it. **RULE.**—*Divide the square of the diameter of the sphere by 3, and extract the square root of the quotient; the root thus found will be the length of one side of the cube.*

To gauge or measure the capacity of a cask. **RULE.**—*Multiply the square of the mean diameter in inches by the length in inches, and this product by .0034; the result will be the capacity in gallons.*

REMARK.—In case the cask is only partly full, stand it on end, find the mean diameter of the part filled, multiply its square by the height, and that product by .0034.

EXAMPLES FOR PRACTICE.

REMARK.—In giving one example under each of the several preceding rules in measurements, the object is as much for reference as for practice in solving.

492 1. How many square feet in the gable end of a house 24 ft. wide and 6 ft 6 in. high?

2. Find the number of square yards in a triangular sail, the sides of which are 36 ft., 45 ft., and 48 ft. respectively.

3. How many acres in a rectangular field 108 rods long and 48 rods wide?

4. A farm stretches across an entire section, being 200 rods wide on the west line and 160 rods wide on the east line. How many acres in the farm?

5. How many feet of fence will inclose a circular pond 82.5 ft. in diameter?

6. What is the diameter of a circle, the circumference of which is 90 rods?

7. The diameter of a circular park is 50 rods. How many acres does the park cover?

8. What is the side of a square having an area equal to that of a circle 100 ft. in diameter?

9. What is the largest square timber that can be hewn from a log 42 inches in diameter?

10. What will be the cost of a sheet-iron smoke-stack 40 ft. high and 2 ft. in diameter, at 15¢ per square foot?

11. Find the capacity in gallons of a tank 14 ft. deep and 18 ft. in diameter?

12. A pyramid has a triangular base 3 ft. on each side, and a slant height of 10 ft. Find the number of square feet in its surface.

13. A tent is in the form of a cone; if its slant height is 16 ft. and its base circumference 30 ft., how many square yards of duck were used in making it?
14. How many square inches of leather will cover a foot ball 8 in. in diameter?
15. How many cubic feet in the contents of a globe 4 ft. in diameter?
16. The diameter of the earth is 7901 miles, and that of the planet Jupiter 85390 miles. How many spheres like the earth are equal to Jupiter?
17. What will be the length of the largest cube that can be cut from a sphere 7901 miles in diameter?
18. A cask 28 in. at each end, and 34 in. at the bilge, is 3 ft. long. How many gallons of water will it hold?
19. If a cask 24 inches at the chime, 30 inches at the bung and 3 feet long, is $\frac{3}{4}$ full, how many more gallons may be put into it?

TABLES AND CUSTOMS IN THE PAPER AND BOOK TRADE.

493. Paper in the stationery trade is sold by the following

Table.

24 sheets	-----	= 1 quire.
20 quires	-----	= 1 ream.
2 reams	-----	= 1 bundle.
5 bundles	-----	= 1 bale.
A bale contains 200 quires, or 4800 sheets.		

REMARKS.—1. In copying, a *folio* is usually 100 words.

2. In type-setting, an *em* is the square of the body of a type, used as a unit by which to measure the amount of printed matter on a page.

494. Books are sometimes classified by their size, or the number of pages in a sheet.

Name.	Sheet folded into.	Pages.
Folio, -----	2 leaves, -----	4
Quarto, 4to. -----	4 leaves, -----	8
Octavo, 8vo. -----	8 leaves, -----	16
Duodecimo, 12mo.	12 leaves, -----	24
16mo.	16 leaves, -----	32
18mo.	18 leaves, -----	36
24mo.	24 leaves, -----	48
32mo.	32 leaves, -----	64

Table for Counting.

12 units = 1 dozen.	12 dozen = 1 gross.
20 units = 1 score.	12 gross = 1 great gross.

Table for Land and Lot Measures.

104 $\frac{1}{2}$ feet square = $\frac{1}{4}$ of an acre.	10 rods \times 16 rods = 1 acre.
147 $\frac{1}{2}$ feet square = $\frac{1}{2}$ of an acre.	8 rods \times 20 rods = 1 acre.
208 $\frac{1}{10}$ feet square = 1 acre.	40 yards \times 121 yards = 1 acre.

THE METRIC SYSTEM.

495. The **Metric System** is a decimal system of denominate numbers. It is in use in nearly all the European States, in South America, Mexico, and Egypt. It is also used somewhat in Asia, and is authorized by law in the United States; but its use here is so limited as to justify only a reference to it, and the presentation of its unit equivalents in our weights and measures, as a reference for interested parties.

496. The **Unit of Length** and basis of the system is the *Meter* = 39.37 + inches, being one ten-millionth of the distance from the equator to the pole. The unit of area is the *Ar* (A.); the unit of solidity is the *Ster* (S.); the unit of weight is the *Gram* (G.); the unit of capacity is the *Liter* (L.). Higher denominations are called *Dek'a* (10), *Hek'to* (100), *Kil'o* (1000), and *Myr'ia* (10000). Lower orders are called *Dec'i* (tenths), *Cen'ti* (hundredths), *Mil'li* (thousandths).

Metric Linear Table.

10 mil'li-me'ters (<i>mm</i>)	= 1 cen'ti-me'ter	cm	= $\frac{1}{100}$ M.
10 cen'ti-me'ters	= 1 dec'i-meter	dm	= $\frac{1}{10}$ M.
10 dec'i-me'ters	= 1 Meter	M.	
10 me'ters	= 1 dek'a-me'ter	Dm	= 10 M.
10 dek'a-me'ters	= 1 hek'to-me'ter	Hm	= 100 M.
10 hek'to-me'ters	= 1 kil'o-me'ter	Km	= 1000 M.
10 kil'o-me'ters	= 1 myr'ia-me'ter	Mm	= 10000 M.

REMARKS.—1. All tables are formed in a similar manner.

2. In naming units, abbreviations are commonly used.

3. The system being on a decimal scale, the full mastery of the names of the higher and lower denominations, with unit equivalents, will be sufficient for practical use.

497. An Act of Congress requires all reductions from the Metric to the common system, or the reverse, to be made according to the following

Tables of Equivalents.

LINEAR MEASURE.

1 inch = 2.54 centimeters.	1 centimeter = .3937 of an inch.
1 foot = .3048 of a meter.	1 decimeter = .328 of a foot.
1 yard = .9144 of a meter.	1 meter = 1.0936 yards.
1 rod = 5.029 meters.	1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.	1 kilometer = .62137 of a mile.

SQUARE MEASURE.

1 sq. inch = 6.452 sq. centimeters.	1 sq. centimeter = .155 of a sq. inch.
1 sq. foot = .0929 of a sq. meter.	1 sq. decimeter = .1076 of a sq. foot.
1 sq. yard = .8361 of a sq. meter.	1 sq. meter = 1.196 sq. yards.
1 sq. rod = 25.293 of a sq. meter.	1 ar = 3.954 sq. rods.
1 acre = 40.47 ars.	1 hektar = 2.471 acres.
1 sq. mile = 259 hektars.	1 sq. kilometer = .3861 of a sq. mile.

CUBIC MEASURE.

1 cu. inch = 16.387 cu. centimeter.	1 cu. centimeter = .061 of a cu. inch.
1 cu. foot = 28.317 cu. decimeter.	1 cu. decimeter = .0353 of a cu. foot.
1 cu. yard = .7645 of a cu. meter.	1 cu. meter = 1.308 cu. yard.
1 cord = 3.624 ster.	1 ster = .2759 of a cord.

MEASURES OF CAPACITY.

1 liquid quart = .9463 of a liter.	1 liter = 1.0567 liquid quarts.
1 dry quart = 1.101 liter.	1 liter = .908 of a dry quart.
1 liquid gallon = .3785 of a dekaliter.	1 dekaliter = 2.6417 liquid gallons.
1 peck = .881 of a dekaliter.	1 dekaliter = 1.135 pecks.
1 bushel = .2524 of a hektoliter.	1 hektoliter = 2.8375 bushels.

MEASURES OF WEIGHT.

1 grain, Troy = .0648 of a gram.	1 gram = .03527 of an ounce, Avoir.
1 ounce, Avoir. = 28.35 gram.	1 gram = .03215 of an ounce, Troy.
1 ounce, Troy = 31.104 grams.	1 gram = 15.432 grains, Troy.
1 pound, Avoir. = .4536 of a kilogram.	1 kilogram = 2.2046 pounds, Avoir.
1 pound, Troy = .3732 of a kilogram.	1 kilogram = 2.679 pounds, Troy.
1 ton (short) = .9072 of a tonneau.	1 tonneau = 1.1023 tons (short)

REMARK.—Metric quantities of any unit are read like ordinary decimals.

FRENCH MONEY.

498. The **Legal Currency of France** is decimal, its unit being the *silver Franc*.

499. The French coins are as follows:

Gold	$\left\{ \begin{array}{l} 100 \text{ francs,} \\ 40 \text{ francs,} \\ 20 \text{ francs,} \\ 10 \text{ francs,} \\ 5 \text{ francs.} \end{array} \right.$	Silver	$\left\{ \begin{array}{l} 5 \text{ francs,} \\ 2 \text{ francs,} \\ 1 \text{ franc.} \end{array} \right.$	Bronze	$\left\{ \begin{array}{l} 10 \text{ centimes,} \\ 5 \text{ centimes,} \\ 2 \text{ centimes,} \\ 1 \text{ centime.} \end{array} \right.$
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Table.

10 millimes (m.)	= 1 centime (ct.)	= \$.00193.
10 centimes	= 1 decime (dc.)	= .0193.
10 decimes	= 1 Franc (fr.)	= .193.

MONEY OF THE GERMAN EMPIRE.

500. The **Unit** is the *Mark* = \$.2385 United States money. It is divided into 100 pfennigs (pennies).

The silver *Thaler* = \$.726 United States money.

501. The German coins are:

Gold	$\left\{ \begin{array}{l} 20 \text{ marks,} \\ 10 \text{ marks,} \\ 5 \text{ marks.} \end{array} \right.$	Silver	$\left\{ \begin{array}{l} 20 \text{ marks,} \\ 1 \text{ mark,} \\ 20 \text{ pfennigs.} \end{array} \right.$	Nickel	$\left\{ \begin{array}{l} 10 \text{ pfennigs,} \\ 5 \text{ pfennigs.} \end{array} \right.$
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MISCELLANEOUS EXAMPLES.

502. 1. What is the value, in English money, of \$1750 in United States gold coin?

2. It required 12 yr. 6 mo. 1 da. to build the Brooklyn bridge. If it was completed July 4, 1882, when was its construction begun?

3. What is the board measure of 7 planks, each 16 ft. long, 15 in. wide, and 3 in. thick?

4. How many acres of land can be bought for \$25000, if a square foot costs 25¢?

5. A cellar is 24 ft. square inside of the wall, which is 9 ft. high, and 2 ft. thick. How many perches of $16\frac{1}{2}$ cu. ft. each does the wall contain?

REMARK.—Sometimes $24\frac{1}{2}$ cubic feet are reckoned as a perch, but this is rarely done by contractors or architects; girt measurements are taken.

6. How many shingles, 4 inches wide, laid 6 inches to the weather, would be required to cover the roof of a barn 60 ft. long and 24 ft. wide on each side?

7. The highest chimney in the world is at Port Dundas, Scotland, it being 450 ft. high. How many rods in height is it?

8. The Italian Government pays out yearly \$2140000 to 32590 monks and nuns. What is the average sum received by each?

9. What will be the cost of the plank, at \$18 per M, that will cover a floor 24 ft. by 13 ft., if the plank is $2\frac{1}{2}$ inches in thickness?

10. A farm having 225 rods fronting the road, is 95 rods wide at one end and 72.5 rods at the other. How many acres does the farm contain?

11. If the capacity of a cask is $64\frac{1}{4}$ wine gallons, how many quarts of berries will it hold?

12. A bird can fly 1° in 1 hr. 10 m. 12 sec. At that rate, in what time can it encircle the earth?

13. What will be the cost in Paris of a cargo of 38500 bu. United States wheat, at 10 fr. 60 cent. per hektoliter?

14. How many francs are equal to \$275.

15. The largest shipping lock in the world is at Cardiff, it being 600 ft. long, 80 ft. wide, and 32 ft. deep. What is its capacity in barrels?

16. When it is noon at the point of your observation, what is the time at a point 1500 statute miles due south-west?

17. If your coal costs \$5.60 per ton, and you use 65 lb. per day, what will be the expense of your fire for the months of the winter of 1891–2?

18. How many barrels of water in a cistern 12.5 ft. long, 10 ft. wide, and 7.5 ft. deep?

19. A carriage wheel 4 ft. 3 in. in diameter will make how many revolutions in going 62.5 miles?

20. If Wm. H. Vanderbilt died worth two hundred millions of dollars, in what length of time could his fortune, in silver dollars, be counted by one person, counting 60 per minute and working 10 hours per day for 365 days each year?

21. What will be the cost of 10 sticks 2 in. by 4 in., 10 sticks 2 in. by 6 in., 10 sticks 4 in. by 4 in., and 10 sticks 2 in. by 10 in., if the sticks are each 16 ft. long and the cost is \$15 per M?

22. How many yards of Axminster carpeting, $\frac{3}{4}$ of a yard in width, and laid lengthwise of the room, will be required to cover a floor $21\frac{3}{4}$ ft. long and $18\frac{3}{4}$ ft. wide, making no allowance for waste in matching the design?

23. How many tons of 324 cu. ft. each, in a mow of hay 36 ft. 3 in. long, 18 ft. 10 in. wide, and 13 ft. 6 in. high?

24. Two astronomers, located at different points, observed at the same instant of time an eclipse of the moon, one seeing it five minutes after 9 P. M., local time, and the other five minutes before midnight. How many degrees of longitude separated the observers?

25. How many Avoirdupois pounds in 10 myriagrams 4 kilograms.

26. If the sun is 93 millions of miles from the earth, and a cannon ball travels nine miles per minute, at what time would a ball fired from the earth at one minute after 3 o'clock P. M., Dec. 25, 1889, reach the sun at that rate?

27. How many francs are equal to £425?

28. Allowing 305 sq. ft. for doors and windows, what will be the cost, at 40¢ per square yard, of plastering the ceiling and walls of a room 45 ft. long, $35\frac{1}{2}$ ft. wide, and 12 ft. 3 in. high?

29. How many German marks are equal to \$1500 United States money?

30. A pile of wood built 10 ft. high and 22 ft. wide must be how long to contain 125 cd.?

31. The main centennial building at Philadelphia in 1876 was 1880 ft. long and 464 ft. wide. What was its area in acres, square rods, and square feet?

32. Reduce 47 mi. 216 rd. 11 ft. 5 in. to metric units.

33. If £2 4 s. 6 d. is paid for a coat and vest, and the coat costs 4 s. more than twice as much as the vest, what is the cost of each, in United States money?

34. From .001 of a section, plus .01 of an acre, take .001 of a quarter section, plus .01 of a square rod.

35. A grocer bought 12 bu. of chestnuts, at \$3.50 per bushel dry measure, and sold them at 15¢ per quart liquid measure. Did he gain or lose, and how much?

36. How many dollars are equal to 2150 francs?

37. How many square feet of sheet lead will be required to line a tank 7 ft. in diameter and 12 ft. deep?

38. If bricks cost \$5.50 per M, what will be the cost of the brick for a wall 12 ft. high and 3 ft. thick, enclosing an acre of land 10 rd. wide and 16 rd. long?

39. The gold coin of the commercial world suffers each year a loss of one ton by wear or abrasion. What is the value, in United States gold dollars, of the loss thus sustained?

40. Reduce 250 hektars to common units.

41. What will be the cost, at \$16.00 per M, of a tapering board 18 ft. long, and 9 in. wide at one end and $16\frac{1}{2}$ in. wide at the other?

42. A German immigrant having 1000 thalers and 500 marks, exchanges them for United States money. How many dollars should he receive?

43. The hight, width, and length of a shed are equal. What are its dimensions, if it will contain 125 cords of wood?

44. A train of 45 cars of Lehigh coal averages, by the long ton, 23 T. 7 cwt 3 qr. 10 lb. per car. What is the value of the coal, at \$5.25 per short ton?
45. How many feet of lumber in a box $6\frac{3}{4}$ ft. long, $5\frac{1}{2}$ ft. wide, and $3\frac{1}{2}$ ft. deep, inside measurements given, and lumber 1 inch in thickness?
46. What will be the cost of carpeting $\frac{3}{4}$ yd. wide, and lining $\frac{7}{8}$ yd. wide, to cover a room 24 ft. long and 20 ft. wide, if the strips of carpet are laid the long way of the room and there is a waste of 9 inches at one end in matching, also an allowance of 10% in width and 6% in length for shrinkage of the lining, the carpet selling at \$2.25 per yd., and the lining at 30¢ per yd.
47. A pile of wood 56 meters long; $18\frac{1}{2}$ meters wide, and $3\frac{3}{4}$ meters high, was sold at \$6 per cord. How much was received for it?
48. A farmer filled a bin 9 ft. wide, 12 ft. long, and $7\frac{1}{2}$ ft. deep, with wheat grown from a field yielding $32\frac{1}{4}$ bu. per acre. How long was the field, if its width was 50 rods?
49. Seasoned pine in freighting is estimated to weigh 3000 lb. per M, and green oak 5000 lb. per M. How much freight must I pay, at \$1 per ton, on a car load of 3265 ft. of pine and 3795 ft. of oak?
50. How long is the side of the largest cube that can be cut from a spherical snow ball 5 ft. in diameter?
51. Glenn's California reaper will in 12 hours cut, thresh, winnow, and put into bags, 30 A. of wheat. How many days, of 15 working hours each, will it require to harvest and thresh the wheat of a field 125 rods wide and 240 rods long?
52. An ounce of gold can be so beaten as to cover 146 sq. ft. What weight of gold would be required for a sheet which will cover an acre of ground?
53. A farmer having 1240 bu. of corn in the ear to store in two rail cribs, builds each 9 ft. square on the inside. If one is built 10 ft. high and filled, how high must the other be built to hold the remainder?
54. The Hercules ditcher, of Michigan, removes 750 cu. yd. of earth per hour. In how many days, of 12 working hours each, can it dig a ditch 7 miles in length, 8 ft. in depth, 24 ft. wide at the surface, and 16 ft. at the bottom?
55. If a car carrying 20 tons of freight is with its couplings 42 ft. long, what would be the length of a train carrying Vanderbilt's two hundred millions of dollars, if it is all in standard silver dollars, and any fractional part of a car load is rejected?

PERCENTAGE.

503. **Percentage** is a term applied to computing by hundredths.

504. The **Elements** of Percentage are, the *Base*, the *Rate*, the *Amount Per Cent.*, the *Difference Per Cent.*, the *Percentage*, the *Amount*, and the *Difference*.

505. The **Base** is the number upon which the *percentage* is computed.

506. The **Rate Per Cent.** denotes how many hundredths of the base are to be taken, and is usually expressed as a decimal.

507. **Per Cent.** is an abbreviation of the Latin words *per centum*, signifying by the hundred, or a certain number of each one hundred parts.

508. The **Sign**, %, is used to denote per cent.

509. The **Rate** may be expressed as a part in a common fractional form, as $\frac{3}{4}$; in the form of an extended decimal, as $.01625 = 1\frac{5}{8}\%$; but only when expressed in *hundredths* can it with strict propriety be considered a *rate per cent.* Thus, $.12$, $.06$, $.15\frac{1}{2}$, $.05\frac{3}{4}$, are each a *rate per cent.*

510. To read per cent., call the first two places *per cent.*, and the added places, if any, fractions of 1 per cent.; as, $.2125$ read as 21 and one-fourth per cent.

511. To express per cent. as a common fraction, write the per cent. for a numerator and 100 for a denominator, and reduce; thus, $25\% = \frac{25}{100} = \frac{1}{4}$.

512. To change a common fraction to an equivalent *per cent.*, apply the decimal explanation, Art. 245. Divide the numerator by the denominator, and give the quotient at least two decimal places.

513. Every rate per cent., being as many hundredths, requires at least two decimals places; hence, if the per cent. be less than 10, a cipher must be prefixed to the figure denoting it; thus, $2\% = .02$.

514. The **Amount Per Cent.** is 100 per cent. increased by the *rate*, or 1 *plus* the rate.

515. The **Difference Per Cent.** is 100 per cent. diminished by the *rate*, or 1 *minus* the rate.

REMARK.—Where the rate per cent. is the equivalent of a common fraction, use in solution whichever is most convenient.

516. The **Percentage** is the sum obtained by multiplying the *base* by the *rate*.

517. The **Amount** is the sum of the *base* and *percentage*.

518. The **Difference** is the remainder after deducting the *percentage* from the *base*.

519. The **Base** is either an abstract or denominate number; the rate per cent. is always abstract, and the percentage, amount, and difference are always like the base.

REMARKS.—1. In all operations where a decimal *rate* is used, too great care cannot be taken to express all decimal terms with exactness.

2. As the greater part of commercial calculations are based upon percentage, the importance of a thorough mastery of its principles will be readily perceived.

520. Since *per cent.* is any number of *hundredths*, it may be expressed either as a *decimal* or as a *common fraction*, and the table of aliquot parts can be used with little variation and to great advantage in many operations in *percentage*. Hence, the rules given under SPECIAL APPLICATIONS may be applied in this subject.

Table.

	Decimal.	Com. Frac.	Lowest Terms.
1 per cent. =	.01	= $\frac{1}{100}$	reducible to $\frac{1}{100}$.
2 per cent. =	.02	= $\frac{2}{100}$	reducible to $\frac{1}{50}$.
3 per cent. =	.03	= $\frac{3}{100}$	reducible to $\frac{3}{100}$.
4 per cent. =	.04	= $\frac{4}{100}$	reducible to $\frac{1}{25}$.
5 per cent. =	.05	= $\frac{5}{100}$	reducible to $\frac{1}{20}$.
6 per cent. =	.06	= $\frac{6}{100}$	reducible to $\frac{3}{50}$.
7 per cent. =	.07	= $\frac{7}{100}$	reducible to $\frac{7}{100}$.
8 per cent. =	.08	= $\frac{8}{100}$	reducible to $\frac{2}{25}$.
9 per cent. =	.09	= $\frac{9}{100}$	reducible to $\frac{9}{100}$.
10 per cent. =	.10	= $\frac{10}{100}$	reducible to $\frac{1}{10}$.
12 per cent. =	.12	= $\frac{12}{100}$	reducible to $\frac{3}{25}$.
14 per cent. =	.14	= $\frac{14}{100}$	reducible to $\frac{7}{50}$.
16 per cent. =	.16	= $\frac{16}{100}$	reducible to $\frac{4}{25}$.
20 per cent. =	.20	= $\frac{20}{100}$	reducible to $\frac{1}{5}$.
25 per cent. =	.25	= $\frac{25}{100}$	reducible to $\frac{1}{4}$.
30 per cent. =	.30	= $\frac{30}{100}$	reducible to $\frac{3}{10}$.
50 per cent. =	.50	= $\frac{50}{100}$	reducible to $\frac{1}{2}$.
75 per cent. =	.75	= $\frac{75}{100}$	reducible to $\frac{3}{4}$.
100 per cent. =	1.00	= $\frac{100}{100}$	reducible to 1.
125 per cent. =	1.25	= $\frac{125}{100}$	reducible to $\frac{5}{4} = 1\frac{1}{4}$.
150 per cent. =	1.50	= $\frac{150}{100}$	reducible to $\frac{3}{2} = 1\frac{1}{2}$.
$1\frac{1}{4}$ per cent. =	.0125	= $\frac{1\frac{1}{4}}{100} = \frac{1.25}{100}$	reducible to $\frac{1}{80}$.
$1\frac{1}{2}$ per cent. =	.0166 $\frac{2}{3}$	= $\frac{1\frac{1}{2}}{100} = \frac{1.5}{100}$	reducible to $\frac{1}{60}$.
$2\frac{1}{2}$ per cent. =	.025	= $\frac{2\frac{1}{2}}{100} = \frac{2.5}{100}$	reducible to $\frac{1}{40}$.
$3\frac{1}{2}$ per cent. =	.033 $\frac{1}{3}$	= $\frac{3\frac{1}{2}}{100} = \frac{3.5}{100}$	reducible to $\frac{1}{30}$.
$6\frac{1}{4}$ per cent. =	.0625	= $\frac{6\frac{1}{4}}{100} = \frac{6.25}{100}$	reducible to $\frac{1}{16}$.
$8\frac{1}{3}$ per cent. =	.0833 $\frac{1}{3}$	= $\frac{8\frac{1}{3}}{100} = \frac{8.33}{100}$	reducible to $\frac{1}{12}$.
$12\frac{1}{2}$ per cent. =	.125	= $\frac{12\frac{1}{2}}{100} = \frac{12.5}{100}$	reducible to $\frac{1}{8}$.
$16\frac{2}{3}$ per cent. =	.166 $\frac{2}{3}$	= $\frac{16\frac{2}{3}}{100} = \frac{16.66}{100}$	reducible to $\frac{1}{6}$.
$33\frac{1}{3}$ per cent. =	.333 $\frac{1}{3}$	= $\frac{33\frac{1}{3}}{100} = \frac{33.33}{100}$	reducible to $\frac{1}{3}$.
$62\frac{1}{2}$ per cent. =	.625	= $\frac{62\frac{1}{2}}{100} = \frac{62.5}{100}$	reducible to $\frac{5}{8}$.
$66\frac{2}{3}$ per cent. =	.66 $\frac{2}{3}$	= $\frac{66\frac{2}{3}}{100} = \frac{66.66}{100}$	reducible to $\frac{2}{3}$.
$87\frac{1}{2}$ per cent. =	.875	= $\frac{87\frac{1}{2}}{100} = \frac{87.5}{100}$	reducible to $\frac{7}{8}$.

521. The relation between the elements of Percentage is such, that by the application of the General Principles of Multiplication and Division, if any two of the elements, except amount per cent. and difference per cent., are given, the other three may be found.

522. To find the Percentage, the Base and Rate being given.

EXAMPLE.—What is 25% of \$468 ?

OPERATION.

\$468 = base.

.25 = rate per cent.

\$117.00 = percentage.

FIRST EXPLANATION.—25 per cent. equals .25; therefore, 25 per cent. of \$468 equals \$468 multiplied by .25, equals \$117.

SECOND EXPLANATION.—\$468 is 100 per cent. of itself; and since 25 per cent. equals $\frac{1}{4}$ of 100 per cent., 25 per cent. of \$468 will be $\frac{1}{4}$ of that sum, or \$117.

Rules.—1. *Multiply the base by the rate expressed decimally.* Or,

2. *Take such a part of the base as the number expressing the rate is part of 1.*

REMARK.—When the rate is an aliquot part of 100, the percentage may be found by taking a like part of the base; thus, for 10% take $\frac{1}{10}$, for 25% take $\frac{1}{4}$, for 33 $\frac{1}{3}$ % take $\frac{1}{3}$, etc.

Formula.—Percentage = Base \times Rate.

EXAMPLES FOR MENTAL PRACTICE.

523. What is

- | | | |
|--------------------------|---|---------------------------|
| 1. 5 per cent. of 100 ? | 4. 20 per cent. of 500 ? | 7. 25 per cent. of 1440 ? |
| 2. 12 per cent. of 600 ? | 5. 25 per cent. of 1200 ? | 8. 8 per cent. of 450 ? |
| 3. 15 per cent. of 800 ? | 6. 33 $\frac{1}{3}$ per cent. of $\frac{1}{10}$ | 9. 50 per cent. of 680 ? |

EXAMPLES FOR WRITTEN PRACTICE.

524. 1. A man owning 250 acres of land, sold 20% at one time, and 25% of the remainder at another time. How many acres did he have left ?

2. If a ranchman having 5450 sheep, lost 20% by a storm and afterwards sold 20% of those remaining, how many sheep did he sell ?

3. A collector deposited \$13500 in coin, and 12 $\frac{1}{4}$ % more than that amount in bank bills. What was the total of his deposit ?

4. Find 11 $\frac{1}{8}$ % of 1680 lb. of wool.

5. Find 16 $\frac{2}{3}$ % of 12 lb. 3 oz. of silver.

6. From a charge of \$675, made for a bill of goods, 8% was deducted. What was the net amount of the bill ?

7. If 526 barrels of salt were bought for \$1.10 per bar., and sold at an advance of 15%, what was gained ?

8. Two men, each having \$12500, made investments, from which one gained 15%, and the other lost 35%. How much did each then have ?

9. How much greater is 12 $\frac{1}{2}$ % of \$1550, than 7 $\frac{1}{2}$ % of \$2150 ?

10. Having raised 1240 bushels of wheat, a farmer used 5% of it for seed and 5% for bread; he then sold to one man 10% and to another 25% of what remained. How many bushels had he left ?

11. Having \$75000 to invest, a gentleman bought United States bonds with $33\frac{1}{3}\%$ of his money, a home with 20%, and invested the remainder equally in farm lands and manufacturing stock. How much did he pay for the farm lands?

12. I owed John Smith \$1750, and paid at one time 20% of the debt, at another time 35% of the remainder, and at another time 25% of what then remained unpaid. How much of the debt did I still owe?

13. A capitalist owning $\frac{3}{4}$ of a coal mine, sold $32\frac{1}{2}\%$ of his share for \$65000. At that rate, what was the entire mine worth?

14. A jobber having bought 2160 bags of coffee, sold at one time $8\frac{1}{3}\%$, at another 25% of what remained, and at a third sale 15% of what still remained. Find the value of what was left, at \$18 per bag.

15. Of a farm containing a half section of land, 15% was in wheat, 32% in oats, 5% in potatoes, and the remainder devoted equally to orchard, corn, beans, and pasture. How many acres were in pasture?

16. A farmer having 156 sheep to shear, agreed to pay for their shearing 4% of the sum received for their wool. If the fleeces averaged $7\frac{1}{2}$ lb. and sold for 30¢ per pound, how much was paid for shearing?

17. A speculator having \$41820, invested 50% of it in oil, on which he lost $16\frac{2}{3}\%$; the remainder he invested in cotton, which he sold at 9% below cost. How much was received from both sales?

18. A trader bought 12 mustangs for \$400, and after selling 25% of the number at a gain of 50%, and $33\frac{1}{3}\%$ of those remaining at a gain of $12\frac{1}{2}\%$, sold those still on hand at \$30 per head. Did he gain or lose, and how much?

525. To find the Base, the Percentage and Rate being given.

REMARK.—Since the base multiplied by the rate produces the percentage, percentage must be a product; if, therefore, it is divided by either factor, the quotient will be the other factor.

EXAMPLE.—By selling 4% of a stock of goods, a merchant realized \$644. What was the value of the entire stock?

OPERATION.

Rate. Percentage.

.04) 644.00

16100 base.

EXPLANATION.—If the value of 4 per cent. is \$644, the value of

1 per cent. will be \$161; and if the value of 1 per cent. is \$161, the value of 100 per cent. will be \$16100.

Rule.—*Divide the percentage by the rate, expressed decimally.*

Formula.—Base = Percentage ÷ Rate.

EXAMPLES FOR MENTAL PRACTICE.

526. 1. $846 = 6\%$ of what number?

2. $2150 = 10\%$ of what number?

3. $543 = 5\%$ of what number?

4. $219 = 33\frac{1}{3}\%$ of what number?

5. $150 = \frac{3}{4}\%$ of what number?

6. A man sold 25% of his farm for \$2120. How much was the farm worth at that rate?

7. What is the value of a house renting for \$360 per year, if the rent equals 9% of its value?
8. How many acres in a farm of which 12.5 acres is but 5%.
9. Of what sum is \$36 but $33\frac{1}{3}\%$?

EXAMPLES FOR WRITTEN PRACTICE.

527. 1. A planter sold 76 bales of cotton, which was 19% of his crop. How many bales did he raise?
2. I paid \$123.48, which was $16\frac{2}{3}\%$ of a debt. What amount did I owe?
3. A lady paid for millinery, \$17.50; for shoes, \$11.40; for jewelry, 113.80; for furs, \$78.55; and had expended but 15% of her money. How many dollars had she at first?
4. A clerk's present salary of \$520 per year is only 75% of what he formerly received. How much was formerly paid him?
5. A grocer, after increasing his stock to the amount of \$6448, found that the new purchase was but 16% of the old stock on hand. What was the value of his old stock?
6. The owner of 68% of a mine, received \$91510 from the sale of 25% of his share. Find the value of the entire mine at that rate?
7. A, B, C, and D are partners; A furnished 15% of the capital, B 25%, C 42%, and D \$16200. What was the capital of the firm?
8. A Wyoming ranchman lost 1684 cattle during a blizzard. How many had he at first, if his loss was only $1\frac{2}{3}\%$ of his herd?
9. The population of a county increased 22% in ten years. If the births exceeded the deaths by 2166, and the county received 13234 immigrants during the time, what must have been its population before the increase?
10. A speculator owned a quarter interest in a mill, and sold one-quarter of his part for \$11250. What was the mill worth, on that basis of value?

528. To find the Rate, the Percentage and Base being given.

REMARK.—The percentage is a product, the base being one of its factors.

EXAMPLE.—What per cent. of 480 is 120?

FIRST OPERATION.

$$\begin{array}{r}
 480) 120.00 \text{ (25 times.} \\
 \underline{960} \\
 2400 \\
 1\% = .01 \ 2400 \\
 \underline{25} \\
 .25 = 25\%.
 \end{array}$$

FIRST EXPLANATION.—Since 480 is 100 per cent. of itself, 1 per cent. of 480 would be $\frac{1}{100}$ part of it, or 4.80; and since 4.80 is 1 per cent. of 480, 120 would be as many times 1 per cent. as 4.80 is contained times in 120, which is 25 times; and 25 times 1 per cent. = 25 per cent.

SECOND OPERATION.

$$\begin{array}{r}
 480) 120.00 \text{ (.25 = 25\%.} \\
 \underline{960} \\
 2400 \\
 \underline{2400}
 \end{array}$$

SECOND EXPLANATION.—Since the percentage is a product of the base and rate, the quotient obtained by dividing the percentage by the base will be the rate. Or, 120 is $\frac{1}{4}\%$, or $\frac{1}{4}$ of 480; and since 480 is 100 per cent. of itself, 120, which is $\frac{1}{4}$ of 480, must be $\frac{1}{4}$ of 100 per cent., or 25 per cent.

Rule.—*Divide the percentage by the base, carrying the quotient to two decimal places.*

Formula. — $\text{Rate} = \text{Percentage} \div \text{Base}.$

EXAMPLES FOR MENTAL PRACTICE.

529. What per cent. is

- | | | |
|----------------|-----------------------------|-----------------------------|
| 1. 25 of 125 ? | 4. $12\frac{1}{2}$ of 100 ? | 7. $37\frac{1}{2}$ of 150 ? |
| 2. 40 of 160 ? | 5. 15 of 45 ? | 8. 200 of 10 ? |
| 3. 18 of 36 ? | 6. 125 of 1000 ? | 9. 120 of 4 ? |

EXAMPLES FOR WRITTEN PRACTICE.

- 530.** 1. From a herd of 1184 cattle, 296 were sold. What per cent. was sold?
 2. R. G. Dun & Co. charged \$21 for collecting an account of \$600. What rate was charged ?
 3. Sold $\frac{3}{4}$ of a stock of goods for what the entire stock cost. What was my rate of gain ?
 4. What per cent. of 12 lb. 8 oz. is 2 lb. 8 oz., Avoirdupois ?
 5. From a half section, 120 acres were sold, and afterwards 80 acres more. What per cent. was sold ?
 6. Of a stock of 800 bushels of potatoes, 240 bushels were sold at one time, and 135 bushels at another. What per cent. was still unsold ?
 7. A merchant failed, owing \$27984, his assets amounting to \$16090.80. What per cent. of his debts can he pay ?
 8. At a normal school there were enrolled 855 male pupils and only 185 female pupils. What per cent. more were the male than the female pupils ?
 9. A girl having \$5.40, expended \$1.35 for gloves, 45¢ for flowers, and one-half of the remainder for a pair of slippers. What per cent. of her money had she left ?
 10. From a cask of lard of 314 lb., 78.5 lb. were sold at one time, and 25% of the remainder at another. What per cent. of the whole remained unsold ?
 11. Of a regiment of men entering battle, 1040 strong, only 260 came out unhurt, $\frac{1}{3}$ of the remainder having been killed. What per cent. of the whole were killed ?

531. To find the Amount Per Cent., the Rate being given.

EXAMPLE.—If the rate be 7%, what is the amount per cent.?

OPERATION.
 $100\% = 1. = \text{a unit}$
 $7\% = .07 = \text{rate}$

 $107\% = 1.07 = \text{amount per cent.}$

EXPLANATION.—Since the *amount per cent.* (definition, page 160), is always 100 per cent. increased by the rate, we may find it by adding 100 per cent., or 1, to the per cent. given. Hence, if the rate is 7 per cent., the amount per cent. will be 107 per cent.

Rule.—*Add the rate to the unit 1.*

Formula.— $\text{Amount Per Cent.} = 1 + \text{Rate}.$

EXAMPLES FOR MENTAL PRACTICE.

532. 1. If the rate be 10%, what will be the amount per cent.?
 2. If the rate be 75%, what will be the amount per cent.?
 3. If the rate be 110%, what will be the amount per cent.?
 4. Find the amount per cent., if the rate per cent. be $16\frac{2}{3}$?
 5. Find the amount per cent., if the rate per cent. be $87\frac{1}{2}$?

EXAMPLES FOR WRITTEN PRACTICE.

533. 1. Goods costing \$1400 were sold for \$1470. Find the amount per cent. of the selling price?
 2. Last month I sold \$2750 worth of coffee, while the previous month I sold \$3000 worth. What was the amount per cent. of my sales for the previous month as compared with those of the last month?
 3. If tea costing $62\frac{1}{2}\phi$ per pound sell at $87\frac{1}{2}\phi$, what amount per cent. do the sales show as compared with the cost?

534. To find the Difference Per Cent., the Rate being given.

EXAMPLE.—If the rate be 5%, what is the difference per cent.?

OPERATION.

$$100\% = 1.00 = \text{a unit.}$$

$$5\% = .05 = \text{rate.}$$

$$\underline{95\% = .95 = \text{dif. \%}}$$

EXPLANATION.—Since the *difference per cent.* (definition, page 160), is equal to 100 per cent., or 1, less the rate, if we take the given rate, 5 per cent., from 100 per cent., the remainder, 95 per cent., will be the answer required.

Rule.—*Subtract the rate from the unit 1.*

Formula.—Difference Per Cent. = 1 — Rate.

EXAMPLES FOR MENTAL PRACTICE.

535. 1. If the rate be 15%, what is the difference per cent.?
 2. If the rate be $37\frac{1}{2}\%$, what is the difference per cent.?
 3. If the rate be $\frac{1}{2}\%$, what is the difference per cent.?
 4. If the rate be $3\frac{1}{3}\%$, what is the difference per cent.?
 5. If the rate be 70%, what is the difference per cent.?

EXAMPLES FOR WRITTEN PRACTICE.

536. 1. The pupils of a school are reduced in number from 112 to 80. What per cent. is the present of the former attendance?
 2. Walter, having 48 marbles, gave Henry 15. What per cent. had he left?
 3. Find the difference per cent., if the rate equals $\frac{1}{2}$ of $\frac{1}{4}$.

537. To find the Amount, the Base and Rate being given.

EXAMPLE.—What is the amount of 550 increased by 8% of itself?

OPERATION.

$$550 = \text{base.}$$

$$.08 = \text{rate.}$$

$$\underline{44.00} = \text{per cent.}$$

$$550 = \text{base.}$$

$$\underline{594} = \text{amount}$$

EXPLANATION.—The amount equals base plus percentage (definition, page 160). The base is 550 and 8 per cent. of 550 equals 44, the percentage; therefore the amount must equal 550 plus 44, or 594; or, since 550 equals 100 per cent. of itself, an increase of 8 per cent. would give 108 per cent. of the original number; and 108 per cent. of 550, or 1.08 times 550 equals 594.

- Rules.**—1. Find the percentage and add it to the base. Or,
2. Multiply the base by 1 plus the rate.

Formula.—Amount = Base + Percentage.

EXAMPLES FOR MENTAL PRACTICE.

- 538.** 1. If the base is 1500, and the rate 10%, what is the amount?
2. If the base is 1356, and the rate 25%, what is the amount?
3. The base is 440 and the rate 5%; find the amount.
4. The base is 1000 and the rate 18%; find the amount.
5. The base is 252 and the rate 10%; find the amount.
6. The base is 2150 and the rate 20%; find the amount.
7. The base is 630 and the rate $33\frac{1}{3}\%$; find the amount.
8. The base is 546 and the rate $16\frac{2}{3}\%$; find the amount.
9. The base is 200 and the rate 125%; find the amount.

EXAMPLES FOR WRITTEN PRACTICE.

- 539.** 1. What amount will be received for a house costing \$13500, if it is sold at a gain of $7\frac{1}{2}\%$?
2. A bought two horses for \$180 each, and sold one at a gain of 20% and the other at a gain of $33\frac{1}{3}\%$. How much did he receive for both?
3. A section of Kansas prairie was bought at \$12.50 per acre, and sold at an advance of 40%. How much was received for it?
4. What is the amount of 768 increased by 25% of $\frac{1}{4}$ of itself?
5. What is the amount of \$3144 increased by $\frac{3}{4}$ of $16\frac{2}{3}\%$ of itself?
6. If the base is \$864.88 and the rate $3\frac{1}{3}\%$ of $\frac{3}{4}$ of itself, what is the amount?

540. To find the Difference, the Base and Rate being given.

EXAMPLE.—What remains after diminishing 450 by 10% of itself?

OPERATION.

100%	= 450	= base.
10%	= .10	= rate.
90% dif. %	45.00	= percentage.
	450 base.	
	45 percentage.	
	405 difference.	

EXPLANATION.—Since 100 per cent. or the number equals 450, 10 per cent. of it will equal 45; and 450 minus 45 equals 405. Or, since 100 per cent. equals 450, 10 per cent. less than 100 per cent., or 90 per cent. will equal 405.

- Rules.**—1. Find the percentage and subtract it from the base. Or,
2. Multiply the base by 1 minus the rate.

Formula.—Difference = Base — Percentage.

EXAMPLES FOR MENTAL PRACTICE.

- 541.** 1. If from a brood of 15 chickens 20% are lost, how many will remain?
2. What number will remain if 225 is diminished by $33\frac{1}{3}\%$ of itself?
3. If the base is 1050 and the rate 10%, what is the difference?

4. 816, less 25% of itself, equals what number?
5. 1440, less $16\frac{2}{3}\%$ of itself, equals what number?
6. 800, less $37\frac{1}{2}\%$ of itself, equals what number?
7. 40, less $87\frac{1}{2}\%$ of itself, equals what number?
8. A boy having 648 ft. of kite string, lost $12\frac{1}{2}\%$ of it. How many feet had he remaining?

EXAMPLES FOR WRITTEN PRACTICE.

- 542.** 1. A speculator lost 35% of $\frac{1}{2}$ of \$16250. How much did he lose?
 2. A planter having 616 acres in rice, lost $\frac{1}{6}$ of $33\frac{1}{3}\%$ of his planting by flood. How many acres had he left for harvest?
 3. Brown deposited \$1147 in a savings bank, and his son deposited 21% less. How much was deposited by both?
 4. An agent earned \$250 in May, 15% less in June, and 20% less in July than in June. What was the amount earned for the three months?

543. To find the Base, the Amount, or Difference, and the Rate being given.

EXAMPLE (first illustration).—What number, increased by 15% of itself, amounts to 345?

OPERATION.			
100%	= 1.00		
15%	= .15	Amount.	Base.
115% amt. %	1.15) 345.00	(300
		345	
		—	00

EXPLANATION.—Since the number must be 100 per cent. of itself, if it has been increased 15 per cent., 345 must be 115 per cent. of that number; if 115 per cent. is 345, 1 per cent. must be $\frac{1}{115}$ of 345, or 3; and 100 per cent. will be 100 times 3, or 300.

EXAMPLE (second illustration).—What number, diminished by 35% of itself, equals 975?

OPERATION.			
100%	= 1.00		
35%	= .35	Diff.	Base.
65% dif. %	= .65) 975.00	(1500
		65	
		—	
		325	
		325	
		—	00

EXPLANATION.—If the number be diminished by 35 per cent. of itself, there will be remaining but 65 per cent. of itself; and if 65 per cent. of the number be 975, 1 per cent. must be $\frac{1}{65}$ of 975, or 15; and if 1 per cent. be 15, 100 per cent. must be 1500.

Rules.—1. Divide the amount by 1 plus the rate. Or,

2. Divide the difference by 1 minus the rate.

Formulas.—1. Base = Amount \div Amount Per Cent.

2. Base = Difference \div Difference Per Cent.

EXAMPLES FOR MENTAL PRACTICE.

- 544.** 1. If the amount is 750 and the rate 25%, what is the base?
 2. What number, increased by 10% of itself, amounts to 440?
 3. After 75% of a number had been added to it, the amount was 525. What was the number?

4. After selling 30% of his apples, a boy had 70 left. How many had he at first?
5. I lost \$600 by a bankrupt, who paid only 85% of his indebtedness. What was the full amount of my claim?

EXAMPLES FOR WRITTEN PRACTICE.

545. 1. A builder gained 15% by selling a house for \$1150. What was its cost?
2. Sold 945 tubs of butter for \$5113, and thereby gained 20%. How much did the butter cost per tub?
3. The income from a tenement house is \$6042 this year, which is 24% less than it was last year. How much was it last year?
4. A liveryman paid \$180 for a horse, which was 40% less than he paid for a carriage. How much did he pay for both?
5. A drover gained $16\frac{2}{3}\%$ on 33 head of cattle sold for \$4081. What was the average cost per head?
6. Smith sold two horses for \$1500 each, gaining 25% on one, and losing 25% on the other. What did the horses cost him?
7. After paying 35% of his debts, a man finds that the remainder can be paid with \$19500. What was his entire indebtedness?
8. A boat load of wheat was so damaged that it was sold for \$8500, which was 15% less than its original value. What was its value before it was damaged?
9. The attendance of pupils at a school during May was 954, which was 6% more than attended during April, and this was 80% more than attended during February. What was the attendance for February?
10. Which is better, to invest in a house that will rent for \$30 per month, at 6% on its value, or to invest the same amount in a farm that in two years will bring \$7000? How much better in the two years?

REVIEW OF THE PRINCIPLES OF PERCENTAGE.

546. 1. To find the percentage, the base and rate being given. RULE.—*Multiply the base by the rate expressed decimally.*
2. To find the base, the percentage and rate being given. RULE.—*Divide the percentage by the rate expressed decimally.*
3. To find the rate, the percentage and base being given. RULE.—*Divide the percentage by the base, carrying the quotient to two decimal places.*
4. To find the amount per cent., the rate being given. RULE.—*Add the rate to the unit 1.*
5. To find the difference per cent., the rate being given. RULE.—*Subtract the rate from the unit 1.*
6. To find the amount, the base and rate being given. RULES.—1. *Multiply the base by the rate, and to the product add the base.* Or, 2. *Multiply the base by 100 per cent. plus the rate.*

7. To find the difference, the base and rate being given. RULE.—*Multiply the base by the rate, and subtract the product from the base. Or, Multiply the base by 100 per cent. minus the rate.*

8. To find the base, the amount and rate being given. RULE.—*Divide the amount by 100 per cent. plus the rate.*

9. To find the base, the difference and rate being given. RULE.—*Divide the difference by 100 per cent. minus the rate.*

547. Percentage is applied to two classes of problems:

First, to those in which time is not an element; as, Profit and Loss, Commission, Brokerage, Insurance, Taxes, Customs or Duties, and Trade Discounts.

Second, to those in which time enters as an element; as, Interest, Bank Discount, True Discount, Equation of Accounts, and Exchange.

REMARK.—The pupil should be drilled in the formulas and rules of simple or abstract Percentage as above, and in their application to problems in applied Percentage to follow.

MISCELLANEOUS EXAMPLES FOR PRACTICE.

548. 1. At the battle of Waterloo, of the 145000 combatants, 51000 were either killed or wounded. What per cent. were uninjured?

2. The pressure on a steam boiler was 61.2 lb., after it had been reduced 10%. What was it before the reduction?

3. A pupil in examination answered correctly 56 questions, which was 20% less than the number asked him. What should be his average, on a basis of 100?

4. By assessing a tax of $\frac{3}{4}\%$, \$175000 was raised in a county. What amount of property was taxed?

5. A benevolent lady gave \$10500 to three charities; to the first she gave \$2500, to the second \$4500, and to the third the remainder. What per cent. did each receive?

6. On attaining his majority, a son finds his age is $62\frac{1}{2}\%$ less than the age of his father. Find the sum of their ages?

7. If 8% of B's money equals 24% of C's, how much has C, if B has \$324?

8. A farmer bought a horse, a mule, and a cow, for \$385. The mule cost 15% less than the horse, and the cost of the cow was $7\frac{1}{2}\%$ of that of the horse. What was the cost of each?

9. A creditor, after collecting $21\frac{2}{3}\%$ of a claim, lost the remainder, which was \$3918.75. What was the sum collected?

10. A woman weaving a rag carpet used 185% more weight of rags than of warp. How many pounds of each in a bale of carpet weighing $96\frac{1}{4}$ pounds?

11. The sum paid for two watches was \$384, and 75% of the sum paid for one equalled 105% of the sum paid for the other. Find the price of each.

12. If A has 35% more money than B, and B has 25% more than C, how much has C, if A has \$192?

13. If a gain of \$4755 was taken out of a business at the end of the first year, and a loss of \$3566.25 was sustained the second year, what was the per cent. of net gain or loss, the investment having been \$63400?

14. After making three of the seven equal annual payments of the face of a mortgage, I find \$5850 to be still unpaid. How many dollars of principal have been paid?

15. After the salary of a book-keeper had been increased 10%, and afterwards 8%, he received \$1242 a year. What was his salary at first?

16. By the United States Census of 1880 the total capital invested in manufactures in the State of Pennsylvania was \$190055904, while the amount invested in Alabama was \$9098181; Arkansas, \$1316610; Delaware, \$5452887; Florida, \$1874125; Georgia, \$10890875; Louisiana, \$7151172; Mississippi, \$4384492; North Carolina, \$9693703; South Carolina, \$6931756; Texas, \$3272450. What per cent. greater was the manufacturing capital invested in Pennsylvania than in the group of the ten other States named?

17. From an estate the widow received \$9250, which was one-third; the remainder was divided among three children, aged respectively 15, 12, and 10 years, and they shared in proportion to their age. What per cent. of the estate did each of the children receive?

18. A herder was asked how many cattle he had, and replied: "My herd increased last year 40%; should it increase at the same rate during this year and next, and I then buy 4 head more, I shall have double my present number." How many head of cattle had he?

19. What per cent. of the amount, at 10%, is 10% of the base?

20. From a farm containing 180 A. 120 sq. rd., one-half was sold at one time, and one-half of the remainder at another time. What per cent. of the whole then remained?

21. A man drew 15% of his deposit from a bank, and with it paid a debt of \$1119.60. What balance was left in the bank?

22. 15% of $\frac{3}{4}$ of a number is what per cent. of $\frac{3}{8}$ of it?

23. A man sold two farms for \$7500 each; for one he received 25% more than it cost, and for the other 25% less than it cost. Did he gain or lose by the sale, and how much?

24. What number is that which, being increased by 35% and 46% of itself and 76 more, will be doubled?

25. A ranchman, when asked how many sheep he had, replied: "If my flock increases next year 20%, the next 25%, and the third year 40%, I can then sell 300, and have left double my present number. How many had he?"

26. The total number of Popes up to 1888 has been 253, of whom 197 have been Italians. What per cent. of all have been of that nationality?

27. By widening a roadway 5%, it was made $10\frac{1}{2}$ yd. wide. What was its original width?

28. Oct. 11, 1888, A bought an engine and mill for \$5250, on six months credit, or 5% off if paid within 90 days, or $7\frac{1}{2}$ % off if paid within 30 days. What amount was required for full settlement Nov. 7, 1888?

29. In settling an estate, an executor found $7\frac{1}{2}$ % of it to be invested in telegraph stock, 15% in railroad stock, $37\frac{1}{2}$ % in city bonds, \$16750 in real estate, and \$7350 cash in bank. Find the total value of the estate.

30. A farm is composed of 20% more grazing than grain land, and the timber is one-half of the area. How many acres of each, if, after deducting 12 acres for lawn and garden, there is left of the farm 1860 acres?

31. A has 20% less money than B, and B has 25% more than C. How much has C, if A has \$192?

32. A and B were heirs of an estate of \$120000, A receiving 15% of the whole more than B. For four years thereafter the property of each increased at an average rate of 9% per annum. How much had each at the end of that time?

33. A man owning $62\frac{1}{2}\%$ of a factory, sold $7\frac{1}{2}\%$ of his share for \$1050. At that rate, what was the value of the factory?

34. What is the per cent. of difference between $16\frac{2}{3}\%$ of $\frac{1}{2}$ of a number, and 25% of $\frac{1}{3}$ of the same number?

35. From a cheese factory, 33630 boxes of cheese were sold in four years; the sales of the second year having been 30% greater than those for the first year, those of the third 30% less than those of the second, and those of the fourth 40% greater than those of the third. What were the sales of each year?

36. In preparing a prize mixture for seeding pastures, Sibley & Co. mixed equal parts of clover seed and timothy with $33\frac{1}{3}\%$ as much orchard grass as clover, and $33\frac{1}{3}\%$ as much red top as orchard grass. How many pounds of each, in a consignment of 1100 pounds of the mixture?

37. Three railroad companies carry six carloads of freight, each weighing 20 T. 6 cwt, a distance of 450 miles; the distance over the first line was 100 miles, and that over the second 125 miles. If the total charge was 15¢ per 100 pounds, how much money should each company be paid?

38. A young man who received \$21000 from his father, had, at the end of five years, only \$3500 left. What average per cent. of his inheritance did he lose yearly?

39. My grocery sales increased 20% the second year, 25% more the third year, and 40% still more the fourth year; during which four years I sold \$131250 worth of goods. What was the amount of my sales the first year?

40. A father located his son upon a farm, expending for the farm, stock, utensils, and household furniture, \$19512.50; the stock cost twice as much as the household furniture, which cost 75% more than the farm utensils, and the cost of the farm was 140% of the cost of the stock. How much was invested in each?

41. The general freight agent of a railroad, when questioned as to the amount of freight carried by his line, replied: "For the past four years our yearly increase over previous business has been 25%; should this be shown for the coming four years, the amount of freight then carried will be 22070 T. 625 lb. more than double the amount carried this year." What was the number of tons carried four years ago?

42. A last will and testament provided that three-eighths of the estate distributed should go to the widow, and the remainder be so divided among two sons and a daughter that the elder son should receive 10% more than the younger, who should receive 25% more than the daughter. What amount was received by each, the estate being valued at \$58000?

PROFIT AND LOSS.

549. **Profit and Loss** treats of gains or losses in business transactions.

550. If, after deducting all expenses of sale, the net price is greater than the cost, the excess is a *Profit* or *Gain*.

551. If the net price received from the sale is less than full cost, the difference is a *Loss*.

552. The **Gross** or **Full Cost** of an article is its first cost, increased by all outlays incident to its purchase and holding to date of sale.

553. The **Net Selling Price** is the gross selling price, less all charges incident to its sale.

554. In ascertaining profit or loss, operations are usually performed by the rules of Percentage heretofore explained; but when the rate is a simple, common, fractional part of 100, it is more convenient to use the equivalent fraction than the decimal per cent.

555. Comparing the elements of Profit and Loss with those of Percentage, the *Cost* corresponds to the *Base*; the *Per Cent. of Gain or Loss* to the *Rate*; the whole *Gain or Loss* to the *Percentage*; the *Selling Price*, if at a gain, to the *Amount*; the *Selling Price*, if at a loss, to the *Difference*.

REMARKS.—For table of ALIQUOT PARTS, convenient for use as common fractional equivalents, refer to page 89.

556. To find the Profit or Loss, the Cost and Rate being given.

EXAMPLE.—An agent paid \$95 for a reaper, and sold it at a profit of 18%. What was his gain?

OPERATION.

\$95 = cost.

EXPLANATION.—Since the agent gained 18 per cent, or 18 cents on

.18 = % of gain. 1 dollar, on the \$95 of cost he would gain 95 times \$.18, or \$17.10.

\$17.10 = gain.

Rule.—*Multiply the Cost by the Rate.*

Formula.—Profit or Loss = Cost \times Rate.

EXAMPLES FOR MENTAL PRACTICE.

557. 1. A set of furniture, costing \$60, was sold at 15% profit. How much was gained?

2. If I pay \$400 for a piano, and gain 12% by its sale, how much is my profit?

3. Having paid \$7500 for a house, I sell it at 10% advance on cost. How much do I gain?

4. After using a carriage which cost me \$250, I was obliged to sell it for 20% less than it cost. What was my loss?
5. How much loss do I sustain by selling a \$200 watch at 16% less than cost?
6. After paying \$1200 for a lot, I built thereon a house costing \$2800, and by selling both lost 8% of my investment. How many dollars did I lose?
7. One of a road team cost \$400 and the other \$500. How much is lost, if the team is sold at 25% below cost?
8. I invested \$10500 in Southern lands. If 20% of the land proved to have a worthless title, how many dollars were lost?
9. Since paying \$14000 for a stock of teas, the price has advanced 5%. How much has the stock increased in value?

EXAMPLES FOR WRITTEN PRACTICE.

558. 1. Three houses, bought for \$5000, \$6500, and \$8250 respectively, were sold so that a gain of 12% was realized on the first, and $7\frac{1}{2}\%$ on the second, while the third was sold at 6% below cost. Find the net gain or loss?
2. A stock of goods costing \$15600 was sold at a loss of $12\frac{1}{2}\%$, and 15% of the selling price was in bad debts. What was the total loss sustained?
3. A canal boat, loaded with 8400 bushels of wheat, collided with a bridge pier and sprung a leak, by which $21\frac{1}{2}\%$ of the cargo sustained a damage equal to $\frac{1}{2}$ of its value. What was the loss sustained, the wheat having been invoiced at 75¢ per bushel?
4. A peddler paid \$46.50 for butter, \$17.60 for eggs, and \$36 for dried berries. He sold the butter at a profit of $16\frac{2}{3}\%$, the eggs at a profit of 20%, and lost 5% on the berries. What was his net gain?
5. Having paid \$1640 for a box of furs, and \$18.50 expressage on the same, I sold 25% of the stock at a gain of 35%, 15% at a gain of 20%, 30% at a loss of 2%, and the remainder at cost. How much did I gain or lose?
6. An agent bought three reapers, paying respectively \$90, \$120, and \$150. He sold the first at 10% loss, the second at cost, and the third at 10% gain. What was his profit by the transaction?
7. A contractor bought 52 M bricks at \$5.60 per M, and sold $\frac{3}{4}$ of them for $\frac{3}{4}$ of their cost, and for the remainder received \$150. What amount did he lose?
8. A grocer bought 7 barrels of sugar, each weighing 315 pounds, at $6\frac{1}{4}\%$ per pound, and sold it so as to gain $16\frac{2}{3}\%$. Find the amount of his gain.
9. How much is gained by purchasing 3 carloads of corn, of 750 bushels each, at $62\frac{1}{2}\%$ per bushel, and selling 40% of it at a gain of $12\frac{1}{2}\%$, and the remainder at a gain of $7\frac{1}{2}\%$?

559. To find the Cost, the Gain or Loss and the Rate of Gain or Loss being given.

EXAMPLE.—An agent gained \$17.10 by selling a reaper at 18% profit. What must he have paid for it?

OPERATION.

Rate.	Gain.
18% = .18	17.10
<hr style="width: 100%;"/>	
	\$95 = cost.

Since the agent's whole gain was \$17.10 and since his gain on 1 dollar of cost was 18% or 18 cents, the cost must have been as many times 1 dollar as \$.18 is contained times in \$17.10, or \$95.

Rule.—*Divide the gain or loss by the per cent. of gain or loss.*

Formula.— $\text{Cost} = \text{Gain or Loss} \div \text{Rate}.$

EXAMPLES FOR MENTAL PRACTICE.

560. 1. What was the cost, if I lost \$15 by selling a machine 15% below cost?
2. By selling a farm at a gain of 10%, I realized a profit of \$350. Find the cost of the farm.
3. A yacht was sold for \$1250 less than cost, its owner thereby losing 12½% of the cost. What was the price paid?
4. By selling a consignment of silk for 11% above the invoice price, a gain of \$484 was realized. Find the invoice price.
5. A suit of clothes, becoming damaged, was sold at a loss of 13%, whereby the tailor lost \$5.20. How much did the suit cost when made?
6. What must have been the cost of a necklace, if its owner, by selling it at a loss of 15%, received \$45 less than it cost?
7. By selling a coach for \$63 above cost, I gained 7% on my purchase price. How much did it cost?
8. Having received \$105 more for a house than its cost, I find my profit to be 10%. How much did I pay for the house?
9. A book-seller lost 65¢ on an album, and thereby sustained a loss of 65%. Find the cost.

EXAMPLES FOR WRITTEN PRACTICE.

561. 1. What must have been the cost of a watch and chain, if \$6.90 was lost by selling them at 12% below cost?
2. A dealer sold a piano at 25% profit, and with the proceeds bought another which he sold at 20% profit, realizing a total gain of \$250. What was the cost of each?
3. By selling a lot for \$1680, I received 40% more than twice its cost. At what price did I purchase it?
4. A sells a horse to B and gains 15%. If B paid 25% of \$420 more for the horse than A did, at what price did A buy it?
5. Having bought a house of A at 12½% less than it cost him, I added \$430 in repairs, and sold it for \$7293, thereby gaining 10% on my investment. How much did the house cost A?
6. A miller's gain in business for four years aggregates 37½% of his capital. If his gain is \$3000, and he withdraws his gain and capital and invests it in a farm, at \$55 per acre, how many acres can he buy?
7. A merchant bought goods and paid freight on them equal to 12% of their first cost; he then sold them at 6¼% profit on the full cost, receiving 60% of the price in cash and a note for \$1309, the amount unpaid. What was the first cost of the goods?
8. A peddler sold 25% of a purchase of butter at 16% profit, and the remainder at 16¾% profit. What was the cost, if the total gain was \$39.60?
9. A dealer sold 35% of a purchase of leather at 14¾% profit, and the remainder at 5% loss. If his net gain was \$87.50, what must have been the cost?

562. To find the Rate of Profit or Loss, the Cost and the Profit or Loss being given.

EXAMPLE.—An agent gained \$17.10 by selling a reaper which cost him \$95. What was his per cent. of gain?

OPERATION.

Cost. Gain.

\$95) 17.10 (.18 = 18%

95

760

760

EXPLANATION.—If 95 dollars of cost gain \$17.10, 1 dollar of cost would gain as much as 95 is contained times in 17.10, or .18, equal to 18 per cent.

Rule.—*Divide the profit or loss by the cost.*

Formula.—Per Cent. of Profit or Loss = Profit or Loss \div Cost.

EXAMPLES FOR MENTAL PRACTICE.

563. 1. I gained \$12.50 on what cost me \$125. Find my rate per cent. of gain.

2. I bought a bicycle for \$150, and sold it for \$.75 below cost. What per cent. did I lose?

3. What per cent. is lost by selling a \$5 book at $62\frac{1}{2}\%$ below its cost?

4. A safe costing \$380 was sold at a loss of \$76. Find the loss per cent.

5. A gold pen cost \$2, and after being tested and found imperfect, was sold as old gold for \$1. Find the per cent. of loss.

6. What per cent. of gain is realized by buying a horse for \$300, and selling it at an advance of \$100?

7. Find the per cent. of gain on a section of Dakota prairie, bought at \$4 per acre, and sold at \$10 per acre.

8. An Ohio river steamer costing \$100000 was sold for \$9500 profit. Find the per cent. of profit.

9. A Vermont manufacturer, having invested \$40000, gained \$8250 each year. What was his per cent. of gain per annum?

EXAMPLES FOR WRITTEN PRACTICE.

564. 1. What per cent. is gained by selling an article for $2\frac{1}{2}$ times its cost?

2. I bought a quantity of cloth at \$1.60 per yard, and sold it at \$2 per yard. What was my per cent. of gain?

3. A speculator bought wheat at 80¢ per bushel, and oats at 32¢ per bushel. If he sold the wheat at 90¢ per bushel, and the oats at 40¢ per bushel, on which would he make the greater per cent., and how much?

4. If a boy sells three apples for what four cost him, what per cent. does he gain?

5. Four-fifths of a stock was sold at 45% loss, and the remainder at 225% profit. What was the per cent. of net loss or net gain on the stock?

6. Paper bought at \$2.70 per ream, and retailed at 1¢ per sheet, will yield what per cent. of profit?

7. Potatoes costing \$1.35 per barrel, and sold at \$1.62 per barrel, will net what per cent. of gain?

8. A wood dealer, after buying 8 car loads of mixed wood, of 16 cords each, at \$5 per cord, sorted it and sold 35% of it at $12\frac{1}{2}\%$ gain, 35% of it at 10% gain, and the remainder at 20% gain? What was his average per cent. of gain?

9. If $33\frac{1}{3}\%$ of a barrel of salt be sold at $33\frac{1}{3}\%$ profit, and the remainder be sold at cost, what per cent. of profit is realized on the whole?

10. An agent sold a sewing machine for \$45.70, and thereby gained \$18.28. What per cent. did he gain?

11. If $\frac{2}{3}$ of an article is sold for what $\frac{3}{4}$ of it cost, what is the loss per cent.?

12. If I sell $\frac{1}{2}$ of an article for what $\frac{1}{2}$ of it cost, what is my rate of gain?

13. A drover, buying 125 beeves at the rate of \$55 per head, and 78 at \$62.50 per head, sold the lot at a profit of \$2115. What was his per cent. of gain?

14. A cargo of lumber cost \$3600. If $\frac{1}{2}$ of it is sold for \$2000, $\frac{1}{2}$ of the remainder for \$1250, and what is left for \$420, what is the per cent. of gain or loss by the transaction?

15. Oil bought at $81\frac{1}{2}\phi$ per barrel is sold at $86\frac{3}{4}\phi$. If $\frac{1}{2}\phi$ per barrel is allowed for expenses, what must have been the investment, the gain having been \$1350?

565. To find the Cost, the Selling Price and the Rate per cent. of Profit or Loss being given.

Rules.—1. Divide the selling price by 1 plus the rate of gain. Or,

2. Divide the selling price by 1 minus the rate of loss.

Formulas.— $\begin{cases} a. \text{ Cost} = \text{Selling Price} \div 1 + \text{Per Cent. of Gain.} \\ b. \text{ Cost} = \text{Selling Price} \div 1 - \text{Per Cent. of Loss.} \end{cases}$

EXAMPLES FOR MENTAL PRACTICE.

566. 1. A buggy was sold for \$105, at a gain of 5%. What the the cost?

2. What must have been the cost of a harness sold at 40% loss, if \$24 were received for it?

3. Find the cost of making a suit of clothes, if 20% is gained by selling it at \$18.

4. Find the cost of a watch that sold at a profit of $16\frac{2}{3}\%$ and brought \$87.50.

5. I sold a house for 125% profit, receiving therefor \$2250. What was the price paid?

6. If \$15360 is realized on a stock of goods after it has been damaged 40%, what was its value before being damaged?

EXAMPLES FOR WRITTEN PRACTICE.

567. 1. One of a pair of horses was sold for \$180, at a loss of $12\frac{1}{2}\%$; the other was sold for \$200, at a gain of 25%. What did the pair cost?

2. A fruit dealer, after losing $16\frac{2}{3}\%$ of his apples by frost, has $147\frac{1}{2}$ barrels left. If he bought his stock at \$2.50 per barrel, what was his outlay?

3. What was the original value of Calumet copper mining stock, which, when sold at a gain of 175%, brought \$20625?

4. A paid 6% tax on his income. What was his income, if, after paying the tax, the remainder equalled \$7050.94.

5. A dairy produced 20% more cheese in March than in February. What was the product for March, if that for the two months was 1980 pounds?

6. I sold a house to A at a profit of 10%; he sold it to B, gaining 15%; and B, by selling it to C for \$6072, gained 20% on his purchase. How much did the house cost me?

REVIEW OF THE PRINCIPLES OF PROFIT AND LOSS.

568. 1. To find the gain, the cost and per cent. of gain being given. *RULE.—Multiply the cost by the per cent. of gain.*

2. To find the loss, the cost and per cent. of loss being given. *RULE.—Multiply the cost by the per cent. of loss.*

3. To find the selling price, the cost and gain being given. *RULE.—Add the gain to the cost.*

4. To find the selling price, the cost and loss being given. *RULE.—Subtract the loss from the cost.*

5. To find the cost, the gain and per cent. of gain being given. *RULE.—Divide the gain by the per cent. of gain.*

6. To find the cost, the loss and per cent. of loss being given. *RULE.—Divide the loss by the per cent. of loss.*

7. To find the selling price, the gain and per cent. of gain being given. *RULE.—Divide the gain by the per cent. of gain, and to the quotient add the gain.*

8. To find the selling price, the loss and per cent. of loss being given. *RULE.—Divide the loss by the per cent. of loss, and from the quotient subtract the loss.*

9. To find the per cent. of gain, the gain and cost being given. *RULE.—Divide the gain by the cost.*

10. To find the per cent. of loss, the loss and cost being given. *RULE.—Divide the loss by the cost.*

11. To find the per cent. of gain, the selling price and gain being given. *RULE.—Subtract the gain from the selling price and divide the gain by the difference.*

12. To find the per cent. of loss, the selling price and loss being given. *RULE.—Add the loss to the selling price, and divide the loss by the sum obtained.*

MISCELLANEOUS EXAMPLES.

569. 1. What is that sum of money of which 50% is \$19.20 more than 37½%?

2. What amount of money must an attorney collect, in order that he may pay over to his principal \$475, and retain 5% for his services?

3. A woman is 72 years old, and 16⅔% of her age is 25% of the age of her daughter. Find the daughter's age.

4. Gunpowder is made of $\frac{3}{4}$ nitre, and the remainder of equal parts of sulphur and charcoal. Find the per cent. of each.

5. A milkman increased his herd of cows by a purchase of 36, which was 45% of the whole number he then owned. How many had he before buying the last lot?
6. If I make a profit of $16\frac{2}{3}\%$ by selling a horse at \$7.50 above cost, how much must I have advanced on the cost to have realized a profit of 25%?
7. Two persons contributed \$2100 towards a business venture, from which their part of the gain was \$350. If of this gain the share of one was \$70 more than that of the other, what part of the original contribution must have been made by each?
8. How much money must be invested in notes, at $4\frac{1}{2}\%$ below their face value, in order that, when sold at 3% above their face, a profit of \$225 may be realized?
9. I bought a warehouse of Brown for $12\frac{1}{2}\%$ less than it cost him, and sold it for $16\frac{2}{3}\%$ more than it cost him, gaining thereby \$963.60. How much did I pay for the warehouse?
10. What per cent. is gained by buying pork at \$17.50 per barrel, and retailing it at 12¢ per pound?
11. A lady wishing to sell her piano, asked 15% more than it cost, but finally sold it at $12\frac{1}{2}\%$ less than her asking price. What did the piano cost, if by its sale she gained \$5? $\$300$
12. Having bought 75 barrels of apples for \$187.50, I sold them at a loss of 20%. How much did I receive per barrel?
13. A sells a steam tug to B, gaining $12\frac{1}{2}\%$, and B sells it to C for \$4130, and makes a profit of 18%. How much did the tug cost A?
14. What per cent. is lost on an article that is sold for two-thirds of its cost?
15. A farmer, after selling 1760 barrels of apples, had 20% of his crop left. How many barrels had he at first?
16. I lost 25% of a consignment of berries. At what per cent. of profit must the remainder be sold, in order that I may gain 10% on the whole? $47\frac{1}{2}\%$
17. A Texas farm of 160 acres was bought at \$15 per acre; \$354 were paid for fencing, \$480 for breaking, \$626 for a house, and \$220 for a barn. At what price per acre must it be sold, to realize a net profit of 25% on the investment?
18. King sold his wheel at $33\frac{1}{3}\%$ gain, and with the money bought another, which he sold at a loss of 25%, receiving therefor \$120. Did he gain or lose, and how much?
19. What per cent. more is $\frac{3}{8}$ than $\frac{5}{8}$?
20. Cloth, bought at \$4 per yard, must be marked at what price in order that the seller may make a reduction of 10% from the asking price and still gain $12\frac{1}{2}\%$ on the cost?
21. If 25% of the selling price is gain, what is the per cent. of gain?
22. I sell $\frac{2}{3}$ of a stock of goods for \$27, thereby losing 20%. For what must I sell the remainder, to make a profit of 20% on the whole? $74\frac{2}{3}\%$
23. If 30% of a farm sold at $33\frac{1}{3}\%$ gain, and 30% of the remainder at 15% gain, how much was the total gain, if the remainder was sold at cost for \$7350?
24. What per cent. of cost is realized on goods marked 25% advance and sold at 20% off from the marked price?

25. A banker bought a mortgage at $7\frac{1}{2}\%$ less than its face value, and sold it for 3% more than its face value, thereby gaining \$981.75. What was the face value of the mortgage?

26. At what price should damaged goods be marked to lose 25% , the first cost having been 36ϕ per yard?

27. A man sold a carriage and gained 25% , and with the proceeds bought another, which he sold at a profit of 10% , thus realizing a total gain of \$75. What did he pay for each?

28. If I sell $\frac{3}{4}$ of an acre of land for what $\frac{5}{8}$ of it cost, what will be my gain or loss per cent.?

29. $21\frac{1}{2}\%$ was lost by selling an engine for \$2355. How much would it have brought had it been sold at a loss of 10% ? 7700

30. What price must be asked for 1000 pounds of coffee, costing 18ϕ per pound, in order that the seller may deduct 10% from the asking price for bad debts, allow $16\frac{2}{3}\%$ for loss in roasting, and still gain 20% on the cost? 288

31. B and C each invested an equal amount of money in business; B gained $12\frac{1}{2}\%$ on his investment, and C lost \$5275; C's money was then 42% of B's. How many dollars did each invest?

32. A trader lost $33\frac{1}{3}\%$ on 20% of an investment, and gained $12\frac{1}{2}\%$ on the remainder, thus realizing a net gain of \$1000. Had he gained 20% on $\frac{1}{3}$, and lost 25% on the remainder, what would have been his net profit?

33. A manufacturing company's per cent. of gain on a self-binder was 25% less than that of the general agent; the general agent's profit was 20% , he thereby gaining \$25.30. What did it cost to make the machine? 110

34. Of a cargo of 8000 bushels of oats, costing 35ϕ per bushel, 25% was destroyed by fire. What per cent. will be gained or lost, if the remainder of the oats are sold at 45ϕ per bushel?

35. For what must hay be sold per ton, to gain $16\frac{2}{3}\%$, if, by selling it at \$18 per ton, there is a gain of 25% ? 1680

36. Jones sold $\frac{1}{2}$ of a stock of goods at cost, $\frac{1}{4}$ at a gain of 35% , $\frac{1}{4}$ at a loss of 25% , and $\frac{1}{6}$ at a gain of 10% . At what per cent. of its cost must he sell the remainder to net cost on the whole?

37. After a carriage had been used two years, it was sold for \$5 less than its cost, the seller thereby sustaining a loss of $3\frac{1}{3}\%$ of the selling price. How much was the first cost of the carriage?

38. If oranges cost \$1.80 per hundred, at what price must they be marked to ensure a gain of 20% , and make allowance for 28% decay, and 25% bad debts in selling? 400

39. Having paid 40ϕ per pound for tea, at what retail price must it be marked, that I may allow $12\frac{1}{2}\%$ for bad debts and gain 40% on the cost?

40. Six wheel-rakes were sold for \$21 each; three of them at a gain of 20% , and the others at a loss of 20% . What was the net gain or loss?

41. A stock of goods is marked $22\frac{1}{2}\%$ advance on cost, but becoming damaged, is sold at 20% discount on the marked price, whereby a loss of \$1186.40 is sustained. What was the cost of the goods?

42. My retail price of Axminster carpet is \$3.50 per yard, by which I gain 25%. If I sell at wholesale, at a discount of 25% from the retail price, how much do I receive per yard. What is my per cent. of gain or loss, and how much is my actual gain or loss by selling 1000 yards at wholesale?

43. If the loss equalled $\frac{1}{3}$ of the selling price, what was the per cent. of loss?

44. A grocer bought 200 quarts of berries, at $11\frac{1}{4}\phi$ per quart, and 150 quarts of cherries, at $6\frac{1}{4}\phi$ per quart. Having sold the cherries at a loss of 30%, for how much per quart must he sell the berries, to gain 15% on the whole?

45. A sells two horses to B at an advance of $16\frac{2}{3}\%$, B sells them to C at an advance of 25%, and C sells them to D for \$735, thereby making a profit of 20%. How much did A pay for the horses? \$420

46. Having bought 48 pounds of coffee, at the rate of $3\frac{1}{2}$ pounds for 91¢, and 84 pounds more at the rate of 7 pounds for \$1.26, I sold the lot at the rate of 9 pounds for \$1.53. What was my per cent. of gain or loss?

47. By selling at a loss of 6¢ per yard I get $87\frac{1}{2}\%$ of the cost of cloth. What per cent. of the cost would I have received had I lost 8¢ per yard?

48. If 15% is lost by selling suits at \$17 each, how much would be gained by selling them at 15% profit? 30

49. The price of a suit of clothes having been marked down 20% or to \$27, the dealer, in order to effect a sale, discounted again 15%, and still by the sale made a profit of $14\frac{3}{4}\%$. What per cent. above cost was the suit originally marked? 68 $\frac{3}{4}$

50. An Iowa farm passed through the hands of five owners, each of whom in succession gained 20% by its purchase and sale. If the average gain was \$1488.32, what was its first cost, and what was its final selling price? 5000, 7446.6

51. By selling a stock of goods at 20% below cost, I received \$150 less than I would have received had I sold the goods at 20% above cost. What should the goods have sold for to gain 20%?

52. The first cost of Parisian goods purchased through an agent was increased 18% by the charges of the agent, the freight, and the import duties; I sold the goods at 25% advance on full cost, thereby gaining \$1785. Find the first cost. 6050

53. After using a carriage for two years, I sold it for $3\frac{1}{3}\%$ of its selling price less than it cost, thereby losing \$5. How much would it have brought, had the amount received for it been $3\frac{1}{3}\%$ of the selling price more than it cost?

54. An agent bought a reaper at 20% off from the wholesale price, and sold it at an advance of 30%, thereby gaining \$37.50. If the wholesale price was 25% above the cost of manufacture, what was the cost to the manufacturer? 125

55. I sold a house at 25% profit, and invested the proceeds in dry goods, on which I lost $12\frac{1}{2}\%$; I invested the proceeds from the sales of dry goods in stocks, on which I lost 10%. What was my net gain or loss per cent. 12.65

56. Having paid a retailer \$138.60 for a set of furniture, I ascertain that by selling to me he gained $12\frac{1}{2}\%$, that the wholesaler of whom he bought gained 10%, that the jobber by selling to the wholesaler gained $16\frac{2}{3}\%$, and that the manufacturer sold to the jobber at 20% above its first cost. How much more than its first cost did I pay? 586

57. I wish to line the carpet of a room 21 ft. long and 18 ft. wide with duck $\frac{5}{8}$ of a yard in width. How many yards will be required, if it shrink 10% in width and 5% in length? If the carpet be laid lengthwise of the room, and be furnished at \$2.25 per yard, $\frac{3}{4}$ of a yard wide, and the duck, before shrinking, at 20¢ persquare yard, and a profit of $16\frac{2}{3}\%$ be realized on both, what will be the gain?

58. If I pay \$3.20 for 20 gal. of vinegar, how many gallons of water must be added, that 40% profit may be realized by selling it at 15¢ per gallon?

59. A huckster sold a quantity of potatoes and onions, gaining $37\frac{1}{2}\%$ on the onions and 25% on the potatoes, $33\frac{1}{3}\%$ of his profit being realized on the potatoes. At what price was each sold, if the total gain was \$450?

60. What price each must be asked for cocoanuts, costing \$4 per C, that an allowance of $16\frac{2}{3}\%$ for breakage, 20% for decay, and $11\frac{1}{3}\%$ for bad debts may be made, and still a profit of $33\frac{1}{3}\%$ be realized?

61. A tree agent sold apple and pear trees for \$2187.50; he gained $16\frac{2}{3}\%$ on the apple, and $37\frac{1}{2}\%$ on the pear trees, receiving for the pear 75% as much money as for the apple trees. Find the cost of each kind.

62. A dry goods house bought a stock of goods, and sold $\frac{1}{4}$ of it at 25% profit, $\frac{1}{4}$ of it at 20% profit, $\frac{1}{4}$ of it at $16\frac{2}{3}\%$ loss, $\frac{1}{8}$ of it at $12\frac{1}{2}\%$ gain, and the remainder, which cost \$4549.25, at 15% gain. What was the net gain or loss, and the per cent. of gain or loss, on the entire stock?

63. A butcher paid equal amounts of money for calves, pigs, and sheep; he cleared 14% on the calves, 10% on the pigs, and lost 30% on the sheep. How many dollars were paid for each kind of stock, the total amount received having been \$1336.50?

64. I sold my house to B and lost 10% of its cost; B expended \$375 for repairs and sold it to C at 120% of its full cost to him; C expended \$525 in enlarging the house, and then sold it for \$6354, thereby making a profit of 20% of its full cost. How much did I pay for the property?

65. A speculator, investing equal sums in corn and wheat, gained \$2713.50 more on the corn than on the wheat. If he gained $10\frac{1}{2}\%$ on the wheat and 15% on the corn, how many bushels of each must have been purchased, the corn having been bought at 60¢ per bushel and the wheat at 80¢ per bushel?

66. A drover bought 50 horses, cows, and sheep for \$870; the number of cows was 600% of the number of horses, and the number of sheep was 300% of the number of cows; the horses cost 200%, and the sheep 20%, as much as the cows. If the entire purchase was sold at a profit of 20%, how much was received per head for each kind?

TRADE DISCOUNT.

570. Discount is the allowance made for the payment of a debt before it becomes due.

571. Trade Discount is the allowance made by manufacturers and merchants upon their fixed or list prices.

REMARKS.—1. It is customary in many branches of business for merchants and manufacturers to have *fixed price lists* of their goods, and when the market varies, instead of changing the price list, to change the *rate of discount*.

2. Business houses usually announce their terms upon their "bill-heads;" as, "Terms, 3 months, or 5% off for cash;" "Terms, 60 days, or 3% discount in 10 days," etc. When bills are paid before maturity, legal interest for the remainder of the time is usually deducted.

572. There may be more than *one* Trade Discount, and they are then known as a *Discount Series*.

573. Trade Discount is computed by the rules of percentage, on the *marked price* as a *base*. When a series of discounts is allowed, the first only is so computed, and in every subsequent discount the remainder, after each preceding discount, is regarded as the base.

574. To find the Selling Price, the List Price and Discount Series being given.

EXAMPLE (first illustration).—The list price of a sewing machine is \$60. What is the net selling price, if a discount of 40% is allowed?

OPERATION.

\$ 60 = list price.

.40 = % of discount.

\$ 24 = discount.

\$ 60 = cost.

24 = discount.

\$ 36 = net selling price.

EXPLANATION.—Since the discount is 40 per cent., and the list price, or base, is \$60, the discount to be deducted will be 40 per cent. of \$60, or \$24; and the net price will be \$60 minus \$24, which equals \$36.

EXAMPLE (second illustration).—The list price of a threshing machine is \$900. What is the net price, if a discount series of 25%, 20%, and 10% is allowed?

OPERATION.

\$900 = list price.

225 = 25%, or $\frac{1}{4}$ = 1st discount.

\$675 = rem. after 1st discount.

135 = 20%, or $\frac{1}{5}$ = 2d discount.

\$540 = rem. after 2d discount.

54 = 10%, or $\frac{1}{10}$ = 3d discount.

\$486 = rem. after 3d discount, or net price.

EXPLANATION.—From the list price take the first discount, and make each remainder the base for the succeeding discount. The last remainder will be the net price.

REMARK.—In like manner treat any series of discounts.

Rule.—*Deduct the first discount from the list price, and each subsequent discount from each successive remainder.*

EXAMPLES FOR PRACTICE.

575. 1. What is the selling price per dozen of hats, listed at \$36, and discounted 20% and 15%?

2. Find the net price of a ton of fence wire, listed at 9¢ per pound, and sold at 70% and 5% off.

3. Find the net cost to the purchaser of a bill of goods invoiced at \$1100, from which discounts of 20% and 25% were allowed.

4. An invoice of silk amounting to \$12000 was sold Sept. 21, 1888, at a discount of 25%, 20%, and 12½%, with a further discount of 10% to be allowed if paid within 30 days. How much cash will pay the bill Oct. 15, 1888?

5. Having bought merchandise at 25% and 15% discount from the list price of \$1500, I sell it at 15%, 15%, and 10% from the same list price. Do I gain or lose, and how much?

6. A wholesale dealer offers cloth at \$2.40 per yard, subject to a discount of 25%, 20%, 10%, and 5%. How many yards can be bought for \$246.24?

7. What is the net cost of a bill of goods invoiced at \$2150, and sold at a discount of 15%, 10%, 5%, and 3%?

8. Three drummers, A, B and C, offer me the same grade of goods at the same list price. A offers to discount 25% and 15%; B 20% and 20%; and C 15%, 15%, and 10%. With which will it be most advantageous for me to deal, and how much would I save from a list price of \$200?

REMARKS.—1. It is often convenient in finding the net price to multiply the list price by 1 minus the first discount, the remainder by 1 minus the next, and so on.

2. The order in which the discounts of any series are considered is not material, a series of 25, 15, and 10 being the same as one of 15, 10, and 25, or of 10, 25, and 15.

576. To find the Price at which Goods must be Marked to Insure a Given Per Cent. of Profit or Loss, the Cost and Discount Series being given.

EXAMPLE (first illustration).—Having bought goods for \$105, at what price must they be marked to allow a discount of 25%, and still make a profit of 10%?

OPERATION

\$105 = the cost.

.10 = % to be gained.

\$10.50 = gain to be insured.

105.00 = cost.

\$115.50 selling price to be insured.

.75) \$115.50

\$154. asking price.

EXPLANATION.—The cost, \$105, is 100 per cent. of itself; the rate of discount to be allowed is 25 per cent.; 100 per cent. minus 25 per cent., or 75 per cent., is the per cent. which the price to be insured is of the price to be asked. And if 10 per cent. must be insured, the goods must actually bring 10 per cent., or \$10.50 more than cost, or \$115.50. And since a deduction of 25 per cent. is to be made from the asking price, the selling price, \$115.50, will be only 75 per cent. of the asking price. Therefore, divide \$115.50 by .75, and the quotient, \$154, will be the asking price.

EXAMPLE (second illustration).—A seal sacque cost a manufacturer \$240. At what price must it be marked, that a discount series of 25%, 20%, and 20% may be allowed, and he still make a profit of 30%?

OPERATION.

\$240 = cost or base.	\$1.00 = % of price realized.
.30 = % to be gained.	.25 = % of 1st discount.
<hr/> 72 = gain to be insured.	<hr/> .75 = % of price to be received in
240 = cost.	order to gain 30% and allow
\$312 = price to be received.	1st discount.
 \$.75) \$312.00	 \$1.00 = % of price.
\$416 = asking price in order	.20 = % of 2d discount.
to pay \$240, gain	<hr/> .80 = % of price to be received in
30%, and allow a dis-	order to gain 30% and dis-
count of 25%.	count 25% and 20%.
 \$.80) \$416.00	 \$1.00 = % of price.
\$520 = asking price in order to	.20 = % of 3d discount.
pay \$240, gain 30%, and	<hr/> .80 = % of price to be received in
discount 25% and 20%.	order to gain 30% and dis-
 \$.80) \$520.00	 count 25%, 20%, and 20%.
\$650 = asking price in order to gain 30% and	
allow the full discount series.	

Rule.—Add to the cost the gain required, and divide consecutively by 1 minus each of the rates in the discount series.

EXAMPLES FOR PRACTICE.

577. 1. What must be the asking price of a watch, costing \$18, that 33 $\frac{1}{3}$ % may be gained, after allowing the purchaser a discount of 20%?

2. Having bought an invoice of lawn mowers at \$15 each, I desire to so mark them that I may gain 20%, and still discount 25% and 20% to my customers. At what price must each be marked?

3. Having paid \$8800 for a stock of goods, what price must be asked for it, in order to gain \$1100 and allow 12 $\frac{1}{2}$ % and 10% discount?

4. After buying velvet at \$5 per yard, I so marked it as to allow discounts of 25%, 20%, and 16 $\frac{2}{3}$ % from the marked price, and yet so sell it as to lose but 10% on my purchase. At what price per yard was the velvet marked?

5. The cost of manufacturing silk hats being \$36 per dozen, how must they be marked, that a gain of 16 $\frac{2}{3}$ % may be realized by the manufacturer, after allowing discounts to the trade of 20% and 12 $\frac{1}{2}$ %?

6. If a carriage be marked 33 $\frac{1}{3}$ % above cost, what per cent. of discount can be allowed from the marked price and realize cost?

7. If the list price of an article is 25% advance on the cost, what other per cent. of discount than 10% must be allowed, to net 10% gain by sale?

8. A merchant purchasing a bill of goods was allowed discounts from the list price of 15%, 10%, 10%, and 6%. If the total discount allowed was \$352.81, what must have been the asking price of the goods?

578. To find a Single Equivalent Per Cent. of Discount, a Discount Series being given.

EXAMPLE.—What single rate of discount is equal to the series 25%, 20%, 10%, and 5%?

OPERATION.

\$1000 = assumed list price or base.

250 = 1st discount.

\$750 = 1st rem. or 2d base.

150 = 2d discount.

\$600 = 2d rem. or 3d base.

60 = 3d discount.

\$540 = 3d rem. or 4th base.

27 = 4th discount.

\$513 = 4th or last rem. or net price.

\$1000 = list price, or base.

513 = net price.

\$487 = total discount on \$1000, which, divided by 1000, gives $48\frac{7}{10}$, the per cent. of discount equivalent to the given series.

EXPLANATION.—Assume \$1000 as the list price, and successively deduct the discounts as by the series, and compare the result with the base assumed.

Rule.—From \$1000 as a list price, or base, take the discounts in order; subtract the final remainder from the base taken, and the result will be the total discount; then point off from its right three places for decimals, and the expression thus obtained will be the equivalent per cent. of discount required.

REMARK.—This is the usual method, and it is more convenient for business men than to compute the net price for each sale through a series of discounts.

EXAMPLES FOR PRACTICE.

579. 1. Find a single discount equivalent to a series of 10% and 10%.
2. Find a single discount equivalent to a series of 25%, 15%, and 5%.
3. Find a single discount equivalent to a series of 30%, 20%, 10%, and 3%.
4. Goods were sold 25%, 35%, 20%, and 15% off; what single discount would have insured the same net price?
5. What is the difference between a single discount of 50% and a series of 20%, 20%, and 10%?
6. What per cent. of the list price will be obtained for goods sold at a discount of 35%, 20%, 15%, 10%, and 5%?
7. From a list price, I discounted 30%, 25%, 20%, 15%, $12\frac{1}{2}$ %, 10%, and 5%. What per cent. better for the purchaser would a single discount of 75% have been?

STORAGE.

580. Storage is a provision made for keeping goods in a warehouse for a time agreed upon, or for an indefinite time, subject to accepted conditions.

581. The term storage is used also to designate the charges for keeping the goods stored.

582. Rates of storage may be fixed by agreement of the parties to the contract, but are often regulated by Boards of Trade, Chambers of Commerce, Associations of Warehousemen, and by legislative enactment.

583. Storage Charges may be made at a fixed price per package or bushel, or at a fixed sum for a term or terms; they may be made for a term of days or months; but usually, if the goods stored are taken out before the storage time expires, the charge made is for the full time.

584. The rates of storage often vary for grains, or goods of different grades or values, and also on account of different modes of shipment.

REMARKS.—Storage Receipts, especially of grains, are frequently bought and sold under the name of "Warehouse Receipts" or "Elevator Receipts," as representing so much value by current market reports.

585. Cash Storage is a term applied to cases in which the payment of charges is made on each withdrawal or shipment, at the time of such withdrawal or shipment, notwithstanding the fact that the owner may still have goods of the same kind in store at the warehouse.

586. Credit Storage is a term applied to cases in which sundry deposits or consignments are received, from which sundry withdrawals or shipments are made, and all charges adjusted at the time of final withdrawal.

REMARKS.—1. When deposits or consignments, and withdrawals or shipments, are made at different times, credit is to be given for the amount of each deposit or consignment, from the date to its final withdrawal or shipment, and credit given to the owner or consignor for each withdrawal or shipment, from date up to the time of settlement.

2. In the private bonded warehouses of the United States, goods may be taken out at any time, in quantities not less than an entire package, or, if in bulk of not less than 1 ton, by the payment of duties, storage, and labor charges. The storage charges are computed for periods of one month each, a fractional part of a month being counted the same as a full month.

3. Drovers sometimes hire cattle fed on account, entering and withdrawing them as circumstances require; such accounts are closed in the same manner as are those for storage.

587. To find the Simple Average Cash Storage.

EXAMPLE.—There was received at a storage warehouse: Oct. 11, 300 bar. apples; Oct. 30, 250 bar. potatoes; Nov. 13, 200 bar. apples; Nov. 20, 60 bar. quinces; Nov. 28, 280 bar. apples. The merchandise was all delivered Dec. 2.

If the contract specified that the rate of storage was 5¢ per barrel for a period of 30 days average storage, what was the storage bill?

OPERATION.

The storage of 300 bar. for 52 da. = the storage of 1 bar. for 15600 da.

The storage of 250 bar. for 33 da. = the storage of 1 bar. for 8250 da.

The storage of 200 bar. for 19 da. = the storage of 1 bar. for 3800 da.

The storage of 60 bar. for 12 da. = the storage of 1 bar. for 720 da.

The storage of 280 bar. for 4 da. = the storage of 1 bar. for 1120 da.

The total storage = the storage of 1 bar. for 29490 da.

Or, 983 periods of 30 days each; $\$.05 \times 983 = \49.15 , storage bill.

EXPLANATION.—The 300 barrels constituting the first deposit or delivery were stored from Oct. 11 to Dec. 2, or for 52 days; the storage of 300 bar. for 52 days equals the storage of 1 barrel for 15600 days; the storage of 250 barrels for 33 days equals the storage of 1 barrel for 8250 days; that of 200 barrels for 19 days equals 1 barrel for 3800 days; that of 60 barrels for 12 days equals 1 barrel for 720 days; that of 280 barrels for 4 days equals 1 barrel for 1120 days. The total storage was equal to that of 1 barrel for 29490 days, or for 983 storage terms or periods of 30 days each. Since the storage charge was 5¢ per barrel for each average period of 30 days, the charge would amount to $\$.05 \times 983$, or \$49.15.

Rule.—*Multiply the number of articles of each receipt by the number of days between the time of their deposit and withdrawal; divide the sum of these products by the number of days in the storage period, and multiply the quotient by the charge per period.*

EXAMPLES FOR PRACTICE.

588. 1. There was received at a warehouse: May 30, 4000 bu. wheat; June 5, 2600 bu. oats; June 24, 3500 bu. barley; July 18, 5000 bu. corn. If all of this was shipped July 20, what was the storage bill, the charge being 1½¢ per bushel per term of 30 days average storage?

2. A farmer received for pasture: April 30, 12 head of cattle; May 15, 14 head of cattle; May 23, 27 head of cattle; June 9, 5 head of cattle; June 30, 8 head of cattle; July 16, 40 head of cattle. All were delivered July 25, and the charges were 75¢ per head for each week of 7 days average pasturage. How much was his bill?

589. To find the Charge for Storage with Credits.

EXAMPLE.—The storage charges being 2¢ per barrel for a month of 30 days average, what will be the bill in the following transaction?

<i>Received.</i>	<i>Delivered.</i>
July 19, 100 bar.; July 31, 240 bar.; Sept. 8, 360 bar.	Aug. 15, 300 bar.; Sept. 12, 200 bar.; Oct. 1, 200 bar.

OPERATION.

From July 19 to July 31 = 12 da.; 100 bar. stored for 12 da. = 1 bar. stored for 1200 da.
 July 31 240 bar. received.

From July 31 to Aug. 15 = 15 da.; 340 bar. stored for 15 da. = 1 bar. stored for 5100 da.
 Aug. 15 300 bar. delivered.

From Aug. 15 to Sept. 8 = 24 da.; 40 bar. remaining for 24 da. = 1 bar. stored for 960 da.
 Sept. 8 360 bar. received.

From Sept. 8 to Sept. 12 = 4 da.; 400 bar. stored for 4 da. = 1 bar. stored for 1600 da.
 Sept. 12 200 bar. delivered.

From Sept. 12 to Oct. 1 = 19 da.; 200 bar. remaining for 19 da. = 1 bar. stored for 3800 da.
 Oct. 1 200 bar. delivered.

000

Total = 1 bar. stored for 12660 da.,

Or, 422 terms of 30 da. each; $\$.02 \times 422 = \8.44 , total storage bill.

EXPLANATION.—100 barrels were stored for 12 days, when 240 barrels were added; these 340 barrels were stored 15 days, when 300 barrels were withdrawn; the 40 remaining barrels were stored 24 days, when 360 barrels were added; these 400 barrels were stored 4 days, when 200 barrels were withdrawn; the remaining 200 barrels were stored 19 days and then withdrawn. The total storage thus equalled that of 1 barrel for 12660 days, or for 422 terms of 30 days each; and since the charge for 1 term is $\$.02$, for 422 terms it would be 422 times $\$.02$, or $\$8.44$, the total amount of the bill.

Rule.—I. *Multiply the number of articles first received by the number of days between the date of their receipt and the date of the next receipt or delivery; add the number of articles of such next receipt, or subtract the number of such delivery, as the case may be, and so proceed to the time of final delivery.*

II.—*Divide the aggregate storage by the number of days in the storage term, and multiply the quotient by the storage charge per term.*

EXAMPLES FOR PRACTICE.

590. 1. What will be the storage charge, at $4\frac{1}{2}\phi$ per barrel, for a term of thirty days average, in the following transaction?

<i>Received.</i>		<i>Delivered.</i>	
Feb. 8,	180 bar. flour.	Mar. 1,	100 bar. apples.
Feb. 27,	100 bar. apples.	Mar. 28,	190 bar. flour.
Mar. 8,	60 bar. potatoes.	Apr. 15,	60 bar. potatoes.
Mar. 13,	300 bar. flour.	Apr. “	60 bar. flour.
		Apr. 29,	230 bar. flour.

2. A drover hired pasture of a farmer, agreeing to pay \$4.20 per head of stock pastured for each average term of 30 days. What was the amount of the bill, the receipts and deliveries being as follows?

<i>Received.</i>		<i>Delivered.</i>	
June 15,	21 head of cattle.	July 1,	30 head of cattle.
June 27,	20 head of cattle.	July 20,	15 head of cattle.
July 5,	15 head of cattle.	July 30,	15 head of cattle.
July 29,	40 head of cattle.	Aug. 21,	the remainder.
July 31,	40 head of cattle.		

591. To find the Storage where Charges Vary.

EXAMPLE.—At a warehouse there was received and delivered flour, as follows:

<i>Received.</i>		<i>Delivered.</i>	
Jan. 3,	150 bar.	Jan. 23,	250 bar.
Jan. 20,	200 bar.	Mar. 1,	400 bar.
Feb. 1,	300 bar.		

The storage charge on the above was, 5¢ per barrel for the first 10 days or part thereof, and 3¢ per barrel for each subsequent period of 10 days or part thereof. What sum must be paid in settlement?

<i>Date.</i>	<i>Receipts and Deliveries.</i>	<i>OPERATION.</i>	<i>Rate.</i>	<i>Storage.</i>
Jan. 3,	received 150 bar.			
" 20,	" 200 "			
	350 "	in store.		
" 23,	delivered 250 "	150 bar. stored 20 days, or 2 terms, 8¢ = \$12.00		
	100 "	100 " 3 " 1 term, 5¢ = 5.00		
Feb. 1,	received 300 "	remainder.		
	400 "	in store.		
Mar. 1,	delivered 400 "	100 bar. stored 40 days, or 4 terms, 14¢ = \$14.00		
	300 "	300 " 28 " 3 " 11¢ = 33.00		
Total storage,				= \$64.00

EXPLANATION.—Of the 250 barrels delivered Jan. 23, 150 barrels had been in store since Jan. 3, 20 days or 2 terms, and the charge was 5 cents plus 3 cents, or 8 cents per barrel, which equals \$12 storage. The remaining 100 barrels of the delivery of Jan. 23, had been in store only since Jan. 20, 3 days or 1 term, at 5 cents per barrel, equal to \$5 storage. Of the 400 barrels delivered Mar. 1, 100 barrels had been in store since Jan. 20, 40 days or 4 terms, at 5 cents plus 3 cents plus 3 cents plus 3 cents, or 14 cents per barrel, equal to \$14 storage; while the remaining 300 barrels had been in store since Feb. 1, 28 days or 3 terms, at 5 cents plus 3 cents plus 3 cents, or 11 cents per barrel, equal to \$33 storage. By addition, the total storage is found to be \$64.

Rule.—*Multiply the number of articles of each delivery by the charge for the term or terms stored, and add the products so obtained.*

EXAMPLE FOR PRACTICE.

592. 1. The receipts and deliveries of goods at a storage warehouse were as follows:

<i>Received.</i>		<i>Delivered.</i>	
Sept. 2,	100 bar.	Sept. 20,	100 bar.
Sept. 25,	200 bar.	Sept. 30,	100 bar.
Oct. 19,	350 bar.	Oct. 10,	100 bar.
Oct. 31,	150 bar.	Oct. 20,	100 bar.
Nov. 7,	200 bar.	Oct. 30,	100 bar.
		Nov. 20,	the remainder.

The contract required the payment of 6¢ per barrel for the present term of 30 days or fraction thereof, and 3¢ per barrel for each subsequent term of 30 days or fraction thereof. Find the storage bill.

COMMISSION.

593. An **Agent** is a person who transacts business for another; as, the purchase or sale of merchandise or real estate, collecting or investing money, etc.

594. An agent who receives goods to be sold is sometimes called a *factor* or *commission merchant*; one employed to buy or sell stocks or bonds, or to negotiate money securities, is called a *broker*.

595. **Commission** is an allowance made to agents or commission merchants for transacting business. It is usually a percentage of the money involved in the transaction, although sometimes it is computed at a certain price per bale, bushel, barrel, etc.

596. The **Agent's Commission** for selling is computed on the gross proceeds, and for purchasing on the *prime* cost.

597. The **Principal** is the person for whom the business is transacted.

598. A **Consignment** is a shipment of goods from one party to another, to be sold on account of the shipper, or on joint account of the shipper and the consignee. The shipper is called the *Consignor*, and the one to whom the goods are shipped is called the *Consignee*.

599. **Guaranty** is a per cent. charged by an agent for assuming the risk of loss from sales made by him on credit, or for giving a pledge of the grade of goods bought; it is computed the same as are commission charges.

600. The **Gross Proceeds** of a sale or collection is the total amount received by the agent before deducting commission or other charges.

601. The **Net Proceeds** is what remains after all charges have been deducted.

REMARKS.—Charges may be for commission, guaranty, freight, inspection, cartage, storage, or any other outlay incident to the sale.

602. An **Account Sales** is a statement in detail rendered by a *Consignee* to his *Consignor*, showing the sales of the consignment, all of the charges or expenses attending the same, and the *net proceeds*.

603. An **Account Purchase** is a detailed statement made by a purchasing agent to his principal, having the quantity, grade and price of goods bought on his account, all the expenses incident to the purchase, and the gross amount of the purchase.

604. *Commission* compares with *Abstract Percentage*, as follows:

The Prime Cost or Gross Selling Price = Base.

The Rate Per Cent. of Commission = Rate.

The Commission for either buying or selling, or for guaranty of quality or credit = Percentage.

The remittance to Purchasing Agent, including both Commission and Investment = Amount.

The Selling Price, *minus* the Commission = Difference.

605.—To find the Commission, the Cost or Selling Price and Per Cent. of Commission being given.

EXAMPLE.—How much commission will be due an agent who buys \$8000 worth of coal, on a commission of 5%?

OPERATION.

\$8000 = investment or base.

.05 = per cent. of commission.

\$400 = commission or percentage.

EXPLANATION.—Since the rate of commission is 5 per cent., the whole commission due the agent will be 5 per cent. of the investment, \$8000, or \$400.

REMARK.—In case of *sales*, proceed in like manner, treating the *selling price* as the base.

Rule.—*Multiply the cost or selling price by the rate per cent. of commission.*

Formula.—Commission = Cost or Selling Price \times Rate per cent. of Commission.

EXAMPLES FOR PRACTICE.

606. 1. An agent sold a house and lot for \$6000, and charged 3% for his services. How much was the commission?

2. Having agreed to pay an agent 3% for all purchases made by him, how much will be due him, if he buys for me goods costing \$2500?

3. If an agent's charges are 2%, how much commission will he earn by selling property valued at \$12500?

4. I owned one-half of a stock of goods sold by an agent for \$10000. If the agent charged $5\frac{1}{2}\%$ for selling, how much commission must I pay?

5. An auctioneer sold a store for \$8500, and its contents for \$7350. How much did his fees amount to, at $1\frac{3}{4}\%$?

6. A real estate agent sold a farm or 91 acres, at \$120 per acre, on a commission of 2%; and the stock and utensils on the farm for \$3150, on a commission of 5%. What was the amount of his commission?

607. To find the Investment or Gross Sales, the Commission and Per Cent. of Commission being given.

EXAMPLE.—If an agent's rate of commission is 2%, what value of goods must he sell to earn a commission of \$50?

OPERATION.

Com.

$2\% = .02 \) \ \$50.00$

\$2500 gross sales.

EXPLANATION.—Since the agent's commission is 2 per cent., he earns 2 cents by selling \$1 worth of goods; the value of the goods sold, therefore, must be as many times \$1 as 2 cents is contained times in \$50; 2 cents is contained in \$50, 2500 times, and 2500 times \$1 is \$2500

REMARK.—When commission for purchase is given and cost required, proceed in like manner.

Rule.—*Divide the commission by the rate per cent. of commission.*

Formula.—Prime Cost or Gross Selling Price = Commission, divided by the Rate Per Cent. of Commission.

EXAMPLES FOR PRACTICE.

608. 1. What amount of merchandise must be purchased on a commission of $3\frac{1}{2}\%$, in order that an agent may receive a commission of \$175?

2. An agent received \$306.25 for selling wheat, on a commission of $1\frac{1}{4}\%$. What was the amount of the sales?

3. A collector's charges of 5% for collecting a note amounted to \$14.10. What sum was collected?

4. A factor charged \$216.80 for selling a consignment of canned fruit. If his commission was $2\frac{1}{2}\%$, what must have been the gross sales?

5. I paid a grain dealer $1\frac{1}{2}\%$ for buying corn for me, at 62¢ per bushel. If his commission amounted to \$83.70, how many bushels did he buy?

6. A Mobile factor earned \$99.75 by selling cotton, at $2\frac{3}{8}\%$ commission. How many bales, averaging 560 lb., did he sell, the price being 15¢ per pound?

609. To find the Investment and Commission, when Both are Included in a Remittance by the Principal.

EXAMPLE.—If \$1050 is sent to a Saginaw agent for the purchase of salt, how much will he invest, his rate of commission being 5%?

OPERATION.

\$1.00 = investment.

.05 = commission.

\$1.05 = actual cost to principal of each dollar invested by agent.

1.05) \$1050.00

\$1000 sum invested in salt.

EXPLANATION.—For each dollar invested

by the agent, the principal supplies the dollar invested and 5 cents for the agent's services; therefore the agent will invest only as many dollars in salt as \$1 plus 5 cents, or \$1.05, is contained times in the remittance, \$1050; 1.05 is contained in \$1050, 1000 times; hence the investment is \$1000.

Rule.—*Divide the remittance by 1 plus the rate per cent. of commission.*

REMARKS.—1. All computations in commission may be made by applying the principles of Percentage.

2. When a charge is made for guaranty, add the per cent. of guaranty to 1 plus the rate per cent. of commission, and proceed as above.

Formula.—Investment = Remittance to Agent ÷ 1 plus the Rate Per Cent. of Commission.

EXAMPLES FOR PRACTICE,

610. 1. An agent receives \$12504.20, with instructions to invest in wool. If his commission is 3%, how many dollars worth of wool will he purchase?

2. How many pounds of wool, at 27¢ per pound, can be bought for \$8424, if the agent is allowed 4% for purchasing?

3. I remitted \$1306.45 to a Boston agent for the purchase of soft hats. If the agent's commission is 4%, and he makes an added charge of 2% for guaranty of quality, how many dozen hats, at \$8.50 per dozen, should he send me?

4. An agent receives \$13760.80 to invest in land, after deducting his charges of 3%. What amount of commission will he receive?
5. A real estate agent, whose stated commission is $2\frac{1}{2}\%$, receives \$8302.50 to invest in Iowa prairie, at \$5.40 per acre. How many acres did he purchase, and how much was his commission.
6. I remitted \$300 to an agent for the purchase of hops. If the agent's charges were 5% for purchase and \$6 for inspection, how many pounds, at 16¢ per pound, ought he to buy?
E R i

MISCELLANEOUS EXAMPLES.

611. 1. A collector obtained 75% of the amount of an account, and after deducting 12% for fees, remitted his principal \$495. What was the amount of his commission?
\$67.50

2. A Hartford fruit dealer sent a Lockport agent \$1946.70, and instructed him to buy apples at \$1.40 per barrel. The agent charged 3% for buying, and shipped the purchase to his principal in six car loads of an equal number of barrels. How many barrels did each car contain?

3. Find the per cent. of commission on a purchase, if the gross cost is \$2048.51, the commission \$87.30, the cartage \$20, and other charges \$1.21. $4\frac{2}{3}\%$

4. 11500 bushels of wheat were bought through an agent, who charged $\frac{1}{3}\%$ for buying. If the agent paid 85¢ per bushel for the wheat, \$762.50 freight, and \$12.50 insurance, what sum should be remitted to him in full settlement?
\$10635.50

5. A collector obtained 75% of a doubtful account amounting to \$1750. How much was his per cent. of commission, if, by agreement with the principal, the commission was to be 50% of the net proceeds remitted?

6. A farmer received from his city agent \$490 as the net proceeds of a shipment of butter. If the agent's commission is 3%, delivery charges \$6.80, and 5% charge is made for guaranty of quality to purchasers, how many pounds, at 27¢ per pound, must have been sold, and how much commission was allowed?
700 lb

7. An agent sold 2000 bu. Alsike clover seed, at \$7.85 per bushel, on a commission of 5%; and 1200 bu. medium red, at \$5.20 per bushel, on a commission of $2\frac{1}{2}\%$; taking the purchaser's 3 month's note for the amount of the sales. If the agent charges 4% for his guaranty of the notes, what amount does he earn by the transaction?

8. An agent bought butter on a commission of 10%, cheese on a commission of 6%, and eggs on a commission of 5%. If his commission for buying the butter was \$21, for buying the cheese \$21.60, and for buying the eggs \$22, and he charges 25% additional for guaranteeing the freshness of the eggs, what sum should the principal remit to pay for purchases and charges?
1184

9. Find the net proceeds of a sale made by an agent charging $3\frac{1}{2}\%$, if incidental charges and commission charges were each \$41.30.

10. From a consignment of 3160 pounds of tea, sold by an agent at 30¢ per pound, the consignor received as net proceeds \$853.74. What was the per cent. of commission charged for selling, if the charges for storage and insurance amounted to \$51.60?
47

11. Find the gross proceeds of a sale made by an agent charging $2\frac{1}{2}\%$ for commission, 5% for guaranty, \$17.65 for cartage, \$11.40 for storage, and \$3.25 for insurance, if the net proceeds remitted amount to \$1714.10. \$1888⁰⁰

12. A Milwaukee agent received \$83195.28, with instructions to invest one-half of it in wheat, at 80¢ per bushel, and the balance, less all commissions, in wool, at 20¢ per pound. If his commission for buying the wheat is 2%, and that for buying the wool is 5%, how many pounds of wool will he buy, and what will be the amount of his commissions?

13. I sent \$3402.77 to my Atlanta agent for the purchase of sweet potatoes, at \$1.60 per barrel; his charges were, for commission, $2\frac{1}{4}\%$; guaranty, 3%; drayage, 1¢ per barrel; and freight, \$200. How many barrels did he buy, and how much unexpended money was left in his hands to my credit?

14. A Texas buyer shipped 33000 lb. of coarse wool to a Boston agent to be sold on commission, and gave instructions for the net proceeds to be invested in leather. If the agent sold the wool at 18¢ per pound, on a commission of 2%, and charged 10% for the purchase and guaranty of grade of the leather, what was the amount of his commissions?

15. I received from Duluth a cargo of 16000 bu. of wheat, which I sold at \$1.10 per bushel, on a commission of 4%; by the consignor's instructions I invested the net proceeds in a hardware stock, for which I charged 5% commission. What was the total commission, and how much was invested in hardware?

16. Having sent a New Orleans agent \$1835.46 to be invested in sugar, after allowing 3% on the investment for his commission, I received 32400 pounds of sugar. What price per pound did it cost the agent?

17. An agent in Providence received \$828 to invest in prints, after deducting his commission of $3\frac{1}{2}\%$. If he paid $7\frac{1}{2}\%$ per yard for the prints, how many yards did he buy?

18. The fees of the general agent of an insurance company are 5% on all sums received, and 5% additional on all sums remaining in his hands at the end of the year, after all losses and the expenses of his office are paid. He receives during the year \$117416.25, paid losses to the amount of \$91140.50, and the expenses of his office were \$3267.70. Find his total fees.

19. An agent sold on commission 81 self-binders, at \$140 each, and 113 mowers, at \$65 each, remitting \$10224.90 to his principal. Find the rate of commission.

20. A commission merchant received a consignment of 660 bales of cotton, of an average weight of 510 pounds, which he sold at $12\frac{1}{2}\%$ per pound, on a commission of 3%, charging 10¢ per bale for cartage. He invested for the consignor \$9416.20 in bacon, charging 5% for buying, and remitted cash to balance consignor's account. How much was the cash remittance?

21. An agent received \$4325, to invest in mess-pork, at \$16 per barrel, after deducting his purchasing commission of 4%. If the charges for incidentals were \$81.40, besides cartage of 75¢ per load of 8 barrels, how many barrels did he buy, and what unexpended balance does he place to the credit of his principal? 253
948

22. A street-car company bought 35 horse-cars through a Troy agent, at \$850 each. If the freight was \$17.50 on each car, and the agent's commission $3\frac{1}{2}\%$ for purchasing, what was the total cost to the company? *31,403.75*

23. I received from Day & Son, of Chicago, a ship load of corn, which I sold for 60¢ per bushel, on a commission of 4%; and, by the shipper's instructions, invested the net proceeds in barley, at 75¢ per bushel, charging 5% for buying; my total commission was \$1350. How many bushels of corn did Day & Son ship, and how many bushels of barley should they receive? *26960 - 19200*

24. An agent sold, on commission, 1750 barrels of mess-pork, at \$16.50 per barrel, and 508 barrels of short-ribs, at \$18 per barrel, charging \$112.50 for cartage, and \$5.55 for advertising. He then remitted to his principal \$36000, the net proceeds. Find the rate of commission. *5%*

25. A Wichita dealer sent 12 car loads of corn, of 825 bushels each, to an agent in Baltimore, where it was sold at $62\frac{1}{2}\%$ per bushel, on a commission of 5%, the agent paying \$682.50 freight. By shipper's instructions, the agent invested the net proceeds in a hardware stock, charging 3% for buying. How much was invested in hardware? *5044.29*

26. The holder of a doubtful claim of \$850, handed it to an agent for collection, agreeing that, for every dollar sent him by the agent, the agent might keep for himself 20¢. The agent succeeded in collecting but 80% of the debt. How much did the agent remit, how much commission did he receive, and what was his per cent. of commission? *Rem 567/3 - Com 113 1/3 - Rate 16 2/3*

27. I remitted \$10500 to a Duluth agent to be invested in wheat, allowing him a commission of 3% for investing. The agent paid 95¢ per bushel for the wheat, and charged me $1\frac{1}{2}\%$ a bushel per month for storage. At the end of 4 months the agent sold the wheat at \$1.10 per bushel, on a commission of 5%. If I paid \$350 for the use of the money, did I gain or lose by the operation, and how much?

28. My Memphis agent sends me an account purchase of 350 bales of cotton, averaging 480 pounds each, bought at 15¢ per pound, on a commission of $2\frac{1}{2}\%$. His charges, other than for commission, were: freight advanced, \$126.50; cartage, \$53.25; and insurance, \$13.75. What sum should I remit to pay the account?

29. A Charleston factor received from Cincinnati a consignment of corn, which he sold at 75¢ per bushel, on a commission of 5%; and by instructions of the consignor invested the net proceeds in cotton, at 20¢ per pound, charging 3% for buying, and 3% additional for guaranty of quality. If the total amount of the agent's commission and guaranty was \$1640, how many bushels of corn were received?

30. A Buffalo brewer remitted \$21500 to a Toronto commission merchant, with instructions to invest 40% of it in barley, and the remainder, less all charges, in hops. The agent paid 60¢ per bushel for barley, and 20¢ per pound for hops, charging 2% for buying the barley, 3% for buying the hops, and 5% for guaranteeing the quality of each purchase. If his incidental charges were \$187.50, what quantity of each product did he buy, and what was the amount of his commission?

CUSTOM-HOUSE BUSINESS.

612. Duties, or Customs, are taxes levied by the Government on imported goods, for revenue purposes and for the protection of home industry.

613. Duties are of two kinds, *ad valorem* and *specific*.

614. An Ad Valorem Duty is a certain per cent. assessed or levied on the actual cost of the goods in the country from which they are imported, as shown by the invoice.

615. A Specific Duty is a tax assessed or levied upon the number, weight, or measure of goods, regardless of their value; as, a fixed sum per bale, ton, barrel, etc.

REMARK.—Upon certain goods both specific and *ad valorem* duties are levied.

616. A Custom-House is an office established by the Government for the transaction of business relating to *duties*, and for the entry and clearance of vessels.

REMARK.—1. The ports at which custom-houses are established are called *ports of entry*.

2. The waters and shores of the United States are divided into collection districts, in each of which there is a port of entry, which is also a port of delivery; other ports than those of entry may be specified as *ports of delivery*. Duties are paid, and entries and clearances made, at ports of entry only; but after vessels have been properly entered, their cargoes may be discharged at any port of delivery.

617. An Invoice, or Manifest, is a written account of the particular goods sent to the purchaser or factor, with the actual cost, or value, of such goods, made out in the currency of the country from which they are imported.

REMARKS—All invoices are made out in the weights, measures, etc., of the place from which the goods are imported.

618. A Tariff is a schedule of goods, and the rates of import duties imposed by law on the same.

619. The Free List includes classes of goods that are exempt from duty.

620. Tonnage is a tax levied upon a vessel independent of its cargo, for the privilege of coming into a port of entry.

621. Duties are collected at the port of entry by a custom-house officer appointed by the United States Government, and known as the Collector of the Port. Under him are deputy collectors, appraisers, weighers, gaugers, etc.

622. The Collector of the Port supervises all entries and papers pertaining to them; estimates all duties, receives all moneys, and employs all weighers, gaugers, etc.

623. Before estimating *specific duties*, allowances are made; these allowances are called *Tare, Leakage, Breakage*, etc.

624. Tare is an allowance made for the box, bag, crate, or other covering of the goods.

625. Leakage, determined by gauging, is an allowance made for waste of liquids imported in barrels or casks.

626. Breakage is an allowance made for loss of liquids imported in bottles.

627. Gross Weight is the weight before any allowances are made.

628. Net Weight is the weight after all allowances have been made.

REMARKS.—1. The ton used at the United States Custom-Houses is of 2240 avoirdupois pounds.

2. Duties are not computed on fractions of a dollar; if the cents in the invoice are less than 50, they are rejected; if 50 or more, they are counted as a dollar.

629. The Naval Officer, appointed only at the more important ports, receives copies of all manifests, countersigns all documents issued by the Collector, and certifies his estimates and accounts.

630. The Surveyor superintends the employees of the Collector, and revises entries and permits.

631. The Appraiser examines imported articles, and determines their dutiable value and also the rate of duty to be charged.

632. The Store-keeper has charge of the warehouse.

REMARKS.—1. *Warehousing* is depositing imported goods in a government or bonded warehouse.

2. A *bonded warehouse* is used for storing goods on which the duties have not been paid.

3. Goods may be withdrawn from a bonded warehouse for export, without the payment of the duties. If goods on which the duty has been paid are exported, the amount of duty so paid is refunded; the sum so refunded is called a *drawback*.

4. *Smuggling* is bringing foreign goods into the country without paying the required duty. This is done either by not entering them at a Custom-House, or by showing less than their real value in the invoice. It is a crime, for the prosecution and punishment of which stringent laws are enacted.

5. Many merchants employ a Custom-House Broker, one familiar with the laws, to enter goods for them.

633. To find Specific Duty.

EXAMPLE.—What is the specific duty on 140 casks of alcohol, of 60 gallons each, at 15¢ per gallon; leakage 5%?

OPERATION.

$$140 \times 60 \text{ gal.} = 8400 \text{ gal.}$$

$$.05 = \% \text{ of leakage.}$$

$$420 \text{ gal.} = \text{leakage.}$$

$$8400 \text{ gal.} = \text{gross quantity.}$$

$$420 \text{ gal.} = \text{leakage.}$$

$$7980 = \text{net quantity.}$$

$$\$.15 = \text{duty per gallon.}$$

$$\$1197.00 = \text{specific duty.}$$

EXPLANATION.—Specific duty is computed on the net quantity; to find the net quantity, take 420 gallons, the allowance for leakage, from 8400, the whole number of gallons, which gives 7980 gallons, on which to charge duty. Since the duty is 15 cents per gallon, for 7980 gallons it will be \$1197.

Rule.—*Multiply the net quantity by the duty per single article of the kind or class considered.*

EXAMPLES FOR PRACTICE.

634. 1. What is the duty on 60 packages of figs, each of 16 lb. weight, at $2\frac{1}{2}\phi$ per pound, tare 5%?

2. Find the duty, at 75ϕ per ton, on an invoice of 897120 lb. of bituminous coal.

3. If the duty on plate glass is 25ϕ per square foot, how much will be charged on an importation of 200 boxes, each containing 20 plates 24×48 in. in size?

4. Find the duty, at \$2 per dozen, on 40 doz. bottles of wine imported from Lyons, if an allowance of 10% is made for breakage.

5. If the duty is 65ϕ per cubic foot, what amount must be paid on an importation of 6 blocks of marble, each 10 ft. long, 3 ft. wide, 2 ft. high?

6. After being allowed 10% for leakage, a wine merchant paid \$864 duty, at \$2 per gallon, on 12 casks of wine. How many gallons did each cask originally contain?

7. Find the duty on 1500 dozen empty bottles, breakage 4%, and rate of duty 10ϕ per dozen.

635. Applying the terms of Percentage to *Ad Valorem* Duties, we observe the following:

The *net Value*, or *Quantity* = the *Base*.

The *Rate Per Cent. Ad Valorem* = the *Rate*.

The *Duty* = the *Percentage*.

636. To find Ad Valorem Duty.

EXAMPLE.—What is the ad valorem duty, at 35%, on 90 boxes of brass rivets, 25 lb. per box, invoiced at 12ϕ per pound, tare being 6 lb. per box?

OPERATION.

$$90 \times 25 \text{ lb.} = 2250 \text{ lb. gross weight.}$$

$$90 \times 6 \text{ lb.} = 540 \text{ lb. tare.}$$

$$1710 \text{ lb. net weight.}$$

$$1710$$

$$\$.12 = \text{cost per pound}$$

$$\$205.20 = \text{net value.}$$

$$.35 = \% \text{ of duty.}$$

$$1025$$

$$615$$

$$\$71.75 = \text{duty.}$$

EXPLANATION.—Find the net weight and value as the base; multiply by the rate of duty.

REMARK.—The cents in the net value, being less than 50, are rejected.

Rule.—*Multiply the value, after all deductions are made, by the per cent. of duty assessed.*

EXAMPLES FOR PRACTICE.

637. 1. What is the duty, at 50%, upon a consignment of 650 dozen kid gloves, invoiced at 90 francs per dozen ?

2. An importation of English crockery was invoiced at £896, 5s. 6d. Find the duty, at 40%.

3. If an importation is invoiced at 17450 francs, what will be the duty, at 35%?

4. If the duty on sperm oil is 20%, what will it amount to in an importation of 600 barrels, of 42 gallons each, invoiced at 45¢ per gallon, $3\frac{1}{2}\%$ being allowed for leakage ?

5. I received by steamer Raglan, from Liverpool, the following invoice of goods : 768 yd. velvet, invoiced at £1 12s. per yd.; 2150 yd. lace, invoiced at 3s. 4d. per yd.; 1200 yd. broadcloth, invoiced at 15s. per yd.; 3520 yd. carpet, invoiced at 11s. 6d. per yd. If the duty on the velvet was 60%, on the lace and broadcloth 35%, and on the carpet 50%, how much was the total duty to be paid?

MISCELLANEOUS EXAMPLES.

638. 1. What is the duty on 1000 yd. of brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yd; duty 44¢ per square yard specific, and 35% ad valorem ?

2. If the duty on flannel is 29¢ per pound specific, and 35% ad valorem, how much must be paid on an invoice of 2150 yd., weighing 420 lb., and valued in Canada, whence it was imported, at 75¢ per yard ?

3. Find the duty on 3 dozen clocks, invoiced at \$21.50 each, and 6 dozen watches, invoiced at \$35 each, if the ad valorem duty was 35% on the clocks, and 25% on the watches.

4. How much duty must be paid on an importation of 27640 lb. of wool, invoiced at £1497 10s. 4d., if the rate of duty is 10¢ per pound specific; and 11% ad valorem?

5. I imported from Canada 7240 bushels of barley, and $17\frac{1}{2}$ tons of hay, invoiced at \$9.50 per ton. What amount of duties had I to pay, at 10¢ per bushel on the barley and 20% on the hay?

6. A merchant imported 300 pieces of three-ply carpet, each piece containing 75 sq. yd., invoiced at 3s. 6d. per square yard, upon which he paid a duty of 17¢ per square yard specific, and 35% ad valorem. What was the total amount of duty paid ?

7. An invoice of woollen cloth, imported from England, was valued at £956 6s. If its weight was 684 lb., how much was the duty, at 50¢ per pound specific, and 35% ad valorem ?

8. Find the duty on 50 cases of tobacco, each weighing 60 lb., and 50000 Havana cigars weighing 550 lb., invoiced at \$75 per M, the duty being 50¢ per pound specific on the tobacco, and \$2.50 per pound specific and 25% ad valorem on the cigars.

TAXES.

639. Taxes are sums of money levied on persons, property, or products, for any public purpose.

640. Capitation or Poll Taxes are levied at a certain amount for each person or head of legal voters not exempt by law.

641. Property Tax is a tax assessed or levied upon property, at a given rate per cent. of the valuation.

642. Property is of two kinds: Personal and Real.

643. Personal Property is *movable* property; as, merchandise, ships, cattle, money, stocks, mortgages, etc.

644. Real Property or Real Estate consists of *immovable* property; as, houses and lands.

645. Assessors are public or government officers, who appraise the value of property to be taxed, and apportion the taxes pro rata; that is, in proportion to the value of each man's property.

646. Collectors are public or government officers, who collect taxes.

REMARK.—Taxes are generally assessed and made payable in money, but in "road taxes" they may be made payable in "day's work."

647. The terms of Percentage, applied to Taxes, are:

The Valuation = the Base.

The tax on \$1.00 = the Rate.

The Sum to be raised = the Percentage.

The Sum, minus the Collector's fees, or commission = the Difference.

648. To find a Property Tax.

EXAMPLE.—The rate of taxation in the city of Des Moines, Iowa, is $1\frac{1}{4}\%$. What amount of tax must a person pay, whose personal property is valued at \$17500, and who owns real estate assessed at \$24900?

OPERATION.

\$17500

24900

$\$42400 \times .01\frac{1}{4} = \$742.$

EXPLANATION.—Since his total valuation was \$42400, and the rate of taxation $1\frac{1}{4}$ per cent., his tax would be $1\frac{1}{4}$ per cent of \$42400, or \$742.

Rule.—Multiply the total assessed value by the rate per cent. of taxation.

EXAMPLES FOR PRACTICE.

649. 1. Mr. B. owns personal property assessed at \$7140, and real estate assessed at \$11100, in a village in which he is taxed one-half of one per cent. Find the amount of his tax.

2. A person having lands valued at \$7500, \$12250 in money, and a stock of goods worth \$6000, pays tax on all at the rate of $1\frac{1}{2}\%$. Find his total tax.

650. To find a General Tax.

EXAMPLE.—A tax of \$2505 is to be assessed upon the village of Livonia; the valuation of the taxable property is \$600000, and there are 324 polls, to be assessed \$1.25 each. What will be the tax on a dollar, and how much will be the tax of Mr. Scott, whose property is valued at \$12500, and who pays for 2 polls.

OPERATION.

$\$1.25 \times 324 = \405 , amount of poll tax.

$\$2505 - \$405 = \$2100$, amount of property tax.

$\$2100 \div \$600000 = .0035$, rate of taxation.

$\$12500 \times .0035 = \43.75 , Mr. Scott's property tax.

$\$43.75 + \2.50 (2 polls) = \$46.25, Mr. Scott's total tax.

EXPLANATION.—Since \$2505, the amount to be raised, includes both the poll and property tax, if \$405, the poll tax, is subtracted from this amount, the remainder, \$2100, will be the Percentage, or sum to be assessed on the Base, or entire property. Divide this Percentage by this Base, and the quotient will be the rate of tax assessed, $3\frac{1}{2}$ mills on the dollar. Multiply \$12500, the assessed valuation of Mr. Scott's property, by .0035, the per cent. expressed decimally, and the result, \$43.75, is his property tax; adding to this \$2.50, the tax on two polls, gives \$46.25, his entire tax.

Rule.—*From the sum to be raised, deduct the poll tax, if any; divide the remainder by the total assessment, and multiply the assessment of each individual by the quotient; add to the product the amount of poll tax to be paid.*

EXAMPLES FOR PRACTICE.

651. 1. A tax of \$125000 is levied on a city, the assessed valuation of which is \$15000000. What is the rate of taxation, and what amount of tax will a person have to pay whose property is valued at \$7500?

2. If a tax of \$120 is assessed on a mill valued at \$24000, what is the valuation of a residence that is taxed \$17.75 at the same rate?

3. The per cent. of tax assessed for state purposes is $\frac{7}{8}\%$, for county $\frac{1}{2}\%$, and for city $1\frac{1}{8}\%$. What will be the amount of my tax, on property assessed at \$21500?

4. The tax assessed upon a town is \$20914.80; the town contains 2580 polls, taxed \$.62 $\frac{1}{2}$ each, and has a real estate valuation of \$4062000, and a valuation of personal property to the amount of \$227400. Find the rate of taxation, and C's tax, who pays for 4 polls, and whose property is assessed at \$15000.

REMARK.—In certain States, the common schools are supported by a tax or rate bill made out on the basis of the total attendance.

5. My son and daughter each attended school 214 days, and the expense, including teacher's wages and incidentals, was paid by a rate bill. How much must I pay, if the teacher's wages amounted to \$440, fuel and repairs \$101.50, and janitor's fees \$74.75, and the total number of day's attendance was 7460?

6. For the year 1888 the rates of taxation in the State of New York were as follows: Schools, 1.085 mills; general purposes, 1.475 mills; new capitol, .6 of a mill; other purposes, .34 of a mill. What was the total rate of taxation, and how much was raised by Livingston County, the valuation of which, as fixed by the State board of equalization, was \$25395180? How much did said county raise for school purposes?

7. The cost of maintaining the public schools of a city during the year 1888, was \$112000, and the taxable property of the city was \$44800000. How many mills on a dollar must be assessed for school purposes? If 10% of the tax assessed cannot be collected, how many mills on a dollar must then be assessed?

8. A tax of \$13943.20 is assessed upon a town containing 860 taxable polls; the real estate is valued at \$2708000, and the personal property at \$151600. If the polls be taxed \$1.25 each, what will be the rate of property taxation, and what will be the tax of Peter Parley, who pays for three polls, and has real and personal estate valued at \$23750?

9. In a school district, the valuation of the taxable property is \$752400, and it is proposed to repair the school house and ornament the grounds, at an expense of \$5000. If old material sells for \$673.70, what will be the rate per cent. of taxation, and what will be B's tax, whose property was valued at \$9400?

10. The assessed value of a town is, on real estate, \$1197500, and on personal property, \$432500. A poll tax of \$.50 per head is assessed on each of 1870 persons. The town votes to raise \$8000 for schools, \$1500 for highways, \$1500 for salaries, \$1000 for support of poor, and \$310 for contingent expenses. How much tax will a milling company have to pay, on a mill valued at \$46500, and stock at \$19750?

11. The total assessed value of a town, real and personal, is \$630000, and the town expenses are \$3913.95. How much tax must be collected to provide for town expenses and allow 3% for collecting? If the same town contains 310 polls, taxed \$1.50 each, what will be the rate of taxation, and how much will be the tax of a man who pays for two polls and owns property assessed at \$14500?

12. The assessed valuation of the real estate of a county is \$1910887, of the personal property, \$921073, and it has 4564 inhabitants subject to a poll tax. The year's expenses are: for schools, \$8400; interest, \$6850; highways, \$7560; salaries, \$5150; and contingent expenses, \$13675. If the poll tax was \$1.50, and the revenue from fairs and licenses \$6200, what tax must be levied on a dollar to meet expenses and provide a sinking fund of \$7000?

INSURANCE.

652. **Insurance** is indemnity secured against loss or damage. It is of two kinds: *Property Insurance* and *Personal Insurance*.

653. **Property Insurance** includes:

1. **Fire Insurance**, or indemnity for loss of or damage to property by fire.
2. **Marine Insurance**, or indemnity for loss of or damage to a ship or its cargo, by any specified casualty, at sea or on inland waters.
3. **Live Stock Insurance**, or indemnity for loss of or damage to horses, cattle, etc., from lightning or other casualty.

654. The **Insured Party** is usually the owner of the property insured, but may be any person having a financial *insurable* interest in the property.

655. The **Insuring Parties** are called *Insurers* or *Underwriters*, and are usually incorporated companies.

656. **Insurance Companies** are distinguished by the way in which they are organized; as *Stock Insurance Companies*, *Mutual Insurance Companies*.

657. A **Stock Insurance Company** is one whose capital has been contributed and is owned by stockholders, who share the profits and are liable for the losses.

658. A **Mutual Insurance Company** is one in which the profits and losses are shared by the insured parties.

REMARKS.—1. Some companies combine the features of both stock and mutual companies, and are called *Mixed Companies*.

2. In mixed companies, all profits above a limited dividend to the stockholders are divided among the policy-holders.

659. **Transit Insurance** refers to risks taken on goods being transported from place to place, either by rail or water or both.

660. The **Policy** is the contract between the insurance company and the person whose property is insured, and contains a description of the insured property, the amount of the insurance, and the conditions under which the risk is taken.

661. The **Premium** is the consideration in the contract, or the sum paid for insurance.

662. The **Term of Insurance** is the period of time for which the risk is taken, or the property insured.

REMARKS.—1. Premium rates are usually given as so much per \$100 of the sum insured, and depend upon the nature of the risk and the length of time for which the policy is issued; insurance is usually effected for a year or a term of years.

2. *Short Rates* are for terms less than one year.

3. It is usual to make an added charge for the policy.

4. Insurance is frequently effected upon plate glass, the acts of employees, etc.

663. An **Insurance Agent** is one who acts for an insurance company, in obtaining insurance, collecting premiums, adjusting losses, reinsuring, etc.

664. An **Insurance Broker** is a person who negotiates insurance for others, for which he receives a brokerage from the company taking the risk; he is considered, however, an agent of the insured, not of the company.

REMARK.—A *Floating Policy* is one which covers goods stored in different places, and generally such as are moved from place to place in process of manufacture.

665. Losses may be *total* or *partial*.

666. **Fire Insurance Losses** are usually adjusted by the insurance company paying the full amount of the loss, provided such loss does not exceed the sum insured; if the policy, however, contains the “average clause,” the payment made is such proportion of the loss as the amount of insurance bears to the total value of the property.

667. When a loss occurs to a vessel, the insurance company pays only such a proportion of the loss as the policy is of the entire value of the vessel.

668. It is an established rule in marine insurance, that insurers shall be allowed one-third for the superior value of the new material, as sails, masts, etc., used in repair of damage; that is, “one-third off new for old.”

REMARK.—Marine policies usually contain the “average clause.”

669. In case a policy is terminated at the request of the insured, he is charged the “short rate” premium; if, however, it be terminated at the option of the company, the lower long rate will be charged, and the company refund the premium for the unexpired time of the policy.

670. A **Valued or Closed Policy** is the ordinary form, and contains a fixed valuation of the thing insured.

671. An **Open Policy** is one upon which additional insurances may be entered at any time from port to port, at rates and under conditions agreed upon.

672. **Policies on Cargoes** are issued for a certain voyage, and on vessels for a voyage or for a specified time.

673. **Salvage** is an allowance made to those rendering voluntary aid in saving vessels or cargoes from marine casualties.

REMARKS.—1. Insurance companies usually reserve the privilege of rebuilding, replacing, or repairing damaged property.

2. Insurance policies ordinarily state that the loss, if becoming a charge upon the company, will be paid *30 days* or *60 days* after due notice and proof of loss. If not then paid, the amount of the claim becomes interest-bearing.

674. The computations in Property Insurance are performed the same as in Percentage, and the terms compare as follows:

The Amount Insured = the Base.

The Rate % of Premium = the Rate.

The Premium = the Percentage.

675. To find the Cost of Insurance.

EXAMPLE.—The mixed stock in a country store is insured for \$750. What is the cost of insurance for one year, at $1\frac{1}{2}\%$ premium, if \$1.25 is charged for the policy?

OPERATION.

\$750. = amount insured.
 $.015 = \% \text{ of premium.}$
 \$11.25 = premium.
 $1.25 = \text{cost of policy.}$
 \$12.50 = full cost of insurance.

EXPLANATION.—Since the amount insured is the base, and the per cent. of premium the rate, if the amount be multiplied by the rate, the product, \$11.25, will be the premium; adding \$1.25, the cost of the policy, gives the full cost, \$12.50.

Rule.—*Multiply the amount of insurance by the rate per cent. of premium, and add extra charges, if any.*

676. To find the Amount Insured, the Premium and Per Cent. of Premium being given.

EXAMPLE.—I paid \$141.50 to insure a stock of goods for three months. If the charge for the policy was \$1.50, and the rate of premium $\frac{7}{8}\%$, for what amount was the policy issued?

OPERATION.

\$141.50 = full cost.
 $1.50 = \text{cost of policy.}$
 $\$140 = \text{premium.}$
 $\frac{7}{8}\% = .00875 = \text{decimal rate.}$
 $\$140 \div .00875 = \$16000, \text{ face of policy.}$

EXPLANATION.—Since \$141.50 was the full cost or premium plus the charge of \$1.50 for the policy, the premium must have been \$140; and since the rate of premium was $\frac{7}{8}\%$ per cent., if \$140 is divided by $\frac{7}{8}\%$ per cent., the quotient, \$16000, will be the face of the policy.

Rule. *From the full cost of insurance, subtract the extra charges, if any; divide the remainder by the per cent. of premium, and the quotient will be the face of the policy.*

EXAMPLES FOR PRACTICE.

677. 1. How much insurance, at $1\frac{1}{4}\%$, can be procured for \$62.50?

2. A ranchman paid a premium of \$75.20 for insuring $\frac{3}{4}$ of his herd of cattle, at 60¢ per \$100. If the cattle were valued at \$40 per head, how many had he?

3. The loss on a property was \$6000, of which \$2000 was insured in the Home, \$3000 in the Phoenix, and \$2500 in the Hartford. How much did each company contribute?

4. If it cost \$663 to insure a certain block for \$44200, what will be the cost, at the same rate, to insure a block valued at \$105000, if \$1.50 extra be charged for the policy in the latter case?

5. How much will it cost to insure a factory for \$42000, at $\frac{3}{4}\%$, and its machinery for \$16500, at $1\frac{1}{4}\%$, charge for policy and survey being \$2.50?

6. A gentleman paid \$35.60 per annum for insuring his house, at 2% on two fifths of its value. What was the value of the house?

7. If a store and its contents are valued at \$27000, for how much must it be insured, at $1\frac{1}{2}\%$, to cover loss and premium in case of total destruction?

8. A cargo of teas, valued at \$33000, was insured for \$18000, in a policy containing an "average clause." In case of damage to the amount of \$21000, how much should the company pay?

9. The steamer Norseman, valued at \$90000, is insured for \$75000, at $2\frac{1}{2}\%$. What will be the actual loss to the insurance company, in case the steamer is damaged to the amount of \$20000?

10. A speculator bought 2000 barrels of flour, and had it insured for 80% of its cost, at $3\frac{1}{2}\%$, paying a premium of \$429. At what price must he sell the flour, to make a net profit of 10%?

11. I insured my grocery store, valued at \$13500, and its contents, valued at \$33000, and paid \$350 for premium and policy. If the policy cost \$1.25, what was the rate per cent. of premium?

12. A canal-boat load of 8400 bushels of wheat, worth 90¢ per bushel, is insured for three-fourths of its value, at $1\frac{3}{4}\%$ premium. In case of the total destruction of the wheat, how much will the owner lose?

13. A stock of goods, valued at \$30000, was insured for 18 months, at $1\frac{1}{2}\%$; at the end of 12 months the owner surrendered the policy. If the "short rate" for 6 months was 65¢ per \$100, what should be the return premium?

14. For how much must a house worth \$6000, and furniture worth \$2000, be insured, at $1\frac{1}{2}$ per cent., to cover the cost of the policy, which was \$2, the amount of premium paid, and $\frac{3}{4}$ of the value of the property?

15. A man owning $\frac{2}{3}$ of a ship, insured $\frac{2}{3}$ of his interest, at $1\frac{1}{2}\%$, and paid \$91.50 for premium and a policy charge of \$1.50. If the ship becomes damaged to the extent of \$12000, how much can be recovered on the policy?

16. A schooner is valued at \$10500, and has a cargo of 3500 barrels of apples, worth \$2.10 per barrel. What amount of insurance must be obtained, at $2\frac{1}{2}\%$, to provide, in case of loss, for the value of the property, the premium, and \$5 additional which the owner paid for survey and policy?

17. A block of stores and contents was insured for \$220000, and became damaged by fire and water to the amount of \$150000. Of the risk, \$40000 was taken by the Hartford Co., \$65000 by the Manhattan, \$35000 by the Aetna, and the remainder was divided equally between the Phoenix and the Provident. What was the net loss of each company, if the premium paid was $1\frac{1}{4}\%$?

18. The furniture in my house is estimated at one-half the value of the house. I got both insured for 5 years, at $2\frac{1}{2}\%$, and find that in case of total destruction the face of the policy will be full indemnity for both the property and premium. Find the value of the house, if the face of the policy and the premium amount to \$7687.50.

19. A factory worth \$45000 is insured, with its contents, for \$62500; \$30000 of the insurance is on the building, \$12500 on machinery worth \$20000, and \$20000 on stock worth \$35000. A fire occurs by which the building and the machinery are both damaged, each to the amount of \$15000, and the stock is entirely destroyed. How much is the claim against the company, if the risk is covered by an "ordinary" policy? How much if the policy contains the "average clause?"

20. The German Insurance Company insured the Field block for \$105000, at 60¢ per \$100; but thinking the risk too great, it re-insured \$40,000 in the Home, at $\frac{3}{4}\%$, and \$45000 more in the Mutual, at $\frac{3}{4}\%$. How much premium did each company receive? What was the gain or loss of the German? What per cent. of premium did it receive for the part of the risk not re-insured?

PERSONAL INSURANCE.

678. Personal Insurance is the insurance of persons. It includes:

1. **Life Insurance**, or indemnity for loss of life.
2. **Accident Insurance**, or indemnity for loss from disability occasioned by accident.
3. **Health Insurance**, or indemnity for loss occasioned by sickness.

679. Policies of Life Insurance are usually either *Life Policies* or *Endowment Policies*.

680. A Life Policy stipulates to pay to the beneficiaries named in it a fixed sum of money on the death of the insured.

681. An Endowment Policy guarantees the payment of a fixed sum of money at a specified time, or at death, if the death occurs before the specified time.

682. Life insurance companies are known as *Stock*, *Mutual*, *Mixed*, and *Co-Operative*.

683. Losses sustained by *Stock* and *Mixed* companies are paid either from "reserve funds" or by assessment on the stockholders; those sustained by *Mutual* and *Co-Operative* companies are paid by pro-rata or fixed contributions of the policy holders.

REMARKS.—1. The money may be made payable to any one named by the insured; if made payable to himself, at his death it becomes a part of his estate and is liable for his debts, if payable to another, that other cannot be deprived of the benefit of the insurance, either by the will of the person taking out the insurance, or by his creditors.

2. A person may insure his life in as many companies as he pleases, and to any amount.

3. Any one having an insurable interest in the life of another, may take out, hold, and be benefited by a policy of insurance upon the life of the other; or he may take out a policy in his own name, and then assign it to any creditor or to any one having an insurable interest.

4. The practical workings of life insurance are fully set forth in documents in general circulation, and all matters of premiums to be paid, cash value of policies surrendered, and manner of becoming insured, are determined from such documents, rendering it unnecessary to require the solution of problems under life insurance,

INTEREST.

684. **Interest** is a compensation paid for the use of money.
685. The **Principal** is the money for the use of which *interest* is paid.
686. The **Amount** is the sum of the *principal* and *interest*.
687. The **Time** is the period during which the principal bears interest.
688. **Interest** is reckoned at a certain per cent. of the principal. It is therefore a *Per Cent.* of which the Base is the Principal.
689. The **Rate of Interest** is the annual rate per cent.
690. **Interest** differs from the preceding applications of Percentage only by introducing *time* as an *element*, in connection with the *rate per cent.*
 The Principal = the Base.
 The Per Cent. per Annum = the Rate.
 The Interest = the Percentage.
 The Sum of the Principal and Interest = the Amount.
691. **Legal Interest** is interest according to the maximum rate fixed by law.
692. **Usury** is interest taken at a rate *higher* than the law allows.
693. **Simple Interest** is interest on the principal only, for the whole time of the loan or credit; and this is generally understood by the term *interest*.
694. **Annual, Semi-Annual, or other Periodic Interest**, is interest computed at a specified rate for a year, half-year or other designated period.
695. **Compound Interest** is interest computed on the amount at regular intervals.
- REMARKS.—1. The payment of *periodic* interest, if specified in a contract, may usually be enforced; and if not paid when due, becomes simple interest bearing, and is not usury.
 2. Neither the paying nor the receiving of compound interest is usury; but its payment cannot ordinarily be enforced, even though it is mentioned in the contract.
696. **Accrued Interest** is interest accumulated on account of any obligation, due or not due.

697. **Common Interest** is interest computed on a basis of 360 days for a year.

REMARKS.—1. This method is generally employed by business men, and in some states has received the sanction of law.

2. In reckoning interest by this method, it is customary to consider a year to be 12 months, and a month to be 30 days.

STATEMENT.—July 22, 1887, at the annual convention of the Business Educators' Association of America, then in session at Milwaukee, Wis., the following resolution was unanimously

adopted: *Resolved*, That, as business educators, we uniformly teach interest and discount on a 360-day basis, finding time by compound subtraction, and calling each month thirty days, except where the day of the minuend time be thirty-one, when it shall be so counted.

REMARK.—In computing interest for short periods of time, it is customary to take the exact number of days.

698. Exact Interest is interest computed for the exact time in days, and regarding the days as 365ths of a year. This method is used by the United States Government and by some merchants and bankers; but as it is inconvenient unless interest tables are used, it is not generally adopted.

REMARKS.—1. Exact interest, for any period of time expressed in days, may be obtained by subtracting $\frac{1}{365}$ part from the common interest for that period of time.

2. Common interest may be obtained from exact interest by adding thereto $\frac{1}{365}$ part of itself.

699. For convenience, the rate of interest should always be expressed decimally; the rules governing the multiplication and division of decimals may then be applied to any product or quotient arising from the use of the decimal rate.

REMARKS.—1. In many of the States a legal rate of interest is established, to save dispute and contention in cases of contracts in which no rate of interest is agreed upon by the parties; still the laws sanction an interest rate higher than the fixed legal rate, if such rate be agreed upon by the parties; in a few of the States, any rate, if agreed upon, is thus made legal.

2. When no particular rate of interest is named in a contract containing a general interest clause, as “with interest,” or “with use,” the legal rate of the place where the contract is made is understood.

2. Debts of all kinds bear interest *after* they become due, but not *before*, unless specified.

SIX PER CENT. METHOD.

700. The following method of computing interest is based upon time as usually reckoned; *i. e.*, 12 months of 30 days each, or 360 days for a year, and is called the *Six Per Cent. Method*. It is convenient for use in all cases where time is not given in days, as for years and months, or for years, months, and days, and where exact interest is not required. Should the rate be any other than six per cent., the change can be easily made. It is a common method of computing interest.

Six Per Cent. Method.

\$1.00 in 1 yr., at 6%, will produce \$.06 interest.

\$1.00 in $\frac{1}{2}$ yr., or 2 mo., at 6%, will produce \$.01 interest.

\$1.00 in 1 mo., or 30 da., at 6%, will produce \$.005 interest.

\$1.00 in 6 da., or $\frac{1}{5}$ mo., at 6%, will produce \$.001 interest.

\$1.00 in 1 da., at 6%, will produce \$.000 $\frac{1}{6}$ interest.

701. To find the Interest on Any Sum of Money, at Other Rates than 6 per cent.:

1. To find the interest at 7%. RULE.—To the interest at 6% add one-sixth of itself.

2. To find the interest at 7 $\frac{1}{2}$ %. RULE.—To the interest at 6% add one-fourth of itself.

3. To find the interest at 8%. *RULE.—To the interest at 6% add one-third of itself.*
4. To find the interest at 9%. *RULE.—To the interest at 6% add one-half of itself.*
5. To find the interest at 10%. *RULE.—Divide the interest at 6% by 6, and remove the decimal point one place to the right.*
6. To find the interest at 12%. *RULE.—Multiply the interest at 6% by 2.*
7. To find the interest at $5\frac{1}{2}\%$. *RULE.—From the interest at 6%, subtract one-twelfth of itself.*
8. To find the interest at 5%. *RULE.—From the interest at 6%, subtract one-sixth of itself.*
9. To find the interest at $4\frac{1}{2}\%$. *RULE.—From the interest at 6%, subtract one-fourth of itself.*
10. To find the interest at 4%. *RULE.—From the interest at 6%, subtract one-third of itself.*
11. To find the interest at 3%. *RULE.—Divide the interest at 6% by 2.*

702. To find the Interest, the Principal, Rate, and Time being given.

EXAMPLE.—What is the interest on \$550, at 6%, for 3 yr. 8 mo. 12 da.?

OPERATION.
 Int. on \$1 for 3 yr. = \$.18
 “ “ “ 8 mo. = .04
 “ “ “ 12 da. = .002
 Int. on \$1 for 3 yr.
 8 mo. 12 da. = \$.222
 $\$550 \times .222 = \$122.10.$

EXPLANATION.—Since the interest on \$1 for 1 year is \$.06, for 3 years it will be \$.18; since the interest on \$1 for 2 months is \$.01, for 8 months it will be \$.04; since the interest on \$1 for 6 da. is \$.001, for 12 days it will be \$.002; therefore the interest on \$1, at 6 per cent., for the full time, is \$.222; and the interest on \$550 will be 550 times the interest on \$1, or the product of the principal and the rate for the given time, which is \$122.10.

Rule.—*Multiply the principal by the decimal expressing the interest of one dollar for the full time.*

EXAMPLES FOR PRACTICE.

703. 1. Find the interest on \$900, for 4 yr. 1. mo. 6 da., at 7%. *258.20*

EXPLANATION.—Find the interest at 6%, and add to it one-sixth of itself.

2. What is the interest on \$400, for 1 yr. 7 mo. 2 da., at $7\frac{1}{2}\%$? *47.62*

EXPLANATION.—Find the interest at 6%, and add to it one-fourth of itself.

3. What is the interest on \$150, for 6 yr. 3 mo. 18 da., at 8%? *75.60*

EXPLANATION.—To the interest at 6% add one-third of itself.

4. Compute the interest on \$1200, for 3 yr. 4 mo. 15 da., at 9%. *364.50*

EXPLANATION.—To the interest at 6% add one-half of itself.

5. Find the interest, at 10%, on \$840, for 5 yr. 5 mo. 9 da. *457.10*

EXPLANATION.—Divide the interest at 6% by 6, to obtain the interest at 1%, and remove the decimal point 1 place to the right.

6. What is the interest, at 12%, on \$366, for 2 yr. 11 mo. 27 da.?

EXPLANATION.—Multiply the interest at 6% by 2%.

7. Find the interest on \$1800, for 6 yr. 9 mo. 25 da., at $5\frac{1}{2}\%$.

EXPLANATION.—From the interest at 6% subtract one-twelfth of itself.

8. Compute the interest, at 5%, on \$1000, for 11 yr. 4 mo. 24 da.

EXPLANATION.—From the interest at 6% subtract one-sixth of itself.

9. What is the interest, at $4\frac{1}{2}\%$, on \$1100, for 6 yr. 6 mo. 6 da.?

EXPLANATION.—From the interest at 6% subtract one-fourth of itself.

10. What is the interest, at 4%, on \$1350, for 9 yr. 8 mo. 12 da.?

EXPLANATION.—From the interest at 6% subtract one-third of itself.

11. Find the interest, on \$546, for 6 yr. 2 mo. 24 da., at 3%.

EXPLANATION.—Divide the interest at 6% by 2.

REMARKS.—1. Interest at any other rate, entire or fractional, can be found by a general application of the methods above explained.

2. When the mills of a result are 5 or more, add 1 cent; if less than 5, reject them.

12. ✓ Compute the interest on \$752.50, for 4 yr. 11 mo. 9 da., at 6%.

13. Compute the interest on \$3560, for 9 yr. 10 mo., at 8%.

14. Compute the interest on \$1540, for 9 mo. 20 da., at 6%.

15. Compute the interest on \$610.15, for 7 yr. 11 da., at 7%.

16. ✓ Compute the interest on \$1116, for 3 yr. 11 mo. 11 da., at 5%.

17. ✓ Compute the interest on \$17500, for 2 yr. 1 mo. 10 da., at $4\frac{1}{2}\%$.

18. Compute the interest on \$350.40, for 5 yr. 5 mo., at 7%.

19. Compute the interest on \$2400, for 7 yr. 1 mo. 19 da., at $10\frac{1}{2}\%$.

20. Find the interest on \$1450, from Aug. 12, 1882, to Nov. 10, 1890, at 6%.

21. What is the amount of \$610, at 8%, for 3 yr. 8 mo. 21 da.?

EXPLANATION.—The Principal plus the Interest equals the Amount.

22. Find the amount due after 1 yr. 10 mo. 20 da., on a 6% loan of \$1941.50.

23. On the 16th of September, 1884, I borrowed \$3500, at 8% interest. How much will settle the loan Jan. 1, 1890?

24. My note, given 2 yrs. 9 mo. 27 da. ago, for \$875.25, and bearing 4% interest, is due to-day. What is the amount due?

25. A manufacturer marks a carriage with two prices; the one for a credit of 6 months on sales, and the other for cash. If the cash price was \$750, and money was worth 8%, what should be the credit price?

26. Borrowed \$2750 July 16, 1887, at 5% interest, and on the same day loaned it at $7\frac{1}{2}\%$ interest. If full settlement is made Jan. 4, 1889, how much will be gained?

27. On goods bought for \$4500, on 6 months credit, I was offered 5% off for cash. If money was worth 6%, how much did I lose by accepting the credit?

28. A man sold his farm for \$16000; the terms were, \$4000 cash on delivery, \$5000 in 9 months, \$3000 in 1 year and six months, and the remainder in 2 years from date of purchase, with 6% interest on all deferred payments. What was the total amount paid?

29. May 16th I bought 300 barrels of flour, at \$7 per barrel; July 28th I sold 50 barrels, at \$8 per barrel; Oct. 30th, 100 barrels, at \$6.75 per barrel; and Feb. 13th following, the remainder, at \$7.80 per barrel. Allowing interest at 6%, what was my gain?

30. John Doe bought bills of dry goods as follows: May 3, \$250; July 1, \$1125; Sept. 14, \$450; Oct. 31, \$150; Dec. 1st, \$680; and on Dec. 21st, he paid in full, with 6% interest. What was the amount of his payment?

31. On March 25, I sold five bills of goods, for amounts as follows: \$1046.81, \$952.40, \$173.50, \$1250, and \$718.25; and on the first day of the following December I received payment in full, with interest at 6%. What was the amount received?

32. A firm bought goods on credit, and agreed to pay 7% interest on each purchase from its date; Oct. 6, 1887, goods were bought to the amount of \$268; Dec. 31, 1887, to the amount of \$765.80; Feb. 29, 1888, to the amount of \$600; Apr. 1, 1888, to the amount of \$325.25. If full settlement was made Aug. 25, 1888, how much cash was paid.

REMARK.—In the following examples, given for teacher's use in class drill, the interest on each separate principal should be computed to its nearest cent; the sum of the results so obtained will be the answer sought.

33. Find the amount of interest at 6%, by the *six per cent. method*,

On \$680, for 2 yr. 6 mo. 10 da.	On \$500, for 3 yr. 1 mo. 27 da.
On \$1895, for 1 yr 7 mo. 7 da.	On \$895, for 5 yr. 11 mo. 11 da.
On \$468, for 5 yr. 5 mo. 1 da.	On \$1650, for 1 yr. 10 mo. 23 da.
On \$1000, for 11 yr. 1 mo. 20 da.	On \$1463, for 9 yr. 1 mo. 9 da.
On \$645, for 4 yr. 4 mo. 5 da.	On \$365, for 4 yr. 1 mo. 25 da.

34. Find the amount of interest, by the *six per cent. method*,

On \$350, for 3 yr. 7 mo. 18 da., at 6%.	On \$538, for 6 yr. 6 mo. 6 da., at 9%.
On \$586.50, for 2 yr. 9 mo. 15 da., at 7%.	On \$1200, for 7 yr. 4 mo. 27 da., at 10%.
On \$1345, for 5 yr. 4 mo. 1 da., at 8%.	

35. Find the amount of interest, by the *six per cent. method*,

On \$550, for 4 yr. 6 mo. 21 da., at 6%.	On \$675, for 5 yr. 5 mo. 25 da., at 10%.
On \$2100, for 1 yr. 11 mo. 3 da., at 7%.	On \$1000, for 11 yr. 11 mo. 11 da., at 5%.
On \$750, for 8 yr. 8 mo. 8 da., at 8%.	On \$2500, for 1 yr. 1 mo. 1 da., at 4½%.
On \$1200, for 3 yr. 3 mo. 1 da., at 7½%.	On \$300, for 2 yr. 2 mo. 2 da., at 4%.
On \$1500, for 7 yr. 7 mo. 9 da., at 9%.	On \$990, for 4 yr. 4 mo. 6 da., at 3%.

36. Find the amount of interest, by the *six per cent. method*,

On \$250, for 3 yr. 4 mo. 29 da., at 8%.	On \$3546.81, for 5 yr. 0 mo. 5 da., at 3%.
On \$967.25, for 7 yr. 0 mo. 27 da., at 6%.	On \$1867, for 2 yr. 0 mo. 2 da., at 7½%.
On \$1305.09, for 1 yr. 11 mo. 7 da., at 7%.	On \$266.60, for 7 yr. 7 mo. 5 da., at 5%.
On \$1255.84, for 9 mo. 1 da., at 10%.	On \$1120.95, for 4 yr. 4 mo. 0 da., at 4%.
On \$316.75, for 5 yr. 11 mo. 0 da., at 4½%.	On \$1000, for 5 yr. 6 mo. 7 da., at 8%.
On \$2100.50, for 1 yr. 1 mo. 19 da., at 9%.	On \$1743, for 2 yr. 3 mo. 6 da., at 6%.

704. To find the Principal, the Interest, Rate, and Time being given.

EXAMPLE.—What principal, in 3 years and 2 months, at 6%, will gain \$47.50 interest?

OPERATION.

$\$.18 = \text{int. of } \$1, \text{ at } 6\%, \text{ for } 3 \text{ yr.}$

$.01 = \text{int. of } \$1, \text{ at } 6\%, \text{ for } 2 \text{ mo.}$

$\$.19 = \text{int. of } \$1, \text{ at } 6\%, \text{ for } 3 \text{ yr. } 2 \text{ mo.}$

$\$47.50 \text{ interest} \div .19 = \$250, \text{ principal.}$

is contained times in \$47.50, or \$250; therefore \$250 is the principal which will, in 3 yr. 2 mo., at 6%, gain \$47.50 interest.

EXPLANATION.—Since \$1 in 3 years, at 6 per cent., will gain \$.18 interest, and in 2 months \$.01 interest, it will in the given time gain \$.19 interest; and if \$1 will in the given time gain \$.19 interest, the principal that will in the given time gain \$47.50 interest must be as many times \$1 as \$.19

Rule.—*Divide the given interest by the interest of one dollar for the given time and rate.*

REMARK.—Whenever the divisor contains a fraction not reducible to a decimal, as in case of some fractional or odd ratio per cent., it is better that the fractional form be retained. Before division in such cases, multiply both divisor and dividend by the denominator of the fractional divisor; the relative value of the terms will not be changed, and greater exactness will be secured in the result.

EXAMPLES FOR PRACTICE.

705. 1. What principal, at 7%, will gain \$154 in 6 yr. 4 mo. 24 da.?

2. What sum of money, loaned at $4\frac{1}{2}\%$, for 7 yr. 11 mo. 15 da., will gain \$1468.21 interest? 4099.71

3. What sum of money, invested at $5\frac{1}{2}\%$, will in 7 yr. 1 mo. 1 da. produce \$131.50 interest? 337.41

4. A money lender received \$221.68 interest on a sum loaned at 8% July 17, 1885, and paid Oct. 11, 1888. What was the sum loaned? $258\frac{2}{3} \div 221.68 = 857.01$

5. A dealer who clears $12\frac{1}{2}\%$ annually on his investment, is forced by ill health to give up his business; he lends his money at 7%, by which his income is reduced \$1512.50. How much had he invested in his business?

6. How many dollars must I put at interest, at 9%, Jan. 27, 1889, so that on the 18th of Nov., 1895, \$506.27 interest will be due? $61275 \div \$826.23$

706. To find the Principal, the Amount, Rate, and Time being given.

EXAMPLE.—What principal, at 6%, will, in 4 yr. 6 mo. 15 da., amount to \$2372.25?

OPERATION.

$\$1.2725 = \text{amount of } \$1.00 \text{ for the time.}$

$\$2372.25 \div 1.2725 = \$1864.24. \text{ principal.}$

EXPLANATION.—Since a principal of \$1 will, in the given time, amount to \$1.2725, it will require a principal of as many times \$1 to amount to \$2372.25 as \$1.2725 is contained times in \$2372.25, or \$1864.24.

Rule.—*Divide the amount by the amount of 1 dollar for the given time and rate.*

EXAMPLES FOR PRACTICE.

707. 1. What sum, put at interest at 7% for 5 yr. 11 mo. 3 da., will amount to \$630.90?

2. A boy is now 15 years old. How much must be invested for him, at $7\frac{1}{2}\%$ simple interest, that he may have \$15000 when he becomes of age?

3. What sum, put at interest June 1, 1888, at 7%, will amount to \$687.50 July 1, 1890?

4. What sum of money, put at interest to-day at 5%, will amount to \$1031.25 in 7 mo. 15 da.?

5. What principal will amount to \$308.34 in 11 mo. 9 da., at 6%?

6. A man loaned a sum of money to a friend from June 13 to Dec. 1, at 7%, when he received \$763.28 in full payment. How much was loaned?

7. Owing a debt of \$2146.18, due in 1 yr. 7 mo. 18 da., I deposited in a bank, allowing me 6% interest, a sum sufficient to cancel my debt when due. Find the sum deposited.

708. To find the Rate Per Cent., the Principal, Interest, and Time being given.

EXAMPLE.—At what rate per cent. must \$750 be loaned, for 2 yr. 5 mo. 6 da., to gain \$164.25 interest?

OPERATION.

\$18.25 = int. of \$750 for the time at 1%.

\$164.25 ÷ \$18.25 = 9 or 9%.

EXPLANATION.—The principal will gain

\$18.25 interest in the given time at 1 per cent.; in order that it may in the given time gain \$164.25, the rate must be as many times 1 per cent. as \$18.25 is contained times in \$164.25, or 9 per cent.

Rule.—Divide the given interest by the interest on the given principal for the given time, at 1 per cent.

REMARK.—When the amount, interest, and time are given, to find the rate per cent., subtract the interest from the amount, thus finding the principal, then proceed as by the above rule.

EXAMPLES FOR PRACTICE.

709. 1. If I pay \$518.75 interest on \$1250, for 5 yr. 6 mo. 12 da., what is the rate per cent.?

2. At what rate would \$710, in 3 yr. 5 mo. 20 da., produce \$172.56 interest?

3. At what rate would \$4187.50 amount to \$4738.68, in 1 yr. 11 mo. 12 da.?

4. If \$1200 amounts to \$2135.80 in 12 yr. 11 mo. 29 da., what is the rate per cent.?

5. A lady deposited in a savings bank \$3750, on which she received \$93.75 interest semi-annually. What per cent. of interest did she receive on her money?

6. A debt of \$480, with interest from August 24, 1886, to Dec. 18, 1888, amounted to \$546.72. What was the rate per cent. of interest?

7. To satisfy a debt of \$1216.80, that had been on interest for 4 yr. 4 mo. 21 da., I gave my check for \$1751.18. What was the rate per cent. of interest?

710. To find the Time, the Principal, Interest, and Rate being given.

EXAMPLE.—In what time will \$540 gain \$74.52 interest, at 6%?

OPERATION.

\$32.40 = int. on \$540 for 1 yr., at 6%.

\$74.52 ÷ 32.40 = 2.3.

2.3 × 1 yr. = 2.3 years.

.3 yr. × 12 = 3.6 months.

.6 mo. × 30 = 18 days.

2 yr. 3 mo. 18 da.

EXPLANATION.—Since in 1 year \$540 will, at 6 per cent., gain \$32.40 interest, it will require as many years for it to gain \$74.52 interest as \$32.40 is contained times in \$74.52, or 2.3 years; find, by the rule for the reduction of a denominate decimal, that 2.3 years equals 2 yr. 3 mo. 18 da.

REMARK.—When by inspection it is apparent that the time is less than a year, divide the given interest by the interest on the principal for the highest apparent unit of time; the quotient will be in units of the order taken, which reduce as above.

Rule.—*Divide the given interest by the interest on the principal for 1 year, at the given rate per cent.*

REMARK.—When the amount, interest, and rate are given to find the time, subtract the interest from the amount, thus finding the principal, and proceed as above.

EXAMPLES FOR PRACTICE.

711. 1. How long will it take \$360 to gain \$53.64, at 6%.

2. How long should I keep \$466.25, at 8%, to have it amount to \$610.48?

3. A debt of \$1650 was paid, with $5\frac{1}{2}\%$ interest, on Aug. 30, 1888, by delivering a check for \$2316.85. At what date was the debt contracted?

4. How long must \$612 be on interest, at 7%, to amount to \$651.27?

5. On April 1, 1888, I loaned \$1120, at 5%, and when the money was due I received \$1202.60 in full payment. What was the date of the payment?

6. In what time will money, bearing 8% simple interest, double itself?

EXPLANATION.—In order to double itself, the interest accumulated must be equal to the principal, or be 100 per cent. of the principal. And since the principal increases 8 per cent. in one year, it will require as many years to increase 100 per cent., or to double itself, as 8 per cent. is contained times in 100 per cent., or $12\frac{1}{2}$, equal to 12 yr. 6 mo.

SHORT METHODS FOR FINDING INTEREST.**712. To find Interest for Days, at 6 per cent., 360 day basis, or Common Interest.**

EXPLANATION.—A principal of \$1 will, in 1 year, at 6 per cent., gain \$.06 interest. A principal of \$1 will, in $\frac{1}{4}$ year, or 2 months, or 60 days, at 6 per cent., gain .01 interest. Since \$.01 equals $\frac{1}{100}$ of the principal, the interest on any sum of money for 60 days, at 6 per cent., can be found by pointing off two integral places from the right; and since 6 is $\frac{1}{10}$ of 60, the interest for 6 days can be found by pointing off three places; and since ten times 60 is 600, the interest for 600 days is ten times that for 60 days, and may be found by pointing off 1 place; and since 6000 is ten times 600, the interest for 6000 days can be found by multiplying the interest for 600 days by 10, or in other words, the interest for 6000 days will equal the principal; the principal thus being shown to double itself in that time at 6 per cent. This may further be proved true from either of two illustrations:

1st. $6000 \text{ da.} \div 360 (12 \times 30) = 16\frac{2}{3}$, or 16 yr. + 8 mo.

2d. $100\% \div 6\% = 16\frac{2}{3}$, or 16 yr. + 8 mo.

Hence, assuming \$2136 as a principal, we form the following

Table.

\$2136 = principal.

\$2.136 = interest at 6% for 6 days.

\$21.36 = interest at 6% for 60 days.

\$213.6 = interest at 6% for 600 days.

\$2136. = interest at 6% for 6000 days.

REMARKS.—1. Observe, as above stated, that the interest for 6000 days equals the principal, or that any sum of money will, at common interest, double itself in 6000 days.

2. Since interest is ordinarily computed on the basis of 360 days, or 12 periods of 30 days each, as illustrated above, all results will be required on that basis, unless otherwise specified.

713.—1. To find the interest of any sum of money, at 6%, for 6 days.

RULE.—*Cut off three integral places from the right of the principal.*

2. To find the interest of any sum of money, at 6%, for 60 days. RULE.—*Cut off two integral places from the right of the principal.*

3. To find the interest of any sum of money, at 6%, for 600 days. RULE.—*Cut off one integral place from the right of the principal.*

4. To find the interest of any sum of money, at 6%, for 6000 days. RULE.—*Write the interest as being equal to the principal.*

REMARK.—Interest is a product of which the rate and time are factors. [Formula.—Interest = Principal \times Rate \times Time.] Since the rate, being a constant factor, may be ignored, it will be observed that it will make no difference if, for convenience, the term *principal* (in dollars), and that of *time* (in days), be interchanged. Illustration: The interest of 500 (dollars) for 93 (days), is the same as the interest of 93 (dollars) for 500 (days); and since 500 is $\frac{1}{12}$ of 6000, the interest required can be found by dividing 93 (dollars) by 12, which gives \$7.75. Again, the interest of 150 (dollars) for 88 (days) equals the interest of 88 (dollars) for 150 (days); and since 150 is $\frac{1}{4}$ of 600, the required interest is obtained by pointing off one place from the right of 88 (dollars), as, \$8.8, and dividing the result by 4, obtaining \$2.2, or \$2.20, as the interest.

714. To find Interest at Other Rates than 6 per cent., 360 Day Basis.

1. To find the interest on any sum of money for 12 days, at 6 per cent. RULE.—*Point off 3 places and multiply by 2.*

REMARKS.—1. For any number of days divisible by 6, proceed in like manner.

2. For other rates, add or subtract fractional parts of results, as in Art. 701.

3. For odd days, add fractional parts to the result.

2. To find the interest for 18 days, at 7%. RULE.—*Point off 3 places, multiply by 3, and to the result add one-sixth of itself.*

3. To find the interest for 24 days, at 5%. RULE.—*Point off 3 places, multiply by 4, and from the result subtract one-sixth of itself.*

4. To find the interest for 36 days, at $4\frac{1}{2}\%$. RULE.—*Point off 3 places, multiply by 6, and from the result subtract one-fourth of itself.*

5. To find the interest for 72 days, at 8%. RULE.—*Point off 3 places, multiply by 12, and to the result add one-third of itself.*

6. To find the interest for 51 days, at 6%. *RULE.—Point off 3 places, multiply by 8, and to the result add one-half of the first result.*

REMARK.—In a similar way all changes of time and rate may be considered.

7. To find the interest for 10 days, at 6%. *RULE.—Point off 2 places, and divide the result by 6.*

8. To find the interest for 20 days, at 7%. *RULE.—Point off 2 places, divide the result by 3, and to the quotient add one-sixth of itself.*

9. To find the interest for 30 days, at $7\frac{1}{2}\%$. *RULE.—Point off 2 places, divide the result by 2, and to the quotient add one-fourth of itself.*

10. To find the interest for 40 days, at 9%. *RULE.—Point off 2 places, subtract from the result one-third of itself, and to the remainder add one-half of itself.*

11. To find the interest for 45 days, at 8%. *RULE.—Point off 2 places, subtract from the result one-fourth of itself, and to the remainder add one-third of itself.*

12. To find the interest for 54 days, at 6%. *RULE.—Point off 2 places, and from the result subtract one-tenth of itself.*

13. To find the interest for 240 days, at 6%. *RULE.—Point off 2 places, and multiply by 4.*

REMARKS.—In a similar manner obtain interest for all terms of 60 days or parts thereof, and at any required rate.

14. To find the interest for 50 days, at 6%. *RULE.—Point off 1 place and divide by 12.*

15. To find the interest for 100 days, at 6%. *RULE.—Point off 1 place and divide by 6.*

16. To find the interest for 150 days, at 6%. *RULE.—Point off 1 place and divide by 4.*

REMARK.—Daily class drill for five or ten minutes, during the time given to the subject of interest and its varied applications, will impart to the class an astonishing degree of accuracy and rapidity in computing interest; and while odd rates are not in common use, valuable drill may be given by their occasional introduction, and the varied changes necessary to obtain interest for odd days will insure the very best results.

EXAMPLES FOR PRACTICE.

715. Find the interest on

1. \$1750, for 15 days, at 6%.

2. \$1125, for 24 days, at 7%. *5²⁵*

3. \$742.50, for 30 days, at 6%. *3²¹*

4. \$900, for 93 days, at $7\frac{1}{2}\%$. *17²⁴*

5. \$660, for 63 days, at 8%. *9⁰⁰*

6. \$136.42, for 33 days, at 9%. *—*

7. \$1000, for 21 days, at 10%. *8²³*

8. \$2000, for 12 days, at 5%.

9. \$351.23, for 40 days, at $4\frac{1}{2}\%$. *1¹⁵*

10. \$1368, for 50 days, at 3%. *5¹⁰*

11. \$93.40, for 150 days, at 6%. *2³⁰*

12. \$550, for 75 days, at 7%. *8⁰²*

13. \$842.50, for 45 days, at 6%. *4⁶³*

14. \$800, for 27 days, at 5%. *3⁰⁰*

15. \$1725, for 57 days, at 9%. *15⁴⁸*

16. \$125, for 55 days, at 6%. *18⁴⁵*

17. \$3741.85, for 6 days, at 7%. *14²⁷*

18. \$5178, for 9 days, at 9%. *13⁴¹*

19. \$732, for 11 days, at 6%. *10⁹⁶*

20. \$1174.51, for 42 days, at 8%.

- | | |
|--|--|
| 21. \$340, for 70 days, at 10%. <i>66</i> | 36. ✓ \$120, for 49 days, at 9%. <i>142</i> |
| 22. ✓ \$1478, for 80 days, at 6%. <i>192</i> | 37. \$60, for 50 days, at 5%. <i>40</i> |
| 23. ✓ \$2150, for 96 days, at 4½%. <i>2585</i> | 38. ✓ \$930, for 83 days, at 6%. <i>1263</i> |
| 24. ✓ \$1200, for 53 days, at 6%. <i>1665</i> | 39. ✓ \$750, for 84 days, at 6%. <i>1125</i> |
| 25. \$1500, for 87 days, at 7%. <i>2520</i> | 40. \$550, for 72 days, at 7%. <i>720</i> |
| 26. ✓ \$420, for 41 days, at 5%. <i>210</i> | 41. \$66.90, for 11 days, at 6%. <i>12</i> |
| 27. ✓ \$360, for 81 days, at 6%. <i>216</i> | 42. \$83.21, for 30 days, at 5%. <i>153</i> |
| 28. \$2347.50, for 18 days, at 7%. <i>272</i> | 43. \$110.25, for 60 days, at 7%. <i>120</i> |
| 29. \$1112.49, for 25 days, at 8%. <i>185</i> | 44. \$77.54, for 54 days, at 6%. <i>111</i> |
| 30. \$1300, for 13 days, at 6%. <i>78</i> | 45. \$300, for 66 days, at 10%. <i>300</i> |
| 31. \$17000, for 3 days, at 5½%. <i>779</i> | 46. \$800, for 93 days, at 8%. <i>1120</i> |
| 32. \$195.50, for 33 days, at 10%. <i>140</i> | 47. \$1110, for 63 days, at 6%. <i>1110</i> |
| 33. \$1050, for 43 days, at 7%. <i>877</i> | 48. \$684, for 50 days, at 6%. <i>1026</i> |
| 34. \$1560, for 44 days, at 7½%. <i>1430</i> | 49. \$1250, for 70 days, at 12%. <i>1500</i> |
| 35. \$180, for 47 days, at 6%. <i>141</i> | 50. \$351.89, for 9 days, at 6%. <i>153</i> |

REMARK.—In the five following examples, compute the interest on each separate principal to the nearest cent; then find the sum total of the interest thus obtained.

716. 1. Find the total amount of interest on

- | | |
|---------------------------------------|-------------------------------|
| \$550, for 18 days, at 6%. <i>165</i> | \$250, for 50 days, at 6%. |
| \$810, for 40 days, at 7%. | \$593.25, for 80 days, at 7%. |
| \$1000, for 41 days, at 7½%. | \$1966, for 75 days, at 5%. |
| \$342.50, for 42 days, at 5%. | \$450, for 83 days, at 8%. |
| \$1362.50, for 45 days, at 6%. | \$990, for 63 days, at 6%. |

2. Find the total amount of interest on

- | | |
|---------------------------------------|-----------------------------|
| \$720, for 9 days, at 10%. <i>180</i> | \$1124, for 15 days, at 3%. |
| \$7500, for 3 days, at 7%. <i>450</i> | \$550, for 45 days, at 7½%. |
| \$216, for 93 days, at 8%. <i>44</i> | \$160, for 27 days, at 6%. |
| \$504, for 54 days, at 6%. | \$240, for 31 days, at 8%. |
| \$600, for 4 days, at 4½%. | \$540, for 41 days, at 9%. |

3. Find the total amount of the interest on

- | | |
|--------------------------------|-----------------------------|
| \$1452, for 8 days, at 3%. | \$1400, for 26 days, at 6%. |
| \$1728, for 10 days, at 6%. | \$1700, for 29 days, at 8%. |
| \$2150.42, for 17 days, at 7%. | \$1900, for 37 days, at 7%. |
| \$519, for 24 days, at 8%. | \$2100, for 43 days, at 6%. |
| \$1600, for 23 days, at 7½%. | \$3100, for 53 days, at 3%. |

4. Find the total amount of interest on

- | | |
|--------------------------------|--------------------------------|
| \$695, for 79 days, at 3%. | \$99, for 59 days, at 5½%. |
| \$546, for 73 days, at 3½%. | \$780, for 101 days, at 6%. |
| \$1382.50, for 69 days, at 4%. | \$1350, for 150 days, at 6½%. |
| \$101.80, for 65 days, at 4½%. | \$775, for 180 days, at 7%. |
| \$500, for 61 days, at 5%. | \$938.20, for 10 days, at 10%. |

5. Find the total amount of interest on

\$285.56, for 11 days, at 11%.

\$372.40, for 21 days, at 7%.

\$519.31, for 27 days, at 7%.

\$3000, for 1 day, at 6%.

\$6000, for 5 days, at 5%.

\$10000, for 16 days, at 8%.

\$400, for 48 days, at 6%.

\$2400, for 54 days, at 5%.

\$730.30, for 33 days, at 9%.

\$100, for 45 days, at 6%.

717. To find Interest for Days at 6 per cent., 365 day basis, or Exact Interest.

REMARKS.—1. Aside from uses in government calculations, *exact* interest is rarely computed; and while it is enforceable, being strictly legal, the greater convenience of the 360 day rules so commend them to public favor as to lead to their common use.

2. On a basis of 12 periods of 30 days each, or 360 days for a year, the year's interest is taken for a period too short, since the year (exclusive of leap year) contains 365 days. The time is, therefore, 5 days or $\frac{5}{365}$, equal to $\frac{1}{73}$, too short, and the interest taken on that basis is proportionally too great; to correct this error and obtain the *exact* interest, subtract $\frac{1}{73}$ part from any interest obtained on a 360 day basis.

EXAMPLES FOR PRACTICE.

718. 1. Find the *exact* interest of \$630, for 50 days, at 6%.

2. Find the *exact* interest of \$954, for 63 days, at 7%.

3. Find the *exact* interest of \$800, for 33 days, at 5%.

4. Find the *exact* interest of \$137.50, for 93 days, at 8%.

5. Find the *exact* interest of \$210.54, for 100 days, at 9%.

6. Find the *exact* interest of \$681.80 for 90 days, at 10%.

7. Find the *exact* interest of \$500, for 48 days, at 6%.

8. Find the *exact* interest of \$1200, for 31 days, at 5%.

9. Find the *exact* interest of \$1500, for 55 days, at $7\frac{1}{2}\%$.

10. Find the *exact* interest of \$811.25, for 45 days, at $4\frac{1}{2}\%$.

REMARK.—In the three following examples, find the *exact* interest on each separate principal to the nearest cent, and then the total of the interest thus obtained.

ADDITIONAL EXAMPLES FOR PRACTICE.

719. 1. Find the total amount of *exact* interest on

\$510, for 63 days, at 7%.

\$615, for 93 days, at 6%.

\$450, for 78 days, at 5%.

\$120, for 96 days, at $7\frac{1}{2}\%$.

\$353, for 80 days, at $10\frac{1}{2}\%$.

\$1935.50, for 75 days, at 5%.

\$2136.88, for 70 days, at 4%.

\$1000, for 73 days, at 6%.

\$2000, for 146 days, at 9%.

\$1500, for 219 days, at $4\frac{1}{2}\%$.

2. Find the total amount of *exact* interest on

\$2150, for 65 days, at 3%.

\$1640, for 14 days, at 4%.

\$900, for 17 days, at $4\frac{1}{2}\%$.

\$182.79, for 24 days, at 5%.

\$605.51, for 33 days, at 6%.

\$890.90, for 45 days, at $7\frac{1}{2}\%$.

\$1100, for 46 days, at 8%.

\$2500, for 54 days, at 10%.

\$720, for 66 days, at 9%.

\$365, for 51 days, at 6%.

3. Find the total amount of *exact* interest on

\$96.60, for 20 days, at 7%.	}	\$615.62, for 93 days, at 6%.
\$138.24, for 15 days, at 6%.		\$730, for 57 days, at 5%.
\$1793.80, for 35 days, at 8%.		\$891.11, for 63 days, at 6%.
\$2000, for 7 days, at 7%.		\$200, for 19 days, at $6\frac{1}{2}\%$.
\$1000, for 1 day, at $4\frac{1}{2}\%$		\$525, for 25 days, at 10%.

PERIODIC INTEREST.

720. Annual Interest is *simple interest* on the principal for each *year* period, and on each year's interest remaining unpaid.

721. Semi-Annual Interest is *simple interest* on the principal for each *half-year* period, and on each period's interest remaining unpaid.

722. Quarterly Interest is *simple interest* on the principal for each *quarter-year* period, and on each period's interest remaining unpaid.

723. In some States annual and other periodic interest is sanctioned by law; but in many States it cannot be legally enforced.

724. When the interest payments are not made when due, *periodic* interest becomes greater than *simple* interest, because of the interest on the unpaid sums.

725. Periodic interest is sometimes secured by a note or series of notes; in such cases the principal only is secured by one of the series (if not by mortgage or otherwise), while each of the other notes is drawn for one interest payment, and matures on the date at which such payment is due. By such arrangement, periodic interest can be enforced in States where it would otherwise be regarded as illegal.

726. In States where periodic interest is legal, the contract should contain the words, "with annual interest," or "with interest payable annually," or "with semi-annual interest," etc.

727. As simple interest cannot be collected until the principal is due, simple and periodic interest are the same up to the end of the first interest period.

REMARK.—When the interest is not paid at the end of the periods, as agreed, much time will be saved in obtaining the amount due, by finding the interest on one over-due payment for the aggregate of the time for which they were all over-due; to this interest add the amount of the principal, at simple interest.

728. To find Periodic Interest, the Principal, Rate, and Time, being given.

EXAMPLE.—What is the interest on \$2500, from July 1, 1885, to Sept. 16, 1888, at 6% interest, due annually, and no payments made until final settlement?

OPERATION.

1888—9—16

1885—7—1

3—2—15 = time.

 $\$2500 \times .06 = \$150 = 1 \text{ yr. int.}$

Remaining unpaid for periods of	{	2 yr. 2 mo. 15 da.	
		1 yr. 2 mo. 15 da.	
		2 mo. 15 da.	

Interest of \$150. for 3 yr. 7 mo. 15 da. = \$ 32.63

3 yr. simple interest on principal = 450.00

2 mo. 15 da. interest on principal = 31.25

Total interest due = \$513.88

Adding to this the interest of the principal for the full time, \$481.25, gives \$513.88, the amount of interest due.

Rule.—*To the simple interest on the principal for the full time, add the interest on one period's interest for the aggregate of time for which the payments of interest were deferred.*

EXAMPLES FOR PRACTICE.

729. 1. What is the annual interest of \$1260, payments due semi-annually from May 21, 1884, to Nov. 9, 1888, at 7%, no interest having been paid?

2. What is the annual interest of \$3416.50, payments due quarterly from Jan. 15, 1882, to Sept. 6, 1889, at 5%, no interest having been paid?

3. Find the amount of interest due at the end of 4 yr. 9 mo. on a note for \$1155, at 6%, interest payable annually, but remaining unpaid.

4. On a note of \$1750, dated Aug. 1, 1882, given with interest payable annually at 10%, the first three payments were made when due. How much remained unpaid, debt and interest, Jan. 1, 1889?

5. Find the amount due Oct. 11, 1891, on a debt of \$11000 under date of July 5, 1888, bearing $4\frac{1}{2}\%$ interest, payable quarterly, notes for the quarterly interest having been given and nothing paid until final settlement.

COMPOUND INTEREST.

730. Compound Interest is the interest on the principal and on the unpaid interest after it becomes due.

731. The Simple Interest may be added to the principal annually, semi-annually, quarterly, or for other agreed periods; when done, interest is said to be compounded *annually, quarterly, etc.*, as the case may be.

732. General Rule.—*Find the amount of the principal and interest for the first period, and make that the principal for the second period, and so proceed to the time of settlement.*

EXPLANATION.—From July 1, 1885, to Sept. 16, 1888, is 3 yr. 2 mo. 15 da. And since the first year's interest, which is \$150, was not paid until 2 yr. 2 mo. 15 da. after it was due, the second year's interest, \$150, was not paid until 1 yr. 2 mo. 15 da. after it was due, and the third year's interest, \$150, was not paid until 2 mo. 15 da. after it was due, the aggregate of the time for which interest should be computed on one year's interest, \$150, is 3 yr. 7 mo. 15 da., and its interest for that time is \$32.63.

REMARKS.—1. If the time contains fractional parts of a period, as months and days, find the amount due for the full periods, and to this add its interest for the months and days.

2. Compound interest is not recoverable by law, but a creditor may receive it if tendered, without incurring the penalty of usury; a new obligation may be taken at the maturity of a compound interest claim, for the amount so shown to be due, and such new obligation will be valid and binding.

733. To find the Compound Interest, when the Principal, Rate, and Time of Computing it are given.

EXAMPLE.—Find the interest of \$750, for 3 yr. 8 mo. 15 da., at 6%, if interest be compounded annually.

OPERATION.

\$45 = int. for 1st yr.
 \$795 = amt. at end of 1st yr.
 \$47.70 = int. for 2d yr.
 \$842.70 = amt. at end of 2d yr.
 \$50.56 = int. for 3d yr.
 \$893.26 = amt. at end of 3d yr.
 \$37.96 = int. for 8 mo. 15 da.
 \$931.22 = amt. for full time.
 \$931.22 — \$750 = \$181.22, comp. int. full time.

EXPLANATION.—Since the interest is to be compounded annually, the amount due at the end of the first year, which is \$795, will be the basis of the interest for the second year; and the amount due at the end of the second year, \$842.70, will be the basis of the interest for the third year; the amount due at the end of the third year, \$893.26, will be the basis of the interest for the remaining 8 mo. 15 da. of the time; and since the compound amount thus found, \$931.22,

is made up of the compound interest and the principal, if from this amount the principal be subtracted, the remainder, \$181.22, will be the compound interest.

Rule.—I. Find the amount of the principal for the first interest period; take this result as a principal for the next period, and so on through the whole time.

II. Subtract the principal from the last amount, and the remainder will be the compound interest.

REMARK.—For half or quarter years, take one-half or one-quarter the rate per cent. for one year.

EXAMPLES FOR PRACTICE.

734. 1. What is the compound interest on \$1200, for 4 years, at 7%, if the interest is compounded annually?

2. What is the compound interest on \$600, for 3 years, at 5%, if the interest is compounded quarterly?

3. What is the compound interest on \$1640, for 2 yr. 6 mo., at 10%, if the interest is compounded quarterly?

4. Find the compound interest on \$1000, for 4 yr. 5 mo. 12 da., at 8%, if the interest is compounded semi-annually? $417\frac{3}{4}$

5. What will be the amount due Feb. 11, 1892, on a debt of \$900, bearing 8% interest, compounded quarterly, if the debt bears interest from July 1, 1888? \$1198.02

6. Oct. 1, 1888, I paid in full a note for \$1350, dated March 15, 1883, and bearing 10% interest. If the interest was compounded semi-annually, what was the amount due at settlement? 3 =

735. The labor of computing compound interest may be greatly shortened by the use of the following

Compound Interest Table,

Showing the amount of \$1 at compound interest at various rates per cent. for any number of years, from 1 year to 50 years, inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 8178
8	1.0828 567	1.1264 926	1.1716 5933	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4237 6089	1.5110 6866	1.6010 3222
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7322	1.5125 8972	1.6186 9452	1.7316 7645
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879
23	1.2571 630	1.4083 772	1.5768 9923	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 5936
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345
39	1.4741 235	1.7872 103	2.1647 4477	2.6195 7443	3.1670 2698	3.8253 7171	4.6163 6599
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9393	4.0978 3381	4.9930 6145
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9537
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5036	3.8950 4372	4.8669 4110	6.0748 2271
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5183	5.2135 8898	6.5705 2824
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7630	4.2562 1944	5.3960 6459	6.8333 4937
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335

Compound Interest Table.

Showing the amount of \$1 at compound interest, at various rates per cent., for any number of years, from 1 year to 50 years, inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1411 6612	1.1576 250	1.1910 160	1.2350 430	1.2597 120	1.2950 290	1.3310 000
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5668 743	1.6771 001	1.7715 610
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390
20	2.4117 1402	2.6532 977	3.2071 855	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6536 004	8.1402 749
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	2.8760 1383	3.2250 990	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434
39	5.5658 9903	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992
43	6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3372 861	72.8904 837
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.8908 529

REMARKS.—1. To find the amount of any given principal, for any required number of years multiply the given principal by the amount of \$1 at the given rate, as shown by the table.

2. For periods beyond the scope of the table: multiply together the amounts shown for periods the sum of which will equal the time required. For example, to find the compound amount of \$1 for 100 years: multiply the amount for 50 years by itself; to find the compound amount for 75 years: multiply the amount for 50 by that for 25; of 30 by 45; of 37 by 38; of 40 by 35, etc.

3. If interest is to be compounded semi-annually, take one-half the rate for twice the time.

4. If interest is to be compounded quarterly, take one-fourth the rate for four times the time.

5. If interest is to be compounded bi-monthly, take one-sixth the rate for six times the time.

6. To find the compound amount of principals of \$100, or less: multiply the principal by the amount as shown in the table, using only 3 of its decimal places. For principals of \$1000, or less, use only 4 of the decimal places, and so on.

736. To find the Principal or Present Worth of an Amount at Compound Interest.

EXAMPLE.—What principal will, in 3 yr., at 6%, amount to \$5955.08, if interest is compounded annually?

OPERATION.

\$5955.08 = total amt.

\$1.191016 = amt. of \$1 for the rate and time.

\$5955.08 ÷ 1.191016 = \$5000, principal.

EXPLANATION.—From the table find the amount of \$1 for 3 years, at 6 per cent. interest, compounded annually, to be \$1.191016; to find the principal that will, at the given rate and time, amount to \$5955.08, divide \$5955.08 by 1.191016.

Rule.—*Divide the compound amount given by the compound amount of one dollar for the time and rate given.*

EXAMPLES FOR PRACTICE.

737. 1. What principal will, in 8 years, at 5%, amount to \$4107.26, if interest is compounded semi-annually?

2. Find that principal which will, in 5 years, at 8% interest, compounded quarterly, amount to \$1516.11.

3. At what rate of interest, compounded annually, must \$1750 be loaned, that in 7 years it may amount to \$2381.51?

REMARK.—Divide the amount by the principal, carrying the quotient to *six* decimal places; refer to the 7 years line, or column, for an amount corresponding to the quotient. The rate column in which it is found will indicate the rate per cent. required.

4. At what rate of interest, compounded annually, must \$2500 be loaned for 12 years, that it may accumulate \$3795.43 interest?

MISCELLANEOUS EXAMPLES FOR PRACTICE.

738. 1. What sum will, on Sept. 5, 1889, discharge a debt of \$550, dated Mar. 19, 1884, bearing 9% interest, if interest is compounded semi-annually, and no payments are made until final settlement?

2. If nothing was paid until final settlement, what amount would pay a debt of \$1450, made July 15, 1885, and paid Dec. 3, 1888, if interest is allowed at the rate of 6%, compounded quarterly?

3. What amount will be due Apr. 15, 1891, on a debt of \$1100, created May 1, 1887, if the interest thereon is at the rate of 10%, compounded semi-annually. 4-

4. Sept. 19, 1886, I borrowed \$5000, at 6%, and agreed that until settlement was made I would permit the compounding of the interest every two months. Under such conditions, what amount will be due Oct. 25, 1890?

5. June 29, 1884, I borrowed some money at 8%, interest to be compounded quarterly; January 5, 1890, I paid \$1361.82 in full settlement. What was the sum borrowed.

REVIEW EXAMPLES IN INTEREST.

739. 1. Smith loaned \$2400, at 6% simple interest, until it amounted to \$3000. For what time was the loan made?

2. At what rate per cent. per annum must \$1080 be loaned for 7 yr. 3 mo. 27 da., that it may amount to \$1611.35? 6.716

3. A man invested \$16000 in business, and at the end of 3 yr. 3 mo., withdrew \$22880, which sum included investment and gains. What yearly per cent. of interest did his investment pay?

4. Find the interest of that sum for 11 yr. 8 da., at $10\frac{1}{2}\%$, which will, at the given rate and time, amount to \$1715.08.

5. Sold an invoice of crockery on 2 mo. credit; the bill was paid 3 mo. 18 da. after the date of purchase, with interest, at 8%, by a check for \$1963.45. How much was the interest?

6. A debt of \$7150, dated Mar. 27, 1885, and bearing 6% interest, payable quarterly, was paid in full July 5, 1888. If no previous payments had been made, how much was due at final settlement?

7. A man having \$21000, invested it in real-estate, from which he received a semi-annual income of \$787.50. He sold this property at cost and invested the proceeds in a business which yielded him \$472.50 quarterly. How much greater rate per cent. per annum did he received from the second investment than from the first.

8. In order to engage in business, I borrowed \$3750 at 6%, and kept it until it amounted to \$4571.25. How long did I keep the money?

9. A bond and mortgage, bearing 8% interest, and dated May 1, 1880, was settled in full Nov. 16, 1888, by the payment of \$17685. For what face amount was the bond and mortgage given?

10. In what time will the interest at 8% be three-fifths of the principal.

11. What sum will be due Jan. 18, 1892, on a debt of \$5100, dated Mar. 17, 1885, bearing interest at 7% per annum, payable semi-annually, if the first five payments were made when due and no subsequent payments were made?

12. A building which cost \$10500, rents for \$87.50 per month. What annual rate of interest on his investment does the owner receive, if he pays yearly taxes amounting to \$102.50; insurance, \$21.25; repairs, \$136.80; and janitor's services, \$56.95? 6.42

13. A merchant sold a stock of glassware on one month's credit; the bill was not paid until 3 mo. 21 da. after it became due, at which time the seller received a draft for \$4716.21 for the bill and interest thereon at the rate of 5%. Find the selling price of the goods. 4644

14. Oct. 12, 1888, I purchased 2700 bushels of wheat, at \$1.05 per bushel, and afterwards sold it at a profit of 6%. On what date was the wheat sold, if my gain was equivalent to 10% interest on my investment?

15. A speculator borrowed \$6250, at $7\frac{1}{2}\%$ interest, and with the money bought a note, the face of which was \$7500, maturing in nine months without interest, but which was not paid until two years from the date of its purchase. If the note drew 6% interest after maturity, did its purchaser gain or lose, and how much?

16. I am offered a house that rents for \$27 per month, at such a price that, after paying \$67.20 taxes, and other yearly expenses amounting to \$24.85, my net income will be $8\frac{1}{2}\%$ on my investment. What is the price asked for the house?

17. Having three girls, Grace, Mabel, and Flora, aged respectively 15, 13, and 2 years, I wish to invest such a sum for each that she may have \$10000 on becoming of age. How much cash will be required to secure a $4\frac{1}{2}\%$ compound interest investment?

18. I loaned a friend a sum of money for 9 months, at 6% per annum, and when the loan was due he paid \$851.50 in cash, which was 75% of the amount due me; the remainder was paid 6 mo. 15 da. later, with interest at the rate of 10%. Find the amount paid at final settlement.

19. Deland owns a summer resort valued at \$45000, for which he receives, for a season of 5 months, \$2400 rent per month. The year's expenses for taxes, repairs, and insurances, average \$6375. If he sells this property and invests the proceeds in a manufacturing business paying quarterly \$1937.50, how much will his rate of interest be increased by the change?

20. Having purchased 1150 barrels of pork, at \$16 per barrel, on 4 months' credit, the dealer, 27 days later, sold it at \$17.50 per barrel, receiving therefor a 6 months' note without interest. When the purchase money became due, he discounted the note on a basis of 7%, and paid his debt. How much was gained?

21. The day Ralph was 6 years old his father deposited for him in a savings bank such a sum of money that, at 4% interest, compounded quarterly, there will be \$7500 to his credit on the day he attains his majority. What sum was deposited?

22. December 11, 1887, a lumber dealer borrowed money and bought shingles at \$4.50 per M.; Sept. 17, 1888, he sold the shingles and paid his debt, and 8% interest, amounting to \$3462.60. How many thousand shingles did he buy?

23. A jobber bought 6000 yd. of Axminster carpet, at \$2.80 per yard, payable in 6 months, and immediately sold it at \$3.15 per yard, giving a credit of 2 months; at the expiration of the 2 months he anticipated the payment of his own debt, getting a discount off of 10% per annum. How much did he gain by the transaction?

24. At the age of 25 a lady invested \$3000, at 7% per annum. What will be her age when the investment, with its interest compounded semi-annually, amounts to \$16754.78?

25. Herbert is 10 and Theodore 7 years old. If 7% compound interest investments can be secured by their father, for what amounts must such investments be made in order that at the age of 21 the boys may each have \$12500?

26. If money be worth 7%, compounded annually, which would be better, and how much, for a capitalist to loan \$25000 for 11 years and 6 months, than to invest it in land that, at the end of the time named, will sell for \$55000 above all expenses for taxes?

27. I loaned a bridge builder \$17500 for 7 years, at 10% per annum, interest payable quarterly, and took a bond and mortgage to secure the debt and its interest. Nothing having been paid until the end of the 7 years, how much was required in full settlement?

28. On the 20th of March, 1888, I borrowed \$13500, at 5% interest; on April 5 I loaned \$5000 of the money until Dec. 20, 1888, at 8%; April 15, I purchased with the remainder a claim for \$10000, due Aug. 1, but which, not being paid at maturity, was extended until the \$5000 became due, at the rate of 6%. How much did I gain, both claims having been paid on the day the loan of \$5000 became due?

29. Having bought a mill for \$12000, I paid cash \$4000 on delivery, and gave a bond and mortgage for 8 years without interest to secure the balance; to secure the interest, which was to be paid semi-annually, at the rate of 7% per annum, I gave sixteen non-interest bearing notes, without grace, for \$280 each, one maturing at the end of each 6 months for the 8 years. If the four of the notes first maturing were paid when due, and no other payment was made until the mortgage became due, how much was required for full settlement?

30. Charles will be 11 years old Dec. 15, 1888, John will be 8 years old July 28, and Walter was 5 years of age April 30. If, on July 1, a 6% compound interest investment be made for each, so that at the age of 21 he may have \$10000, what amount of cash will be required, the interest being compounded quarterly?

TRUE DISCOUNT.

740. Discount is an abatement or allowance made from the amount of a debt, a note, or other obligation, or a deduction from the price of goods for payment before it is due.

741. The Present Worth of a debt payable at a future time without interest, is its value *now*; hence, is such a sum as, being put at simple interest at the legal rate, will amount to the given debt when it becomes due.

742. True Discount is the difference between the *face* of a debt due at a future time and its *present worth*.

REMARKS.—1. To find *present worth*, apply the principles given in **INTEREST**. The debt corresponding to the amount; the rate per cent. agreed upon to the rate; the time intervening before the maturity of the debt, to the time; and the present worth, which is the unknown term, is the principal.

2. When payments are to be made at different times, without interest, find the present worth of each payment separately, and take their sum.

3. With debts bearing interest, and discounted at the same or at a different rate of interest, the face of the debt plus its interest as due at maturity becomes the base.

743. To find the Present Worth of a Debt.

EXAMPLE.—Find the present worth and true discount of a claim for \$871.68, due 2 yr. 3 mo. hence, if money is worth 6% per annum.

OPERATION.

.135 = int. on \$1 for 2 yr. 3 mo. at 6%.
 \$1.135 = amount “ “ “ “
 $\$871.68 \div 1.135 = \768 , present worth.
 $\$871.68 - \$768 = \$103.68$, true discount.

EXPLANATION.—The amount of the debt at the end of 2 yr. 3 mo. is \$871.68; and since \$1 would in that time, at 6 per cent., amount to \$1.135, the present worth must be as many times \$1 as \$1.135 is contained times in \$871.68, or \$768. If the face of the debt is \$871.68, and its present worth is only \$768, the true discount will be \$871.68 minus \$768, or \$103.68.

Rule.—Divide the amount of the debt, at its maturity, by one dollar plus its interest for the given time and rate, and the quotient will be the present worth; subtract the present worth from the amount, and the remainder will be the true discount.

EXAMPLES FOR PRACTICE.

744. 1. What is the present worth of \$661.50, payable in 3 yr. 9 mo., discounting at 6%?

2. Find the present worth and true discount of a debt of \$138.50, due in 5 yr. 6 mo. 18 da., if money is worth 7% per annum.

3. Find the present worth of a debt of \$1750, \$1000 of which is due in 9 mo. and the remainder in 15 mo., money being worth 6% per annum.

4. Which is greater, and how much, the interest, or the true discount on \$516, due in 1 yr. 8 mo., if money is worth 10% per annum?
5. Which is better, and how much, to buy flour at \$6.75 per barrel on 6 months time, or to pay \$6 cash, money being worth 6%?
6. When money is worth 5% per annum, which is preferable, to sell a house for \$20,000 cash, or \$21,000 due in one year?
7. A farmer offered to sell a pair of horses for \$420 cash, or for \$475 due in 15 months without interest. If money is worth 8% per annum, how much would the buyer gain or lose by accepting the latter offer?
8. If money is worth 6%, what cash offer will be equivalent to an offer of \$1546 for a bill of goods on 90 days credit?
9. An agent paid \$840 cash for a traction engine, and after holding it in stock for one year, sold it for \$933.80, on eight months' credit. If money is worth 6%, what was his actual gain?
10. A stock of moquette carpeting, bought at \$1.95 per yard, on 8 months' credit, was sold on the date of purchase for \$1.80 per yard, cash. If money was worth 6% per annum, what per cent. of gain or loss did the seller realize?
11. Marian is now fifteen months old. How much money must be invested for her, at 6% simple interest, that she may have \$15000 of principal and interest when she celebrates her eighteenth birthday?
12. A thresher is offered a new machine for \$480 cash, \$500 on 2 months credit, or \$525 on 1 year's credit. Which offer is the most advantageous for him, and how much better is it than the next best, with money worth $7\frac{1}{2}\%$?
13. After carrying a stock of silk for 4 months, I sold it at an advance of 30% on first cost, extending to the purchaser a credit of one year without interest. If money is worth 5% per annum, what was my per cent. of profit or loss?
14. Having bought a house for \$5048 cash, I at once sold it for \$7000, to be paid in 18 months without interest. If money is worth 8% per annum, did I gain or lose, and how much?
15. Goods to the amount of \$510 were sold on 6 months' credit. If the selling price was \$30 less than the goods cost, and money is worth 6% per annum, how much was the loss and the per cent. of loss?
16. How much must be discounted for the present payment of a debt of \$8741.50, \$2000 of which is on credit for 5 months; \$3000 for 8 months, and the remainder for 15 months, money being worth 10% per annum?
17. What amount of goods, bought on 6 months time or 5% off for cash, must be purchased, in order that they may be sold for \$4180, and net the purchaser 10% profit, he paying cash and getting the agreed discount off?
18. A dealer bought grain to the amount of \$2700, on 4 months' credit, and immediately sold it at an advance of 10%. If from the proceeds of the sale he paid the present worth of his debt at a rate of discount of 8% per annum, how much did he gain?
19. A merchant bought a bill of goods for \$2150, on 6 months' credit, and the seller offered to discount the bill 5% for cash. If money is worth $7\frac{1}{2}\%$ per annum, how much would the merchant gain by accepting the seller's offer.

20. The asking price of a hardware stock is \$5460, on which a trade discount of 25%, 15%, and 10% is offered, and a credit of 90 days on the selling price. If money is worth $5\frac{1}{2}\%$, what should be discounted for the payment of the bill ten days after its purchase?

21. A merchant sold a bill of goods for \$1800, payable without interest in three equal payments, in 3 months, 6 months, and 9 months respectively. If money is worth 5% per annum, how much cash would be required for full settlement on the date of purchase?

22. A stationer bought a stock worth \$768, at a discount of 25% on the amount of his bill, and 4% on the remainder for cash payment. He at once sold the stock on 4 months' time, at 10% in advance of the price at which it was billed to him. How much will the stationer gain if his purchaser discount his bill on the date of purchase by true present worth, at the rate of 7% per annum?

23. I sold my farm for \$10,000, the terms being one-fifth cash, and the remainder in four equal semi-annual payments, with simple interest at 5% on each from date; three months later the purchaser settled in full by paying with cash the present worth of the deferred payments, on a basis of 10% per annum for the use of the money. How much cash did I receive in all?

24. What amount of goods, bought on 4 months' time, 10% off if paid in 1 month, 5% off if paid in 2 months, must be purchased, in order that they may be sold for \$11480, and $\frac{1}{2}$ the stock net a profit of 15% and the remainder a profit of 20% to the purchaser, if he cashes his purchase within 1 month and gets the agreed discount off?

BANK DISCOUNT.

745. A **Bank** is a corporation chartered by law for the receiving and loaning of money, for facilitating its transmission from one place to another by means of checks, drafts, or bills of exchange, and, in case of banks of issue, for furnishing a paper circulation.

REMARK.—Some banks perform only a part of the functions above mentioned.

746. **Negotiable Paper** commonly includes all orders and promises for the payment of money, the property interest in which may be negotiated or transferred by indorsement and delivery, or by either of those acts.

747. **Bank Discount** is a deduction from the sum due upon a negotiable paper at its maturity, for the cashing or buying of such paper before it becomes due.

748. The **Proceeds of a Note** or other negotiable paper is the part paid to the one discounting it, and is equal to the face of the note, less the discount.

REMARK.—In *true* discount, the *present* worth is taken as the principal; in *bank* discount the *future* worth is taken as the principal.

749. The **Face of a Note** is the sum for which it is given.

750. The **Discount** may be a fixed sum, but is usually the interest at the legal rate, and taken in advance.

751. The **Time** in bank discount is always the number of days from the date of discounting to the date of maturity.

752. The **Term of Discount** is the time the note has to run after being discounted.

REMARK.—Bank discount is usually reckoned on a basis of 360 days for a year.

753. A **Promissory Note** is a written, or partly written and partly printed, agreement to pay a certain sum of money, either on demand or at a specified time.

REMARK.—In general, notes discounted at banks do not bear interest. If the note be interest-bearing, the discount will be reckoned on and deducted from the amount due at maturity.

754. **Days of Grace** are the three days usually allowed by law for the payment of a note, after the expiration of the time specified *in the note*.

755. The **Maturity** of a note is the expiration of the days of grace; a note is *due* at maturity.

REMARKS.—1. Notes containing an interest clause will bear interest from date to maturity, unless other time be specified.

2. Non-interest bearing notes become interest bearing if not paid at maturity.

3. The maturity of a note or draft is indicated by using a short vertical line, with the date on which the note or draft is nominally due on the left, and the date of maturity on the right; thus. Oct. 21/24.

756. The **Value** of a note at its maturity is its face, if it does not bear interest; if the note is given with interest, its value at maturity is the face plus the interest for the time and grace.

REMARKS.—1.* Grace is given on all negotiable time paper unless “*without grace*” be specified.

2. In some States, as Minnesota, Pennsylvania, and others, drafts drawn payable *at sight* are entitled to days of grace, and should be *accepted* in the same form as time drafts; while in such States drafts payable on *demand* have no days of grace, and like the *sight* drafts of most of the States, are dishonored if not paid on demand. Other States, as New Jersey and Pennsylvania, have statutory requirements as to the phraseology of the note; as to include the phrase “*without defalcation or discount,*” etc. In such matters State laws should be observed.

757. Notes given for months, have their maturity determined by adding to their date the full months, regardless of the number of days thereby included, and also the three days of grace.

758. Notes given for days have their maturity determined by counting on from their date the expressed time, plus three days of grace. This is done regardless of the number of months compassed by the days so counted.

759. In some States, the bank custom is to take discount for both the day of discount and the day of maturity, which is excessive.

REMARKS.—1. In general, the laws of the different States provide that, if a note matures on Sunday, it shall be paid on Saturday; if Saturday be a legal holiday, then the note shall be paid on Friday; but the laws of different States vary, and should be carefully studied and fully observed, in order to hold contingent parties responsible.

2. Notes maturing on a legal holiday must be paid on the day previous, if the legal holiday occurs on Monday, payment must be made on the preceding Saturday.

760. Banks, in many of the larger cities, loan money on collateral securities, such as stocks, bonds, warehouse receipts, etc. Such loans, being made payable on demand, or on *one* day’s notice, are termed “call loans” or “demand loans.” On such the interest is usually paid at the end of the time.

REMARK.—Variations in practice among banks, and at the same bank with different patrons, are very common and subject to no rule of law.

GENERAL REMARKS ON COMMERCIAL PAPER.

761. **Commercial, or Negotiable Paper,** includes promissory notes, drafts, or bills of exchange, checks, and bank bills, warehouse receipts, and certain other evidences of indebtedness; but notes and time drafts are the only two kinds entering largely into the operations of bank discount.

762. If there is no admixture of fraud in the transaction, any negotiable paper may be bought and sold at any price agreed upon by the parties, and the purchaser thus have full right of recovery.

763. The purchaser of a negotiable paper is protected in his right of recovery of its amount against all original and contingent parties thereto, if he can show three conditions:

1st. That he gave value for the paper.

*Days of grace have been abolished by statute in California, Connecticut, Georgia, Idaho, Illinois, Montana, New Jersey, New York, North Dakota, Oregon, Pennsylvania, Utah, Vermont. Wisconsin.

2d. That he bought it before its maturity.

3d. That he did not, at the time of its purchase, know of the existence of any claim or condition affecting its validity.

764. Indorsements are made on notes for three purposes:

1st. To secure their payment.

2d. To effect their transfer.

3d. To make a memorandum of a partial payment.

765. Persons indorsing for security or transfer are liable for the payment of the paper indorsed, unless the holder of the paper fails to demand payment of its maker at maturity, and, in case of its non-payment, gives the indorser or indorsers, within a reasonable time, notice of its dishonor by the maker.

766. If the dishonored paper be *foreign*—i. e., the parties to it being of different states or countries—to hold contingent parties, a formal notarial protest, mailed to the indorsers, is required by the laws of most States; but a verbal or other informal notice of dishonor is sufficient if the paper is *domestic*.

767. No demand notice or protest is necessary to hold the maker; he, being a principal debtor, is only released from his obligation by the outlawing of the note, or by his payment of it.

768. A **Protest** is a written, or partly written and partly printed, statement, made by a notary public, giving legal notice to the maker and indorsers of a note of its non-payment.

769. The laws governing negotiable paper are not uniform throughout the United States, and a careful observance of the laws of all the States wherein one does business is necessary to avoid risks of loss.

770. It is lawful to compute and take interest for *all three* of the days of grace, although the debtor may thus lose the interest for *one* or *two* days by the fact that the note matures on Sunday or on a legal holiday.

771. Interest charges for time of transfer of notes to distant places for demand, and for the return of the remittance therefor, is a matter wholly of custom with banks, as is also an added charge or fee for services in relation to such demand and remittance.

772. Patrons of good standing at banks are often given credit for the face of interest bearing notes discounted.

773. When a note is discounted at a bank, the payee indorses it, thus making it payable to the bank; both maker and payee are then responsible to the bank for its payment.

774. Indorsements for transfer are at the same time indorsements for surety, unless made "without recourse."

775. Negotiable papers may be transferred:

1st. By indorsement *in full*—i. e., by the payee writing on the back of the note, in substance, as follows: "Pay to the order of John Doe, Richard Roe"

(payee). In which event Doe becomes the legal owner of the note, and possesses a right to receive payment on it, or, in case of its non-payment at maturity, to sue and recover from the maker; and if he follows the statute law of the State as to demand and notice, he may recover, either jointly or severally, from either the maker, or from Roe, the indorser, as such indorser is also a surety.

2d. By indorsement "in blank"—*i. e.*, by the payee writing, on the back of the note, simply his name. After this is done, the holder is *presumed* to be the owner, and he may, in case of default, recover by suit from the maker; and if he observes the requirement of the law of the place, he may also hold the indorser, as such indorser becomes a surety.

3d. By indorsement "without recourse"—*i. e.*, by the payee writing, on the back of the note, in substance, "Pay John Doe, or order, without recourse to me, Richard Roe" (payee); or by writing simply "Without recourse, Richard Roe." A note so indorsed is fully transferred from the payee, but he rests under no obligation as to its payment.

776. The corresponding terms of Bank Discount and Percentage are as follows:

The Face of the note = the Base.

The Rate Per Cent. = the Rate.

The Bank Discount = the Percentage.

777.—To find the Discount and Proceeds, the Face of a Note, Time, and Rate Per Cent. of Discount, being given.

EXAMPLE.—Find the bank discount and proceeds of a note for \$580, due in 63 days, at 6%.

OPERATION.

\$580.00 = face.

6.09 = dis. for 63 da.

\$573.91 = proceeds.

EXPLANATION.—The bank discount of a note being its interest for the time *plus* grace, and the proceeds being the face of a note *minus* the bank discount, it is only necessary to compute the interest on the face for the *full* time to obtain the discount, and to subtract such discount from the face to find the proceeds; thus, \$6.09 being the discount, \$580, minus \$6.09, equals \$573.91, proceeds.

Rule.—*Compute the interest for the time and rate, for the bank discount; and subtract this bank discount from the face of the note, to find the proceeds.*

REMARK.—If the note is on interest, find the discount on the amount of the note at maturity.

EXAMPLES FOR PRACTICE.

778. 1. Find the bank discount and proceeds of a note for \$750, due in 90 days, at 5%.

2. Find the bank discount and proceeds of a note for \$286.50, due in 30 days, at 7%.

3. Find the bank discount and proceeds of a note for \$1325, due in 60 days, at 10%.

4. What is the discount on a note for \$1000, discounted at a bank for 23 days, at 7%?

5. What are the proceeds of a 90-day note for \$1000, discounted at a bank at $4\frac{1}{2}\%$?

6. I paid in cash \$950 for an engine, and sold it the same day for \$975, taking a 60-day note, which I discounted at a bank at 8%. What was my gain or loss?

7. Find the bank discount and proceeds of a note for \$1240, dated Sept. 3, 1888, payable in 4 months, with interest at 6%, and discounted Nov. 1, 1888, at the same rate.

8. What are the proceeds of a note for \$1750, due in 63 days, bearing interest at 10%, and discounted at a bank at the same rate?

9. Find the maturity, term of discount, and proceeds of the following note:
\$286.00.

Buffalo, N. Y., Oct. 25, 1888.

Three months after date, I promise to pay to the order of Smith & Bro., Two Hundred Eighty-six Dollars, at the Erie County National Bank.

Value received.

THOMAS BROWN, JR.

Discounted Jan. 1, 1889, at 6%.

10. Find the maturity, term of discount, and proceeds of the following note:
\$300.00.

Cleveland, O., Jan. 31, 1888.

One month after date, without grace, we promise to pay to the order of Hale & Bly, Eight Hundred Dollars, with interest at 5 per cent.

Value received.

HART & COLE.

Discounted Feb. 10, 1888, at 10%.

11. Find the maturity, term of discount, and proceeds of the following note.
\$660.90.

Albany, N. Y., May 5, 1888.

Ninety days after date, I promise to pay to the order of H. H. Douglas, Six Hundred Sixty and $\frac{9}{100}$ Dollars, with interest.

Value received.

CLAYTON S. MEYERS.

Discounted June 1, 1888, at 5%.

12. Find the maturity, term of discount, and proceeds of the following note:
\$2400.00.

St. Paul, Minn., Aug. 31, 1888.

Six months after date, we promise to pay to the order of John W. Bell, Two Thousand Four Hundred Dollars, with interest at 8 per cent. after one month.

Value received.

OLIVER & JONES.

Discounted Sept. 5, 1888, at 8%.

REMARKS.—1. If discount be required on a basis of 365 days for the year, compute the discount first on a basis of 360 days, and from the discount so obtained, subtract $\frac{1}{3}$ of itself.

2. The following three examples are to be worked on a discount basis of 365 days.

13. Paul Harmon's bank account is overdrawn \$3596.11; he now discounts, at 6%: a 90-day note for \$450; a 60-day note for \$1754.81; a 30-day note for \$851.95; a 20-day note for \$345.25; a 10-day note for \$100; proceeds of all to his credit at the bank. What is the condition of his bank account after he receives these credits?

14. Swick & Sons' bank account is overdrawn \$11546.19; they now discount, at 6%: a 90-day note for \$3975.21; a 60-day note for \$5514.25; a 30-day note for \$1546.19; a 20-day note for \$2546.85; proceeds of all to their credit at the bank. What is the condition of their bank account after they receive credit as above?

15. Philo Perkins & Co.'s bank account is overdrawn \$12,916.47; they now discount, at 6%: a 90-day note for \$2428.40; a 60-day note for \$6311.25; a 30-day note for \$1120.50; a 20-day note for \$4500; a 10-day note for \$1550.50; Proceeds of all to their credit at the bank. What is the condition of their bank account after they receive the above credits?

779. To find the Face of a Note, the Proceeds, Time, and Rate Per Cent. of Discount, being given.

EXAMPLE.—What must be the face of a note, payable in 60 days, that, when discounted at 6%, the proceeds may be \$573.91?

OPERATION.

\$1.00 = face of note of \$1.

.0105 = dis. of note of \$1.

\$.9895 = proceeds of note of \$1.

$\$573.91 \div .9895 = \580 , face required.

EXPLANATION.—If the discount of \$1, at 6 per cent., for 63 days, is \$.0105, the proceeds of \$1 of the note would be \$1 minus .0105, or \$.9895; and if the proceeds of \$1 are \$.9895, it would require as many dollars face of note to give \$573.91 proceeds as \$.9895 are contained times in \$573.91, or \$580.

Rule.—*Divide the proceeds of the note by the proceeds of one dollar for the given rate and time.*

REMARK.—If the note be interest-bearing, find the proceeds of one dollar of such note, and proceed as above.

EXAMPLES FOR PRACTICE.

780. 1. What must be the face of a 90-day note that will give \$315.04 proceeds, when discounted at 6%?

2. What face of a 30-day note, discounted at 7%, will give \$1241.98 proceeds?

3. Wishing to borrow \$900 of a bank, for what sum must my 90-day note be drawn, to obtain the required amount, discount being at 10%?

4. Having bought goods to the amount of \$2431.80 cash, I gave my 60-day note in settlement. If discount be at 7%, what should have been the face of the note?

5. What must be the face of a note dated Aug. 16, 1888, and payable 6 months after date, that when discounted at a bank Oct. 1, 1888, at 6%, it will bring \$2100.55 proceeds?

6. A note dated Sept. 1, 1888, payable in 90 days, with interest at $7\frac{1}{2}\%$, was discounted 21 days after date, at 10%. If the proceeds were \$690.42, what must have been the face?

7. You have \$328.40 to your credit at the bank; you give your check for \$936.20, after which you discount a 30-day note for \$425.40, proceeds to your credit at the bank; you also discount a 90-day note made by H. C. Davis, proceeds to your credit; you now find yourself indebted to the bank \$12.37. If discount be at 6% what must have been the face of the Davis note?

PARTIAL PAYMENTS.

781. A **Partial Payment** is a *part payment* of the amount of a note, mortgage, or other obligation existing at the time such payment is made.

782. Part payments, or payments, are usually acknowledged, and should always be by indorsement on the back of the note or other obligation, but sometimes special receipts are given for the sums paid. Indorsements should give date and state amount paid; they are then equivalent to receipts.

783. Partial payments may apply to obligations, either before or after their maturity.

784. A debtor, his attorney, or other authorized agent, may make a payment either partial or in full of any obligation, and such payment may be received and receipted for by the creditor, his attorney, authorized agent, or even by one not authorized, if such a person occupies his place and is so apparently his agent as to deceive a debtor making a payment in good faith.

785. Various rules are in use for finding the balance due on claims on which partial payments have been made; but only the United States Rule and the Merchants' Rule have more than *local* application.

786. The **United States Rule** is very generally used. It has the sanction of the law, being the rule adopted by the Supreme Court of the United States, and has been adopted by most of the States.

REMARKS.—1. It was held by the Supreme Court of the United States, in its decision adopting or making the above-mentioned rule, that the payment should first be applied to cancel the interest; that what is left, if anything, after paying the interest, should be used to diminish the principal. In case the payment is not large enough to cancel the interest, it fails of its object, and is to be passed as directed by the rule.

2. If at the time of the making of a partial payment of a debt, the debtor renew his obligation by taking up the *old* note or bond, and giving a *new* one bearing interest for the unpaid part of his debt, no taint of usury can be shown affecting the validity of the *new* note, even though it may be clearly shown that a payment credited was less than the interest due at the time such payment was made.

787. Principles.—1. *Payments must be applied, first, to the discharge of accrued interest, and then the remainder, if any, toward the discharge of the principal.*

2. *Only unpaid principal can draw interest.*

788. The **Merchants' Rule** is used by most banks and business houses, where computations are on short time obligations, as such rule is regarded as the most convenient for business purposes.

REMARK.—The merchants' rule is varied in its use by different creditors, and hence is rather more an agreement, founded upon custom or otherwise, between debtor and creditor as to mode of settlement, than a strict rule of law.

789. United States Rule for Partial Payments.

REMARK.—Settlements by this rule are made as follows:

EXAMPLE.—A note, the face of which was \$3600, bearing interest at 6%, was given Oct. 17, 1884, and settled Feb. 14, 1889. Find the balance due, the following payments having been made: Mar. 3, 1885, \$600; Oct. 25, 1886, \$1000; Dec. 6, 1888, \$2400.

OPERATION AND EXPLANATION.

REMARK.—Find the time by compound subtraction.

Face of note.....	\$3600.00
Interest to date of first payment (4 mo. 16 da.).....	81.60
Amount of principal and interest at time of first payment.....	\$3681.60
First payment (of Mar. 3, 1885).....	600.00
Remainder after deducting first payment.....	\$3081.60
Interest to date of second payment (1 yr. 7 mo. 22 da.).....	304.05
Amount due at time of second payment.....	\$3385.65
Second payment (of Oct. 25, 1886).....	1000.00
Remainder after deducting second payment.....	\$2385.65
Interest to date of third payment (2 yr. 1 mo. 11 da.).....	302.58
Amount due at time of third payment.....	\$2688.23
Third payment (of Dec. 6, 1888).....	2400.00
Remainder after deducting third payment.....	\$288.23
Interest to time of settlement (2 mo. 8 da.).....	3.27
Balance due at time of settlement (Feb. 14, 1889).....	\$291.50

Rule.—Find the amount of the principal to the time when the payment, or the sum of the payments, shall equal or exceed the interest then due; from this amount deduct the payment or payments made; and with the remainder as a new principal, proceed as before, to the time of settlement.

SECOND OPERATION AND EXPLANATION

REMARK.—Find the time by compound inverse subtraction.

1884—10—17	= date of note. Face of note	\$3600.00
	= 4 mo. 16 da. = time first payment, which was.....	\$600.00
1885— 3— 3	Interest on principal to time of first payment.....	81.60
	Remainder after deducting proceeds of first payment.....	\$3081.60
	= 1 yr. 7 mo. 22 da. = time to second payment which was.....	\$1000.00
1886—10—25	Interest on new principal to date of second payment.....	304.05
	Remainder after deducting proceeds of second payment.....	\$2385.65
	= 2 yr. 1 mo. 11 da. = time to third payment which was.....	\$2400.00
	Interest on new principal to date of third payment.....	302.58
1888—12— 6	Remainder after deducting proceeds of third payment.....	\$288.23
	= 2 mo. 18 da. = time to settlement.	
1889— 2—14	Interest on new principal to settlement.....	3.27
	Balance due at time of settlement.....	\$291.50

Rule.—Find the interest on the principal to the time when the payment, or the sum of the payments, shall equal or exceed the interest then due. Deduct the interest from the payment or payments made, and this difference subtract from the principal. Treat the remainder thus found as a new principal, with which proceed as before, to the time of settlement.

EXAMPLES FOR PRACTICE.

790. 1. On a loan of \$2000, made Mar. 19, 1884, and bearing 6% interest, payments were made as follows: Nov. 1, 1885, \$500; May 3, 1887, \$700; Feb. 1, 1888, \$1000. How much will be required for settlement in full, Mar. 2, 1888?

2. Oct. 1, 1885, a note for \$1000 was given, payable in 4 years, with 6% interest. A payment of \$50 was made 1 yr. from date; a payment of \$250 was made 1 yr. 6 mo. from date; a payment of \$224 was made 2 yr. from date; a payment of \$20 was made 2 yr. 8 mo. from date; a payment of \$110 was made 2 yr. 10 mo. from date. How much remained due at the maturity of the note?

3. On a claim for \$3000, dated Aug. 12, 1885, and bearing interest at 7%, payments were made as follows: Dec. 15, 1885, \$30; Apr. 1, 1886, \$550; Jan. 20, 1887, \$85; June 12, 1887, \$1651.50. How much was due May 30, 1888?

4. I gave a mortgage for \$10000, May 9, 1881, bearing 6% interest, and made thereon the following payments: Sept. 19, 1881, \$500; Jan. 1, 1883, \$500; Apr. 25, 1883, \$4000; Oct. 15, 1885, \$4000; May 1, 1888, \$3525. How much was due at final settlement, June 2, 1888?

5. The following note was settled Oct. 13, 1888; a payment of \$25 having been made Feb. 15, 1887; one of \$300, July 12, 1887; and one of \$200, Apr. 1, 1888. If money be worth 8%, how much was due at final settlement?

\$585.50.

Elmira, N. Y., Aug. 1, 1886.

Six months after date, I promise to pay to James H. Kingsbury, or order, Five Hundred Eighty-five and $\frac{50}{100}$ Dollars, value received.

SIMEON G. FREEMAN.

6. On a mortgage for \$5500, dated Aug. 13, 1882, and bearing 6% interest, the following payments were made: Jan. 1, 1883, \$100; Mar. 2, 1883, \$25; Aug. 13, 1885, \$2500; Dec. 19, 1887, \$2500; Mar. 1, 1889, \$500. How much was required for full settlement, Mar. 11, 1889?

7. On the following note payments were endorsed as follows: Nov. 3, 1886, \$50; Mar. 16, 1887, \$50; Oct. 1, 1887, \$50; Dec. 30, 1887, \$1000; Apr. 1, 1888, \$625. How much was due, if paid in full May 8, 1888, money being worth 6%?

\$1600.00.

Dayton, Ohio, Apr. 1, 1886.

Three years after date, I promise to pay to the order of Silas Hopkins, One Thousand Six Hundred Dollars, value received, with use.

PETER S. BRYANT.

8. On the following note indorsements were made as follows: Aug. 1, 1883, \$350; Nov. 3, 1883, \$1000; Mar. 20, 1885, \$600; Mar. 31, 1885, \$2500; Dec. 11, 1888, \$2000. What was the balance due Jan. 30, 1889?

\$6500.00.

Chicago, Ill., Mar. 19, 1882.

On demand, we promise to pay to the order of Ames & Adams, Six Thousand Five Hundred Dollars, with interest at 6 per cent.

Value received.

HURD & HOUGHTON.

791. Merchants' Rule for Partial Payments.

EXAMPLE.—Find the balance due Oct. 13, 1888, on a note for \$1500, dated July 1, 1887, bearing 6% interest, and on which the following payments had been made: Oct. 1, 1887, \$300; Feb. 12, 1888, \$420; June 13, 1888, \$700.

OPERATION AND EXPLANATION

REMARK.—Find the time by compound subtraction.

Face of note, dated July 1, 1887.....	\$1500.00
Interest to date of settlement (1 yr. 3 mo. 12 da.).....	115.50
Amount of note at date of settlement.....	\$1615.50
First payment (of Oct. 1, 1887).....	\$300.00
Interest of first payment to date of settlement (1 yr. 12 da.).....	18.60
Second payment (of Feb. 12, 1888).....	420.00
Interest on second payment to date of settlement (8 mo. 1 da.).....	16.87
Third payment (of June 13, 1888).....	700.00
Interest on third payment to date of settlement (4 mo.).....	14.00
Total amount of the payments.....	\$1469.47
Balance due.....	\$146.03

Rule.—*Find the amount of the principal to the time of settlement; also find the amount of each payment, from the time it was made to the time of settlement; subtract the sum of the payments from the amount of the principal debt; the remainder will be the balance due.*

REMARK.—This rule is mainly used in case of short notes or business accounts.

EXAMPLES FOR PRACTICE.

792. 1. What is the balance due, Apr. 27, 1889, on a note for \$1050, dated Jan. 24, 1888, bearing 7% interest, if the following indorsements were made thereon: July 1, 1888, \$150; Oct. 15, 1888, \$400; Jan. 21, 1889, \$300; Mar. 27, 1889, \$60.

2. Find the balance due at the maturity of the following note, payments having been made as follows: Apr. 1, 1888, \$500; Aug. 25, 1888, \$1250; Nov. 3, 1888, \$240; Dec. 30, 1888, \$300; Feb. 1, 1889, \$200.

\$3000.00.

St. Louis, Mo., Dec. 3, 1887.

Eighteen months after date, I promise to pay to the order of Ezra R. Andrews, Three Thousand Dollars, with interest at 5 per cent.

Value received.

GEO. J. BRAYER.

3. How much was due at the maturity of the following note, payments having been made as follows: Sept. 11, 1888, \$75; Sept. 19, 1888, \$225; Sept. 26, 1888, \$159; Oct. 1, 1888, \$155.

\$650.00.

Wichita, Kan., Sept. 6, 1888.

Thirty days after date, I promise to pay to Gideon Platt & Co., Six Hundred and Fifty Dollars, with interest at 10 per cent., without grace.

Value received.

BENJ. F. COLEMAN.

4. Find the balance due on the following note, payments having been made as follows: May 28, 1888, \$255.50; June 13, 1888, \$168.41; Aug. 31, 1888, \$50; Oct. 30, 1888, \$500; Nov. 1, 1888, \$684.25.

\$2150.00.

Denver, Colo., May 1, 1888.

Six months after date, we promise to pay to the order of Wm. H. Sanford, Two Thousand One Hundred Fifty Dollars, with interest at 8 per cent.

Value received.

MARTIN F. RIGNEY,
RICHARD M. PECK.

EQUATION OF ACCOUNTS.

793. Equation of Accounts, or Equation of Payments (called also *Averaging Accounts* or *Averaging Payments*), is the process of finding the date on which a *single* payment can be made of two or more debts falling due at different dates, or when the *balance* of an account having both *debits* and *credits* can be paid without loss of interest to either party.

794. Accounts having entries on but one side, either *debit* or *credit*, are appropriately called *simple accounts*, and the process of equating such accounts may be called *Simple Equation*.

795. Accounts having both *debit* and *credit* items may likewise be called *compound accounts*, and the process of equating such accounts may be called *Compound Equation*.

796. The **Average Date of Payment, or Due Date**, is the date on which such payment or settlement may be equitably made; called also the *Equated Time*.

797. The **Focal Date** is *any assumed date* of settlement, with which the dates of the several accounts are compared for the purpose of finding the *average time* or *due date*.

REMARKS.—1. Any date conceivable may be taken as a focal date, and interest may be computed at any rate per cent., and either on a common or exact basis, without varying the result; providing only that the dates of all items be compared with such focal date, and uniformity in rate and manner of computing interest be observed throughout.

2. In practice it is vastly better to observe a simple method, by assuming the latest date in the account as a focal date, computing all interest at 6% by the short method, on a 360 day basis.

3. The importance of uniformity, simplicity, accuracy, and rapidity in the equation of payments and accounts is such as to justify the use and repetition of the above suggestions as a

General Rule.—*In all equations, extend time if credit or time paper be involved; select the latest date as a focal date, find actual time in days, and compute interest at 6 per cent., on a 360 day basis.*

798. The **Term of Credit** is the time to elapse before a debt becomes due; if given in days, it is counted on from the date of purchase or sale the *exact* number of days of the term; if given in months, it is counted on the number of months, regardless of the number of days thus included.

REMARKS.—1. Book accounts bear legal interest after they become due, and notes, even if not containing an *interest clause*, bear interest after maturity.

2. The importance of a thorough knowledge of both the theory and practice of Equation of Accounts, on the part of bookkeepers and accountants, can hardly be over-rated, as much of this class of work is to be found in every wholesale and commission business.

799. The equity of a settlement of an account by equation rests in the fact that, by a review of such account, one of the parties owes the other a balance to

which certain interest should be added or from which certain interest (discount) should be subtracted.

800. To find the Equated Time, when the Items are all Debits or all Credits and have no Terms of Credit.

EXAMPLE.—When does the (face) amount of the following account become due by equation?

Peter Dunn,

To Robt. S. Campbell, *Dr.*

1888.

Sept. 5.	To Mdse.	\$ 60
" 26.	" "	100
Oct. 8.	" "	200
Nov. 1.	" "	120
		<hr/>
		\$480

DIRECTIONS.—1. Take Nov. 1 as the focal date.

2. Find the exact time in days from the date of each item to the focal date.

3. Compute interest at 6 per cent., 360 day basis, on each item for its *time*.

4. Find the total of interest

5. Divide the total interest by the interest on the face amount for one day; the quotient will be the average time in days.

6. Count *back* from the focal date the number of days average time thus found

REMARK.—Compute interest by rules on page 217

OPERATION.

1888.	Items.	Time.	Int.
Sept. 5.	\$ 60	$\times 57 =$	\$.57
" 26.	100	$\times 36 =$.60
Oct. 8.	200	$\times 24 =$.80
Nov. 1.	120	$\times 0 =$.00
	\$480		\$1.97

Int. on \$480 for 1 day = \$.08

$\$1.97 \div .08 = 24\frac{3}{4}$ or 25 days,
the average time; 25 days back
from Nov. 1 is Oct. 7.

interest also on that item from its date to Nov. 1, or for 57 days, or \$.57 Now, on Nov. 1, Dunn owes Campbell not only the \$480, the total face amount of the debt, but also \$1 97 interest; and if a cash balance were required Nov. 1, Dunn would owe \$481.97.

But the question was not, what is the cash balance due Nov. 1, but when was the \$480, the face amount of the account, due; that is, from what date should such face amount draw interest, in order that neither party gain or lose.

Now observe that we have the principal, \$480, the interest as found, \$1.97, and the rate as assumed and used, 6 per cent., to find the time. The interest on \$480 for 1 day is \$.08. Since it takes the principal 1 day to accumulate \$.08, it must have taken it as many days to accumulate \$1.97—or the account was due as many days back from Nov. 1, the focal date—as \$.08 is contained times in \$1.97, or 25 days. Count back 25 days from Nov. 1, 1888, and obtain Oct. 7, 1888, the equated date of payment, or the date on which Dunn could pay Campbell \$480, the face of the debt, without loss of interest to either party.

Again: the same example solved, when assuming Sept. 5, the earliest date, as a focal date, or by the discount method.

REMARK.—Explanations like the following are based upon a settlement of accounts, none of which are due at the date of settlement or adjustment, as in case of the giving of an interest bearing note or bond for the equitable amount due, or for anticipating the payments of debts thus requiring a cash balance.

OPERATION.			
1888.	Items.	Time.	Disct.
Sept. 5.	\$ 60 ×	0 =	\$.00
" 26.	100 ×	21 =	.35
Oct. 8.	200 ×	33 =	1.10
Nov. 1.	120 ×	57 =	1.14
	\$480		\$2.59

EXPLANATION.—Assume the earliest date (Sept. 5) as the focal date, and reason as follows: If, on Sept. 5, Dunn pays the \$60 due on that date, he will neither have to pay interest on it nor be allowed discount; but if, on Sept. 5, he pays the \$100 due Sept. 26, he should be allowed discount on that item for the 21 days between Sept. 5 and Sept. 26, or \$.35 discount. If, on Sept. 5, he pays the \$200 not due until Oct. 8, he should be allowed discount on that item for the 33 days between Sept. 5 and Oct. 8, or \$1.10 discount; and if, on Sept. 5, he pays the \$120 not due until Nov. 1, he should be allowed discount on that item for the 57 days between Sept. 5 and Nov. 1, or \$1.14 discount. Therefore, assuming Sept. 5 as the date of settlement, Dunn does not owe on that date the face amount of the account, but such amount, \$480, less the amount of the above discounts, \$2.59, or really a cash balance of \$480, minus \$2.59, or \$477.41. But the question is not, what was the cash balance Sept. 5, but on what date would the payment of the face amount, \$480, have been equitable? We have thus a condition similar to that found in the first operation, viz.: the principal, \$480, the rate, 6 per cent., and the discount (interest) given, to find the time; and, as before, divide the discount by the discount on the principal for 1 day, and the quotient, 32, will be the average time in days. And reason, in conclusion, that from Sept. 5 Dunn is entitled to retain the face amount of his debt, \$480, for 32 days, or until it has accumulated \$.59 interest in his hands; or, in other words, in equity, he should pay such amount 32 days after Sept. 5, or Oct. 7.

Again: same example, explained with an intermediate date (Oct. 1) assumed as a focal date.

OPERATION.

Interest on \$60 from Sept. 5 to Oct. 1, 26 days =	\$.26
Interest on \$100 from Sept. 26 to Oct. 1, 5 days =	.0833+
Total interest, - - - -	\$.3433+
Discount on \$200 from Oct. 8 back to Oct. 1, 7 days =	\$.2333+
Discount on \$120 from Nov. 1 back to Oct. 1, 31 days =	.62
Total discount, - - - -	\$.8533+
.8533+ — .3433+ = \$.51, excess of discount. \$.51 ÷ .08 = 6 days.	
Oct. 1 + 6 days = Oct. 7.	

EXPLANATION.—Assume Oct. 1 as the focal date, and reason as follows: If, on Oct. 1, Dunn pays the \$60 due Sept. 5, he should also pay interest on that item for the 26 days between Sept. 5, when it became due, and Oct. 1, when it was (assumed to have been) paid, or he should pay or be charged with \$.26 interest. If, on Oct. 1, he pays the \$100 due on Sept. 26, he should also pay interest on that item for the 5 days between Sept. 26, when it became due, and Oct. 1, when it was (assumed to have been) paid, or he should pay \$.0833+ interest; thus we have a total interest charge against him of \$.3433+ on the two items of his account not paid until after they were due. But if, on Oct. 1, he pays the \$200 not due until Oct. 8, he should be allowed a discount for the 7 days between Oct. 8, when it became due, and Oct. 1, when it was paid, or he should be allowed a discount of \$.2333+ on that item; and if, on Oct. 1, he pays the \$120 not due until Nov. 1, he should be allowed a discount on that item for the 31 days between Nov. 1, when it became due, and Oct. 1, when it was paid, or he should be allowed a discount of \$.62. Thus we have a total discount to be allowed him of \$.8533+ off from the two items of his account which he paid before they were due. The difference between the amount of interest charged to him, \$.3433+, and the amount of discount for which he is given credit, \$.8533+, is \$.51, an excess of discount, showing that at the date assumed

(Oct. 1) he does not owe the face amount of the account, \$480, but \$480, the face amount, less \$.51 discount, or only \$479.49, which sum is the cash balance due on that date (Oct. 1). But since, as before, the question is not as to the cash balance, but is the date on which equitable settlement could have been effected by the payment of the face amount of the account, \$480, we have, as before, the principal, rate, and discount (interest) given, to find the time. Divide the discount, \$.51 by \$.08, and find Dunn to be entitled to withhold or delay the payment of the \$480 until it accumulates \$.51 interest (discount) in his hands, or that he keep the \$480 for 6 days after Oct. 1, thereby in equity paying it on Oct. 7, as already twice shown.

REMARKS.—1. The above explanation is given in addition to the former two, in order to illustrate that *any* date may be used as a *focal* date, and for the object of aiding the teacher in imparting to the pupil a full understanding of the underlying principles involved, and it gives added assurance that the solutions before given led to a correct result. Yet no *one* of them, nor *all* of them taken together, can be accepted as being anything beyond *assurances*. They are *not proofs*.

2. If settlement on Oct. 7 be equitable, the interest on such of the accounts as *fall due before* that date *must* be *offset* or *balanced* by the discount (interest) of such of the accounts as *fall due after* that date, to within less than one-half of the interest (discount) of the face amount of the account for one day; otherwise the due date as determined would be proven wrong.

PROOF.—Oct. 7 as a focal date.

OPERATION.		
	Days to Oct. 7.	Interest.
Sept. 5, \$ 60	32	\$.32
Sept. 26, 100	11	.1833+
		.5033+
		Discount.
Oct. 8, \$200	1	\$.0333+
Nov. 1, 120	25	.50
		\$.5333+
		.5033+
		\$.03

EXPLANATION.—Assume Oct. 7 as a focal date, and reason as follows. If, on Oct. 7, Dunn pays the \$60 due Sept. 5, he should pay interest also on that item for the 32 days between Sept. 5 and Oct. 7, or \$.32 interest; and if, on Oct. 7, he pays the \$100 due Sept. 26, he should pay interest on that bill for the 11 days between Sept. 26 and Oct. 7, or \$.1833+ interest; being thus charged \$.5033+ interest on the two items not paid until after they were due. But if, on Oct. 7, he pays the \$200 not due until Oct. 8, he should be allowed a discount on that item for the 1 day between Oct. 7 and Oct. 8, or \$.0333+ discount; and if, on Oct.

7, he pays the \$120 not due until Nov. 1, he should be allowed a discount on that item for the 25 days between Oct. 7 and Nov. 1, or \$.50 discount; being thus allowed a total discount of \$.5333+ for the pre-payment of the items of the account coming due after Oct. 7. The difference between the amount of the interest on the items of the account falling due before Oct. 7, from their respective dates down to Oct. 7, and the amount of the discounts on the items of the account coming due after Oct. 7 from their respective dates back to Oct. 7, is only \$.03, or is less than one-half the interest (or discount) on the face amount of the account for one day, thus proving Oct. 7 to be the date on which the payment of the face amount of the account, \$480, will effect an equitable settlement between Dunn and Campbell.

Rule.—I. *Select the latest date as a focal date; find the time in days from the date of each item of the account to the focal date, and compute the interest on each of the respective items for its time as found.*

II. *Divide the amount or sum of the interest on the items by the interest on the face amount of the account or items for one day; the quotient will be the number of days average time.*

III. *Count back from the focal date the number of days so found; the date thus reached will be the due date of the face amount of the account or the date on which such face amount could be paid without loss to either party.*

REMARKS.—1. In finding the average time of credit in days, fractions of a day of one-half or greater are counted as a full day; fractions less than one-half are rejected

2. In business, odd days, odd cents, and even odd dollars, are often rejected in the interest calculations in equating the time, it being correctly reasoned that, in the *long run*, any losses or gains thereby shown would fairly balance; and therefore business men, so settling, may *cut off* as they please. But for class-work, *exact* money, *exact* time, and interest computed to four decimal places, should be required.

3. Any date *between* the extremes, or within the account, may be taken as a *focal* date, the only question involved being a balance of the interest or discount; but, except for illustrative purposes by the teacher, or test exercises for advanced pupils, the selection of any date except the latest for a focal date is not recommended.

4. The selection of the latest date saves one interest computation, and removes the objection often raised in case an earlier or the earliest date be chosen, that an account is not likely to have been settled *before it was made*.

5. The product method of equating accounts, often used, and in many cases capable of producing correct results, is not recommended, because:

First. It is much more difficult to comprehend than the interest method.

Second. It usually involves a greater number of figures.

Third. By it, a cash balance, often desirable, is only obtainable by an additional operation, and with difficulty and perplexity.

Fourth. Equation of accounts having *debit* and *credit* items is impossible by that method, in case, as frequently happens, the face amounts of the two sides chance to be equal; *i. e.*, the debtor having paid the *face* amount of his obligation; while there may still be an important balance of interest or discount, which can be readily adjusted if the interest method be used.

Fifth. A book-keeper, equating by the interest method, can readily exhibit to his employer the equity conditions of an excess of interest or discount, even though the employer be unfamiliar with the formal work of the equation.

SUGGESTION TO THE TEACHER.—Place on the blackboard, as an example, an account with a dozen or more items, having different dates, and each for a simple amount, and so assign the example that each pupil may have a different focal date from which to work; then require each pupil to prove his result and withhold the announcement until called for. Such exercises will stimulate the pupils to accuracy and speed in their work, and will result in imparting a very thorough knowledge of the subject.

EXAMPLES FOR PRACTICE.

801. When are the following accounts due by equation:

REMARK.—The teacher should require that each result be proved.

1. Warren Pease,		3. Parker H. Goodwin,	
1888. To Calvin Gray, Dr.		1888. To Perkins & Hawley, Dr.	
Oct. 1, To Mdse.	\$ 75	Aug. 7, To Mdse.	\$200.00
" 6, " "	50	" 25, " "	180.55
" 14, " "	80	Sept. 30, " "	35.60
" 25, " "	120	Oct. 12, " "	100.00
" 31, " "	40	Dec. 3, " "	50.25
2. Norman Colby,		4. Wm. P. Dugan,	
1888. To Seth Stevens & Sons, Dr.		1889. To Godfrey, Son & Co., Dr.	
Oct. 1, To Mdse.	\$300	Jan. 6, To Mdse.	\$200
" 5, " "	150	Feb. 1, " "	100
" 11, " "	120	" 27, " "	100
" 16, " "	200	Apr. 3, " "	300
" 28, " "	100	" 20, " "	300
" 30, " "	180	" 27, " "	200

5. Gerald, Jones & Co.,		7. Felix Peterson & Bro.,	
1888.	<i>To Samuel Smith, Dr.</i>	1888.	<i>To Paul Paulson & Co., Dr.</i>
Oct. 13,	To Mdse. \$500.00	Dec. 1,	To Mdse. \$1500
Nov. 7,	" " 821.75	" 16,	" " 2000
" 28,	" " 150.00	1889.	
Dec. 17,	" " 205.25	Jan. 19,	" " 7000
" 30,	" " 33.00	Mar. 1,	" " 500
1889.		" 21,	" " 1000
Jan. 12,	" " 300.00		
6. Theodore Stanley,		8. Philip Darling,	
1887.	<i>To Paul Fleming, Dr.</i>	1887.	<i>To Jacob V. Hall, Dr.</i>
Nov. 6,	To Mdse. \$500	Oct. 6,	To Mdse. \$300
" 28,	" " 200	" 31,	" " 150
Dec. 17,	" " 150	Nov. 17,	" " 150
" 29,	" " 150	Dec. 1,	" " 450
1888.		1888.	
Jan. 13,	" " 300	Jan. 20,	" " 300
" 30,	" " 100	Feb. 16,	" " 600
Feb. 11,	" " 200	Mar. 3,	" " 300
Mar. 31,	" " 200	Apr. 6,	" " 300

802. To find the Equated Time, when the Items have Different Dates, and the Same or Different Terms of Credit.

EXAMPLE (requiring time extension).—When does the face amount of the following account become due by equation?

John Price,		<i>To Volney Clark, Dr.</i>	
1888.			
Sept. 14,	To Mdse., 1 mo.	\$1000	
" 30,	" " 5 mo.	500	
Nov. 10,	" " 60 da.	700	
" 29,	" " 30 da.	200	
Dec. 31,	" " 2 mo.	600	

If the time for the payment of each of the several items of the above account be extended for the term of credit indicated, the account will stand as follows:

John Price,		DIRECTIONS.—1. Assume the latest date as a focal date.	
<i>To Volney Clark, Dr.</i>		2. Star the focal date to distinguish it.	
1888, Oct. 14 \$1000	3. Observe general directions for example on page 244.	
1889, Feb. 28 500		
1889, Jan. 9 700		
1888, Dec. 29 200		
1889, Feb. 28 600		

Rule.—I. *Extend the time of credit of such items as are sold on credit.*

II. *Select the latest date as a focal date, and find the interest on each item from its maturity date to the focal date.*

III. *Divide the aggregate of interest thus found by the interest on the face amount of the account for one day; the quotient will be the time in days to be counted back from the focal date to determine the due date or average date.*

EXAMPLES FOR PRACTICE.

803. On what dates are the face amounts of the following accounts due by equation?

REMARKS.—1. Extend the time, by adding the term of credit to the date of each item, before proceeding with the work.

2. Should two or more items mature on the same date, their sum may be found, and one computation of interest serve for all.

1. Herbert G. Williams,			
1888. To Brewster & Brewster, <i>Dr.</i>			
Aug. 15,	To Mdse., 2 mo.,	-----	\$180
" 29,	" " " "	-----	300
Sept. 20,	" " " "	-----	200
Oct. 4,	" " " "	-----	120
Nov. 1,	" " " "	-----	100

2. Samuel S. Sloan,			
1888. To A. D. Wilton, <i>Dr.</i>			
Sept. 12,	To Mdse., 1 mo.,	-----	\$1000
" 30,	" " 5 mo.,	-----	500
Nov. 10,	" " 60 da.,	-----	700
" 29,	" " 30 da.,	-----	200
Dec. 31,	" " 2 mo.,	-----	300

3. H. C. Colvin,			
1888. To Jas. Fowler, <i>Dr.</i>			
Nov. 3,	To Mdse., 30 da.,	-----	\$550
" 23,	" " " "	-----	800
Dec. 1,	" " " "	-----	90
" 28,	" " " "	-----	210
1889.			
Jan. 11,	" " " "	-----	600
" 31,	" " " "	-----	300

4. Oliver H. Brown,			
1888. To Stephen Brackett, <i>Dr.</i>			
Oct. 3,	To Mdse., 30 da.,	-----	\$ 319.50
" 31,	" " 4 mo.,	-----	750.00
Dec. 1,	" " " "	-----	280.50
" 31,	" " 2 mo.,	-----	400.00
1889.			
Feb. 3,	" " 60 da.,	-----	250.50
" 28,	" " " "	-----	216.75
Mar. 12,	" " " "	-----	80.25
Apr. 30,	" " 1 mo.,	-----	150.00

5. T. L. King & Son,			
1888. To Groves & Co., <i>Dr.</i>			
Jan. 30,	To Mdse., 1 mo.,	-----	\$ 300
Feb. 28,	" " 60 da.,	-----	300
Mar. 25,	" " 2 mo.,	-----	1200
June 29,	" " 30 da.,	-----	1500

6. John Jennings,			
1889. To Richard Smith, <i>Dr.</i>			
Jan. 17,	To Mdse.,	-----	\$ 50
" 31,	" " 1 mo.,	-----	100
Feb. 9,	" " 2 mo.,	-----	600
Mar. 3,	" " " "	-----	200
June 20,	" " 3 mo.,	-----	120
July 8,	" " 1 mo.,	-----	300

7. Porter Cass & Sons,			
1888. To Phelps Bros., <i>Dr.</i>			
Feb. 19,	To Mdse., 60 da.,	-----	\$519.22
" 29,	" " 60 " "	-----	211.50
Mar. 14,	" " 30 " "	-----	120.00
" 25,	" " 30 " "	-----	181.75
May 1,	" " 2 mo.,	-----	80.00
" 31,	" " 1 " "	-----	69.78
June 24,	" " 3 " "	-----	127.75

8. H. B. Spencer & Co.,			
1888. To Wood, Son & Co., <i>Dr.</i>			
Sept. 14,	To Mdse., 1 mo.,	-----	\$ 1000
" 30,	" " 5 mo.,	-----	500
Nov. 10,	" " 60 da.,	-----	700
" 29,	" " 30 da.,	-----	200
Dec. 31,	" " 2 mo.,	-----	600
1889.			
Jan. 30,	" " 1 " "	-----	300
Feb. 28,	" " 60 da.,	-----	300
Mar. 25,	" " " "	-----	1200
June 29,	" " " "	-----	1500

804. To find the Equated Time, when an Account has both Debits and Credits

EXAMPLE.—What is the balance of the following account, and when due by equation.

<i>Dr.</i>				JAMES B. GREENE.				<i>Cr.</i>			
1889.											
Jan.	15	To Mdse.,	600	1889.	Feb.	1	By Cash,		300		
Feb.	28	“ “	300		Mar.	31	“ “		300		

- DIRECTIONS.—1. Select the latest date as a focal date.
 2. Find the time from the date (maturity) of each item to the focal date.
 3. Compute the interest on each item for its time.
 4. By addition, determine the sum of the interest on each side.
 5. Find the difference between the Dr. and Cr. interest for an interest balance.
 6. Divide this interest balance by the interest on the balance of the account for one day.

OPERATION.

Dr.

1889. Jan. 15, \$600.	75 days to focal date = \$7.50, interest.
“ Feb. 28, 300.	31 “ “ “ “ = 1.55, “
Total Dr., \$900.	\$9.05, total Dr. interest.

Cr.

1889. Feb. 1, \$300.	58 days to focal date = \$2.90, interest.
“ * Mar. 31, 300.	0 “ “ “ “ = 00, “
Total Cr., \$600.	\$2.90, total Cr. interest.

Dr. balance, \$300.

Interest on \$300 for 1 day = \$.05.

\$9.05 — \$2.90 = \$6.15, excess Dr. interest.

\$6.15 ÷ \$.05 = 123, or 123 days equated time.

123 days *back* from Mar. 31, 1889, gives Nov. 28, 1888.

* Focal date.

EXPLANATION.—Assume the *latest* date, Mar. 31, as a *focal* date, and reason as follows: If, on Mar. 31, Greene receives credit for the \$300 paid on that day, he should not receive credit for any interest, because the money was paid on the day it fell due; but if, on Mar. 31, he receives credit for the \$300 that he paid Feb. 1, he should receive credit also for the interest on that payment for the 58 days between Feb. 1, when he paid it, and Mar. 31, when he received credit for it, or he should be credited for \$2.90 interest; and if there were no debits or charges against him, he would be entitled, Mar. 31, 1889, to a net credit of \$602.90, as a cash balance in his favor. But we have the *debit* of the account to be considered, as follows: If, on Mar. 31, Greene be charged with \$300, the value of Mdse. sold to him Feb. 28, he should also be charged with its interest for the 31 days between Feb. 28 and Mar. 31, because he did not pay for the Mdse. when the amount of it was due; or he should, on this item, be charged \$1.55 interest; and if, on Mar. 31, he be charged with \$600, the value of Mdse. sold him Jan. 15, he should also be charged with its interest for the 75 days between Jan. 15 and Mar. 31, because he did not pay for the Mdse. when the amount of it was due; or he should, on this item, be charged \$7.50 interest, thus being charged a total of \$9.05 interest, and showing his total debt to be \$909.05 on Mar. 31, in case he had received no credit for payments made. But since he had received credit for payments amounting to \$600, and for interest thereon amounting to \$2.90, his debt, on Mar. 31, was not \$900, as the sum of the items charged, plus

\$9.05, the sum of the interest charged, but was \$900, the sum charged, less \$600, the sum credited, or only \$300 of principal debt or charge unpaid, and \$9.05, less \$2.90, or \$6.15, interest balance due. And if the cash balance due was required, it would thus be found to be \$306.15. But the question is not concerning the cash balance due Mar. 31, 1889, but on what date was the \$300 balance of account due by equation? And to determine this, proceed as in the earlier explanation of this subject; having given the principal (balance of account), \$300, interest (balance), \$6.15, and rate, to find the time. Divide the balance of interest by the interest on the balance for 1 day, and find the time to be 123 days, and reason in conclusion that, since on Mar. 31, Greene owed not only the \$300, but also \$6.15 interest, he had at that date been owing the \$300 for a time sufficient to enable it to accumulate \$6.15 interest, or for 123 days; and if he had, on Mar. 31, 1889, been owing the \$300 for 123 days, that debt must have been due by equation 123 days prior to Mar. 31, 1889, or since Nov. 28, 1888.

For reference, and to give assurance of the correctness of the above conclusion, the same example is taken and solved with the earliest date assumed as a *focal* date.

OPERATION.

Dr.

1889. * Jan. 15, \$600. 0 days to focal date = 00, discount.
 " Feb. 28, 300. 44 " " " " = \$2.20, total Dr. discount.
 Total Dr., \$900.

Cr.

1889. Feb. 1, \$300. 17 days to focal date = \$.85, discount.
 " Mar. 31, 300. 75 " " " " = 3.75, "
 Total Cr., \$600. \$4.60, total Cr. discount.
 2.20, total Dr. discount.
 Dr. balance, \$300. \$2.40, excess Cr. discount.

Interest or discount on \$300 for 1 day, = .05.

\$2.40 ÷ \$.05 = 48 = number of days equated time.

48 days *back* from Jan. 15, 1889, gives Nov. 28, 1888.

* Focal date.

EXPLANATION.—Assume Jan. 15, 1889, the earliest date, as a *focal* date, and reason as follows: If, on Jan. 15, Greene pays the \$600, the value of Mdse. bought on that day, he pays his debt when due, and should neither be charged with interest nor credited with discount; but if, on Jan. 15, he pays the \$300 not due until Feb. 28, he should be credited with discount on that item for the 44 days between Jan. 15, when he paid it, and Feb. 28, when it becomes due; or he should be credited with \$2.20 discount for the pre-payment of this item. Thus we find that, on Jan. 15, he did not owe the \$900, the face amount of his debt, but only \$900, the face, less \$2.20 discount. If there were no credits to be considered, he would, on Jan. 15, 1889, owe \$897.80 as a cash balance. But we have to consider the Cr. of his account, and do so as follows: If, on Jan. 15, he be credited for the \$300 not paid until Feb. 1, he should be charged discount on that sum for the 17 days between Jan. 15, when he received credit for its payment, and Feb. 1, when such payment was actually made, or he should be charged discount on this item of \$.85; and if, on Jan. 15, he receives credit for the \$300, the payment not made until Mar. 31, he should be charged discount on this item for the 75 days between Jan. 15, when he received credit for its payment, and Mar. 31, when it was actually paid, or he should be charged discount on this item of \$3.75; thus we find that, on Jan. 15, he should have received credit for the sum of his payments, \$600, less the sum of the discount, \$4.60, charged against him, or for \$595.40 as a cash balance; or

that, on Jan. 15, he owed \$300 and stood charged with discount balance of the difference between \$4.60 and \$2.20, or \$2.40; in other words that, on Jan. 15, 1889, he not only owed the \$300, balance of items, but also the \$2.40 balance of discount, or had been owing the \$300 for a length of time sufficient to enable that sum to accumulate \$2.40 in the creditor's hands. We have thus, as before found, the principal, interest (discount), and rate given to find the time; and divide the interest (discount) balance, \$2.40, by the discount on the balance of the account for 1 day, and find that, on Jan. 15, 1889, Greene had been owing the \$300 for 48 days. Counting *back* 48 days from Jan. 15, 1889, find, as before, the balance, \$300, to have been due by equation Nov. 28, 1888.

REMARKS.—1. While the result, being the same in both the foregoing operations, gives assurance of the correctness of both, it is assurance *only*, it is *not* proof.

2. If the conclusions drawn from the above explanations be correct, and the balance be due Nov. 28, 1888, as found, then the sum of the discount of the Dr. items from their respective dates back to Nov. 28, 1888, must be offset or balanced by the sum of the discount of the Cr. items from their respective dates back to Nov. 28, 1888, to within less than one-half of the discount of the balance, \$300, for 1 day, or to within less than \$.02½.

PROOF.—Take the example as above explained, and assume Nov. 28, 1888, as a focal date.

OPERATION.

Dr.

1889. Jan. 15, \$600.	48 days back to focal date = \$4.80, discount.
" Feb. 28, 300.	92 " " " " " = 4.60, "
	\$9.40, total Dr. discount.

Cr.

1889. Feb. 1, \$300.	65 days back to focal date = \$3.25, discount.
" Mar. 31, 300.	123 " " " " " = 6.15, "
	\$9.40, total Cr. discount.

Focal date, Nov. 28, 1888.

EXPLANATION.—Assume Nov. 28, 1888, as a focal date, and compute the discount on each item of the account for the time between the date of such item and the focal date, and find that the total of the Dr. discount exactly balances the total of the Cr. discount. Hence it is proved that the balance of the account considered was due by equation Nov. 28, 1888, as twice shown.

REMARKS.—1. In case a cash balance is required at any given date, it may be ascertained in either of the two following ways: First, compute interest or discount on all debit items; add the interest to these items, or subtract the discount from them; find the total amount of these items for a total debit; in the same way find the total amount of the credit items for a total credit; then by subtraction, find the balance due. Second, find the date on which the *face* balance of the account is due by equation, then if the due date comes before the actual date of settlement, add to the face balance of the account the interest upon it from the due date to the actual date of settlement; or if the due date comes after the actual date of settlement, subtract the discount on the face balance for the same time.

2. After the due date is determined, the rate of interest or discount allowed should be determined by the law of the place, but it may be determined by agreement of the parties. Local interest and usury laws would prevail in disputed cases.

3. In proving the equation of accounts, the equitable settlement of which is found to come at a date within the account or between its extreme dates, the difference between the interest and discount of the Dr. items from their respective dates to the due date (by equation) must be offset or balanced by the difference between the interest and discount of the Cr. items, from their respective dates to the *due* date, within one-half of the interest or discount on the balance for one day

805. EXAMPLE.—What is the balance of the following account, and when is it due by equation ?

(Student's Ledger.)

Dr.				CHARLES S. WILLIAMS.				Cr.			
1886.					1886						
Feb.	1	To Mdse.,	600	Feb.	19	By Cash,				300	
"	10	" "	1800	"	28	" "				300	
				Mar.	6	" "				300	

OPERATION.

Dr.

June 19, 1886, focal date.

1886. Feb. 1, \$ 600. 138 days to focal date = \$13.80, interest.

" Feb. 10, 1800. 129 " " " " = 38.70, "

Total Dr., \$2400. \$52.50, total Dr. interest.

Cr.

1886. Feb. 19, \$300. 120 days to focal date = \$6.00, interest.

" Feb. 28, 300. 111 " " " " = 5.55, "

" Mar. 6, 300. 105 " " " " = 5.25, "

Total Cr., \$900. \$16.80, total Cr. interest.

\$52.50, Dr. interest.

Dr. balance \$1500.

16.80, Cr. interest.

Interest of \$1500 for 1 day = \$.25.

\$35.70, excess Dr. interest.

\$35.70 ÷ \$.25 = 142 $\frac{2}{5}$ = 143 days equated time.

143 days back from June 19 = Jan. 27, 1886.

REMARK.—Since debit and credit accounts are accounts wherein both debtor and creditor are represented by certain purchases (debts) and payments, and since the items constituting the *Dr.* on the Ledger of one of the parties would constitute the *Cr.* on the Ledger of the other party, and *vice versa*, it follows that an account equated from both these views must show like conclusions; *i. e.*, the above account *reversed*, so that its *Cr.* shall appear a *Dr.*, and its *Dr.* appear a *Cr.*, and equated from *any* date as a *focal* date, must show the same conclusion as before.

EXAMPLE.—Same as before, *reversed*, and with May 1 assumed as a focal date.

(Charles S. Williams' Ledger.)

Dr.				"STUDENT."				Cr.			
1886.					1886.						
Feb.	19	To Cash,	300	Feb.	1	By Mdse.,				600	
"	28	" "	300	"	10	" "				1800	
Mar.	6	" "	300								

OPERATION.

*Dr.*May 1, 1886, *focal date.*

1886. Feb. 19, \$300. 71 days to focal date = \$3.55, interest.

" Feb. 28, 300. 62 " " " " = 3.10, "

" Mar. 6, 300. 56 " " " " = 2.80, "

Total Dr., \$900.

\$9.45, total Dr. interest.

Cr.

1886. Feb. 1, \$ 600. 89 days to focal date = \$ 8.90, interest.

" Feb. 10, 1800. 80 " " " " = 24.00, "

Total Cr., \$2400.

\$32.90, total Cr. interest.

9.45, total Dr. interest.

\$23.45

Cr. balance, \$1500.

Interest of \$1500 for 1 day = \$.25.

\$23.45 ÷ \$.25 = 93 $\frac{1}{5}$ = 94 days equated time.

94 days back from May 1, 1886 = Jan. 27, 1886, as before found.

EXAMPLE (same as first illustrated). — Proof, assuming Jan. 27, 1886, as a focal date.

OPERATION.

Dr.

1886. Feb. 1, \$ 600. 5 days after focal date = \$.50, discount.

" Feb. 10, 1800. 26 " " " " = 4.20, "

4.70, total Dr. discount.

Cr.

1886. Feb. 19, \$300. 23 days after focal date = \$1.15, discount.

" Feb. 28, 300. 32 " " " " = 1.60, "

" Mar. 6, 300. 38 " " " " = 1.90, "

\$4.65, total Cr. discount.

Cr. balance, \$1500.

\$4.70, total Dr. discount.

Discount on \$1500 for 1 day, \$.25.

4.65, " Cr. "

\$.05, difference.

EXPLANATION.—The difference between the Dr. discount and the Cr. discount is 5 cents, or $\frac{5}{100} = \frac{1}{20}$ of the discount on the \$1500 balance for 1 day, or less than one-half of 1 day's discount, thus proving the balance to have been due since Jan. 27, 1886, as determined by both the former operations, and rendering an explanation which could be made in the usual form quite unnecessary.

Rule.—Find the face balance of the account, and also the excess of interest from the latest date as a focal date. If the balance of account and excess of interest be on the same side, date back; if on opposite sides, date forward.

EXAMPLES FOR PRACTICE.

506. 1. When is the balance of the following account due by equation ?

Dr.				FRANK H. BARNARD.				Cr.			
1887.					1887.						
Jan.	15	To Mdse.,	600	Feb.	1	By Cash,	300				
Feb.	28	" "	300	Mar.	31	" "	300				

2. What is the balance of the following account, and when due by equation ?

Dr.				BENJ. F. HAWKINS.				Cr.			
1887.					1887.						
Jan.	14	To Mdse.,	600	Jan.	20	By Cash,	1000				
"	28	" "	300	Feb.	10	" "	700				
Feb.	3	" "	500								
"	15	" "	600								

3. If money be worth 7% per annum, what was the cash balance due on the following account July 1, 1887 ?

Dr.				VICTOR E. BROWN & Co.				Cr.			
1887.					1887.						
Jan.	31	To Mdse.,	450	Jan.	2	By Mdse.,	600				
Mar.	30	" "	450	Feb.	13	" Cash,	300				
				Mar.	29	" Mdse.,	300				

4. What was the cash balance due on the following account Jan. 1, 1889, if money be worth 8% per annum ?

Dr.				HENRY J. SANFORD & BRO.				Cr.			
1888.					1888.						
Aug.	4	To Mdse., 1 mo.,	200	Oct.	1	By Cash,	150				
Sept.	1	" " 2 mo.,	400	Nov.	1	" "	150				
Oct.	31	" " 4 mo.,	600	Dec.	1	" "	150				
Dec.	3	" "	300								
				1889.							
				Jan.	1	" "	150				
				Feb.	1	" "	150				
				Mar.	1	" "	150				

5. Find the balance of the following account, and when due by equation.

Dr.				LOUIS K. GOULD.				Cr.			
1888.					1888.						
Sept.	21	To Mdse., 1 mo.,	100	Nov.	1	By Cash,	75				
Oct.	5	" " 60 da.,	150	"	28	" Mdse., 1 mo.,	110	50			
"	30	" " 30 da.,	116	Dec.	31	" " 2 mo.,	120				
Dec.	18	" " 60 da.,	251								
			45	1889.							
Jan.	31	" " 1 mo.,	80	Jan.	15	" Cash,	175				
Feb.	28	" "	100	Mar.	1	" "	200				
			10								

6. What is the balance of the following account, and when due by equation ?

Dr.				REED & Co.				Cr.			
1888.					1888.						
June	14	To Mdse.,	300	July	1	By Cash,	100				
"	29	" Cash,	150	Aug.	1	" "	100				
Aug.	4	" Mdse.,	200	Sept.	1	" "	100				
Oct.	31	" Cash,	100	Oct.	1	" "	100				
				1889.							
				Jan.	1	" Mdse.,	450				

REMARK.—Interest may be computed on one of the four similar Cr. items for the aggregate of their days.

7. When is the balance of the following account due by equation ?

Dr.				KING & SHERWOOD.				Cr.			
1888.					1888.						
Nov.	3	To Mdse.,	750	Dec.	20	By Cash,	500				
Dec.	31	" "	1000	1889.							
1889.				Jan.	1	" Mdse.,	500				
Jan.	11	" "	600	Feb.	1	" "	1500				
Mar.	4	" "	150	May	3	" Cash,	500				

8. When is the balance of the following account due by equation ?

Dr.				SAMUEL PECK & SON.				Cr.			
1887.					1887.						
Mar.	3	To Mdse.,	60	Apr.	1	By Cash,	150				
Apr.	24	" "	100	June	1	" "	150				
May	1	" "	150	Aug.	1	" "	150				
"	30	" "	90	Oct.	1	" "	90				
Aug.	17	" "	200								

9. Find, 1st, the balance of the following account; 2d, when due by equation.

Dr.				WALTER L. PARKER.				Cr.			
1888.					1888.						
May	11	To Mdse., 2 mo.,	108 40	June	1	By Cash,	124 27				
July	1	" " 30 da.,	225	Oct.	31	" 4 mo. note (no					
Aug.	31	" "	280 80			interest),	167 91				
Oct.	1	" "	137 50	Dec.	1	" Cash,	305 05				

10. Find, 1st, when the following account is due by equation ; 2d, the cash balance due Jan. 1, 1888, if money be worth 5% per annum. Prove the result.

Dr.				JOHN MONTGOMERY & Co.				Cr.			
1887.					1888.						
Dec.	15	To Mdse.,	200	Jan.	2	By Cash,	300				
"	28	" " 2 mo.,	300	Mar.	1	" 60-da. note (no					
1888.						interest),	150				
Jan.	14	" " 30 da.,	300								

REMARK.—In case a negotiable paper is given, its maturity is determined in the usual way, by adding to its express time three days of grace. If the paper bear interest, its *value* is equivalent to its *face as cash* at its date; while if the paper be non-interest bearing, its *value* is equivalent to cash at its *full* maturity.

11. Find, 1st, the balance of the following account; 2d, when due by equation; 3d, cash balance due Jan. 1, 1888, if money be worth 6% per annum. Prove the result.

Dr.				R. E. ROGERS & BRO.				Cr.	
1887.				1887.					
May	14	To Mdse., 1 mo.,	300	May	31	By 2-mo. note (no			
June	3	“ “ 60 da.,	200			interest),	240		
July	31	“ “ 2 mo.,	400	July	15	“ 30-da. note, on			
						interest.	150		
				1888.					
				Jan.	1	“ Cash,		100	

12. Find, 1st, the balance of the following account; 2d, when due by equation; 3d, the cash balance due Jan. 1, 1888, if money be worth 10% per annum. Prove the result.

Dr.				KING, SON & Co.				Cr.	
1887.				1887.					
Oct.	1	To Mdse., 1 mo.,	150	Nov.	1	By Cash,		200	
Nov.	3	“ “ 2 mo.,	150	Dec.	1	“ 3-mo. accept. (no			
Dec.	14	“ “ 60 da.,	300			interest),	200		
				1888.					
1888.				Feb.	15	“ Cash,		200	
Jan.	15	“ “	300						

13. When is the balance of the following account due by equation?

Dr.				SPAULDING & Co.				Cr.	
1888.				1888.					
Oct.	15	To Mdse., 30 da.,	278 50	Nov.	20	By Cash,		210	
“	31	“ “ 4 mo.,	147 50	Dec.	31	“ 2-mo. accept. (no			
						interest),	175		
1889.				1889.					
Jan.	5	“ “	160 25	Mar.	1	“ 60-da. note, on			
Feb.	14	“ “ 60 da.	311 50			interest,	220 50		

14. Find, 1st, the balance of the following account; 2d, when due by equation; 3d, the cash balance due Mar. 1, 1889, if money be worth 5% per annum. Prove the result.

Dr.				ABRAHAM BRADLEY.				Cr.	
1888.				1888.					
Aug.	31	By Mdse., 1 mo.,	150	Oct.	2	By 30-da. note (no			
Sept.	5	“ “ 60 da.,	200			interest),	100		
Oct.	31	“ “ 4 mo.,	600	“	30	“ Cash,		200	
Dec.	19	“ “ 30 da.,	150	Dec.	1	“ 60-da. note, on			
						interest,	300		
1889.				1889.					
Jan.	1	“ “ 1 mo.,	100	Jan.	25	“ 1-mo. accept. (no			
						interest),	500		

15. Find, 1st, the balance of the following account; 2d, when due by equation; 3d, the cash balance due Apr. 1, 1889, if money be worth 7% per annum. Prove the result.

Dr.				LEE & POWERS.				Cr.
1888.					1889.			
Sept.	9	To Mdse.,	600	Jan.	1	By Cash,		500
Oct.	1	" " 2 mo.,	300	Mar.	16	" 2-mo. note, on		
Dec.	13	" " 1 mo.,	150			interest,	100	
1889.				Apr.	30	" 3-mo. note (no		
Jan.	31	" " 1 mo.,	450			interest),	300	
				May	1	" Cash,	200	

16. When are the net proceeds of the following account sales due by equation?

Kansas City, Mo., Oct. 3, 1888.

Account Sales of Flour,

Sold for account of HENRY H. GRINNELL & Co.,

Burlington, Iowa.

By C. H. BRAYTON.

1888.									
Sept.	23	95 barrels to Hudson & Son,	@ \$5.60, cash,						
Oct.	1	200 " " Chas. H. Knapp,	@ \$5.75, 1 mo.,						
"	18	65 " " " " " "	@ \$5.80, 60 da.,						
Nov.	3	110 " " Wm. Clark & Bro.,	@ \$5.80, 30 da.,						
"	25	130 " " Clinton McPherson,	@ \$5.75, cash,						
		<i>Charges.</i>							
Sept.	24	Freight, -----				62	50		
"	26	Cartage, -----				30	00		
Oct.	28	Cash advanced on consignment, -----				2000	00		
Nov.	15	Cooperage, -----				5	00		
"	25	Commission, 4%, -----				137	78		

REMARKS.—1. In rendering *Accounts Sales*, the expenses (freight, storage, commission, etc.) charged constitute the *Debits* of the account, while the gross sales constitute the *Credits*. Equate such accounts in the usual manner.

2. After extension of time to determine actual due (or just Cr.) dates of the items on both sides of the account, should it then be found that certain items of the Dr. have dates corresponding to those of certain items of the Cr., such items, if of equal amount, may be cancelled the one against the other; if of unequal amounts, they may be offset for like amounts, and only their difference enter into the work of the equation.

RATIO.

807. Ratio is the relation of one number to another of the same denomination. It is of two kinds, *Arithmetical* and *Geometrical*.

808. Arithmetical Ratio is the difference of the two numbers; as, the arithmetical ratio of 7 and 3, or $7 - 3 = 4$.

REMARK.—Arithmetical ratio indicates subtraction, and is or shows a *difference*.

809. Geometrical Ratio is the quotient of one number divided by another; as, the ratio of 6 to 2, or $6 \div 2 = 3$.

810. The Sign of Ratio is the colon (:), and is considered to be the division sign with the horizontal bar omitted, and is read *is to*. Thus, $6 : 2$ is read, *6 is to 2*.

811. The Terms of a ratio are the two numbers compared, and taken together they are called a *couplet*.

812. The left hand term of an arithmetical ratio is called the *antecedent*, and stands in the relation of a minuend; the right hand term is called the *consequent*, and stands in the relation of a subtrahend.

813. In geometrical ratios, the antecedent corresponds to the dividend, and the consequent to the divisor; and to such ratios the General Principles of Division apply, as follows:

- 1st. Any change in the antecedent produces a *like* change in the ratio.
- 2d. Any change in the consequent produces an *opposite* change in the ratio.
- 3d. A similar change effected in both terms will not change the ratio.

814. Reverse, indirect, or reciprocal ratios are formed by reversing the position or order of the terms.

815. Simple Ratio is the ratio of two numbers; as $20 : 5$.

816. Compound Ratio is the ratio of the products of the corresponding terms of two or more ratios; as, $20 : 5$ and $15 : 3$ may be compounded and read $20 \times 15 : 5 \times 3$, which, when the multiplication is performed, becomes a simple ratio.

$$\text{Formulas.} \left\{ \begin{array}{l} \text{Arithmetical.} \left\{ \begin{array}{l} \text{Ratio} = \text{Antecedent} - \text{Consequent.} \\ \text{Consequent} = \text{Antecedent} - \text{Ratio.} \\ \text{Antecedent} = \text{Consequent} + \text{Ratio.} \end{array} \right. \\ \\ \text{Geometrical.} \left\{ \begin{array}{l} \text{Ratio} = \text{Antecedent} \div \text{Consequent.} \\ \text{Consequent} = \text{Antecedent} \div \text{Ratio.} \\ \text{Antecedent} = \text{Consequent} \times \text{Ratio.} \end{array} \right. \end{array} \right.$$

PROPORTION.

817. Proportion is an equality of ratios, and is indicated in two ways:

- 1st. By placing the sign of equality between the ratios; thus, $8:2 = 12:3$; or,
- 2d. By placing a double colon ($::$) between the ratios; thus, $8:2::12:3$, which reads, *8 is to 2 as 12 is to 3.*

REMARKS.—1. The *first* and *fourth*, or outside terms, of a proportion are called the *extremes*; the *second* and *third*, or inside terms, are called the *means*.

2. Observe that, in the arithmetical proportion, $7:3::12:8$, the *sum* of the extremes equals the *sum* of the means. If, then, either extreme be wanting, it may be found by subtracting the *given* extreme from the sum of the means. If either mean be wanting, it may be found by subtracting the *given* mean from the sum of the extremes. If the extremes be equal and both wanting, each must equal one-half of the sum of the means, and if the means be equal, and both wanting, each must equal one half of the sum of the given extremes. This is shown and its use made valuable in proportions of three terms; as, $9:6:3$, in which 6 is a mean proportional term, the extended form being $9:6::6:3$.

3. Observe that, in the geometrical proportion, $12:4::15:5$, the *product* of the extremes equals the product of the means. If, then, either extreme be wanting, and both means given, the wanting extreme can be found by dividing the product of the means by the given extreme; and if one mean be wanting, and both extremes given, the wanting mean can be found by dividing the product of the extremes by the given mean. And if the extremes be equal and both wanting, each must equal the square root of the product of the means; and if the means be equal, and both wanting, each one must equal the square root of the product of the extremes. This is again shown and its use made valuable in proportions of three terms; as, $27:9:3$, in which 9 is a mean proportional term, the extended form being $27:9::9:3$.

SIMPLE PROPORTION.

818. A Simple Proportion is an equality of two simple ratios; thus, $27:3::45:5$, consisting of four terms, the relations of which, as above explained, are such that, if any three of them are given, the fourth may readily be found; for this reason, solutions by proportion were said, by the old writers, to come under “the Rule of Three.”

819. Take the proportion $27:3::45:5$, and suppose the last extreme unknown, and indicate its value by x . The proportion would read, $27:3::45:x$, in which the value of x is found by dividing 3×45 (the product of the means) by 27 (the given extreme); $3 \times 45 = 135$; $135 \div 27 = 5$; hence, $x = 5$.

Rules.—1. *Divide the product of the given means by the given extreme; the quotient will be the other extreme.* Or,

2. *Divide the product of the given extremes by the given mean; the quotient will be the other mean.*

REMARK.—Since the unknown term and its given multiplier (mean or extreme) constitute the factors of the *divisor*, and the remaining two terms the factors of the dividend, the rules for CANCELLATION apply, and their use will simplify the work.

EXAMPLES FOR PRACTICE.

820. Find the unknown term in each of the following proportions:

- | | |
|--------------------|--|
| 1. $39:3::52:x$. | 4. $x:9::45:5$. |
| 2. $105:15::x:4$. | 5. $96 \text{ yd.}:x::\$134.50:\403.50 . |
| 3. $42:x::54:9$. | 6. $x:\$77.50::8 \text{ bu. } 2 \text{ pk.}:153 \text{ bu.}$ |

7. If a post $7\frac{1}{2}$ ft. high casts a shadow $1\frac{1}{2}$ ft., what is the height of a tower that casts a shadow 150 ft. at the same time?

8. If 15 bushels of wheat can be bought for \$13.50, how many bushels can be bought for \$430.20?

9. An insolvent debtor owes \$14400, and has an estate valued at \$10800. How much will A receive, on a claim of \$3750?

10. A friend loaned me \$750, for 3 yr. 4 mo. 15 da. For what period of time should I loan him \$900 to fully repay his favor?

COMPOUND PROPORTION.

821. A **Compound Proportion** is a proportion, any of the terms of which have been compounded—i. e., in which such terms are made up of factors; as, the simple proportions $6:2::15:5$ and $21:3::28:4$, become compound when expressed $6 \times 21:2 \times 3::15 \times 28:5 \times 4$. This is more conveniently expressed as follows:

$$\begin{array}{c} 6 \cdot 2 \cdot 15 \cdot 5 \\ 21 \cdot 3 \cdot 28 \cdot 4 \end{array}$$

REMARK.—In compound proportions, wherein the number of factors in the couplets are more than two, it is well to substitute lines for colons, as follows:

$$\begin{array}{c} 6 \mid 2 \mid 15 \mid 5 \\ 21 \mid 3 \mid 28 \mid 4 \end{array}$$

822. Every question of proportion involves the principle of *cause and effect*. That is, work done for pay, cash given for goods, wood cut by labor performed, investments made resulting in gains or losses, etc.; and to keep theorizing as simple as possible in a subject rarely used, it seems best to adhere to some one of the many logical statements of the principles of proportion, as follows:

1st Cause : 1st Effect :: 2d Cause : 2d Effect.

This will apply, whichever term may be unknown, and will apply as well to groups of causes or groups of effects, as may be shown in compound proportions. Take for illustration the following:

EXAMPLE.—If 10 men, working 12 days, of 8 hours each day, can cut 200 cords of wood, how many cords should be cut by 12 men in 15 days, if they work 6 hours per day?

EXPLANATION.—Observe that the cutting of 200 cords of wood is an effect produced, the cause of which was 10 men, working for 12 days of 8 hours per day; and that the working of 12 men for 15 days of 6 hours per day was a cause, the effect of which is unknown; but from the application of the logical statement of the principles of proportion (1st Cause : 1st Effect :: 2d Cause : 2d Effect), we have the statement of the example given in form as follows:

1st Cause.	1st Effect.	2d Cause.	2d Effect.
10		12	
12	200	15	x
8		6	

And since, as before shown, the extreme (or outside) terms constitute the factors of the *divisor*, and the mean (or inside) terms constitute the factors of the *dividend*, any factor of the divisor may be cancelled against any factor of the dividend,

or *vice versa*. Reproducing the above statement, and effecting possible cancellations, we have:

$$\begin{array}{c|c|c|c}
 10 & & 12 & \\
 12 & 200 & 15 & \\
 8 & & 6 &
 \end{array}
 \quad x = 225 \text{ cords.}$$

$$\begin{array}{c|c|c|c}
 12 & 200 & 15 & \\
 8 & & 6 &
 \end{array}
 \quad 5 \times 15 \times 3 = 225.$$

REMARK.—All problems in proportion, simple or compound, by some called the “single rule of three” or the “double rule of three,” can be solved as above.

EXAMPLES FOR PRACTICE.

823. 1. If 5 men, working 6 days of 12 hours per day, can cut 24 acres of corn, how many acres of corn should 8 men cut in 5 days, if they work 10 hours per day?

2. If 6 men, working for 12 days, dig a ditch 80 rods long, how many rods of such ditch should 15 men dig in 21 days?

REMARK.—When any term or terms is fractional, either common or decimal in form, treat them in the usual manner, or reduce such fractions to a common denominator and compare their numerators.

3. If 15 men earn \$607.50 in 18 days, how much should 21 men earn in 12 days?

4. If \$1600, invested in a business for 3 years, gain \$960, how much should \$2150 gain in the same time.

5. If \$145.35 interest accrue on \$510, at 6%, in 4 yr. 9 mo., how much interest will accrue at the same rate and time on \$1350?

6. If 40 yards of carpet, $\frac{3}{4}$ of a yard in width, will cover a room 18 feet long and 15 feet wide, how many yards of carpet, $\frac{1}{3}$ of a yard in width, will cover a room 35 feet long and 28 feet in width?

7. If \$684, at interest for 3 yr. 3 mo. 18 da., at 5%, accrue \$112.86 interest, at what rate per cent. must \$1800 be put at interest for the same time to accrue \$445.50 interest?

8. If \$760, put at interest at 10%, accrue \$9.50 interest in 45 days, in how many days will \$1140 accrue \$17.67 interest at 6%?

REMARK.—The subjects of RATIO AND PROPORTION have been briefly discussed as above for the sole purpose of the introduction and use of the analysis of the principles involved in them, in the division of the gains or losses in partnerships.

PARTNERSHIP.

824. Partnership is the association resulting from an agreement between two or more persons to place their money, effects, labor, and skill, or some or all of them, in some enterprise or business, and divide the profits and bear the losses in certain proportions.

825. Partnerships may be formed by written agreement, sealed or unsealed, by oral agreement, or by implication.

REMARK.—Important partnerships should be formed by written agreements, in which all of the conditions of the partnership should be fully stated.

826. The business association is generally called a *Firm*, but is sometimes designated as a House.

827. The **Capital** consists of the money or other property invested.

828. The **Resources** or **Assets** of a firm consist of the property it owns and the debts due the firm.

829. The **Liabilities** of a firm are its debts.

830. The **Net Capital** is the amount which the resources exceed the liabilities.

831. The **Net Insolvency** is the amount which the liabilities exceed the resources.

832. The **Net Investment** of a partner is the amount of the firm's capital which he has invested, less the amount which he may have withdrawn from the business.

833. The **Net Gain** is the excess of the total gains over the total losses, for a given period.

834. The **Net Loss** is the excess of the total losses over the total gains, for a given period.

835. Partners are of four classes:

1. Real or ostensible.
2. Dormant, silent, or concealed.
3. Limited.
4. Nominal.

836. A **Real or Ostensible Partner** is one who appears to the world to be and who actually is a partner.

837. A **Dormant or Silent Partner** is one whose name does not appear in the firm name, whose relation is purposely concealed, but who yet profits by an investment.

REMARK.—The rule concerning silent partners is, that, being sharers in the firm's profits, they are liable the same as real partners to all creditors of the firm who, either before or after trusting the firm, learn of their connection therewith.

838. A **Limited Partner** is one who, according to the requirements of statute law, publishes his connection with the firm, names the limit of his responsibility thereby assumed, and in that manner escapes general responsibility.

839. A **Nominal Partner** is one whose name appears to the public, but who has no investment, and receives no share of the gains.

REMARK.—The rule of law concerning nominal partners is, that false appearances have been held out by them, and that all persons trusting the firm, on account of the association of their names with it, are entitled to hold them the same as if they were real partners.

840. To Divide the Gain or Loss, when each Partner's Investment has been Employed for the Same Period of Time.

REMARK.—In determining the division of the gains or losses in partnership, the principles of Proportion will be found applicable, as in the following:

EXAMPLE.—A and B together bought a house for \$8750, of which A paid \$5000, and B paid \$3750. If the house rents for \$560 a year, how many dollars of the rent should each receive?

REMARK.—By reference to conditions heretofore given, it will be observed that money invested is a *cause*, and profit therefrom is an *effect*.

From the above example, we have the following

STATEMENT.

Investment of	\$8750, first cause.
Gain, in rent, of	560, first effect.
A's investment of	5000, second cause.
The unknown term	second effect.

From which relations we have, by application of the principles and use of the explained forms of Proportion, the following

OPERATION.

1st C.	1st E.	2d C.	2d E.
\$8750 : 560 ::	5000 :	(A's part of the rent).	

Reproducing and canceling, we have:

$$\$750 : \$60 :: \$000 : (A's \text{ part}).$$

$$\cancel{7} \quad 8 \quad 40$$

$$8 \times 40 = \$320, A's \text{ part of the rent.}$$

B's part is the difference between the whole rent, \$560, and the part to which A is shown to be entitled; or it may be obtained by application of the same form as that used to determine the gain of A, viz.:

$$\$750 : \$60 :: \$750 : (B's \text{ part}).$$

$$\cancel{7} \quad 8 \quad 30$$

$$8 \times 30 = \$240, B's \text{ part of the rent.}$$

Rule.—The whole capital is to the whole gain, as each partner's share of the capital is to his share of the gain.

REMARKS.—1. Should the result of the investment be a loss, the share to be sustained by each can be determined in the same manner as above.

2 If investments are made for different periods of time, compute the investment of each partner for one period of that time, day, month, or year, then make the proportion as above.

EXAMPLES FOR PRACTICE,

841. 1. Two men bought a mine for \$20000, of which sum A paid \$12500, and B paid the remainder; they afterwards sold the mine for \$42000. How much of the selling price was each partner entitled to receive?

2. The condition of the business of Hadley & Hunt is as follows: Mdse. on hand, \$28240; notes and accounts due the firm, \$21416.54; cash on hand, \$1619.62; total liabilities of the firm, \$23186.75. Hadley's investment was \$9000, and Hunt's \$12500. What has been the gain or loss, and what is the share of each?

3. A, B, C, and D, engaged in a business, in which D invested \$8400, which was also the amount of the net gain; if A's share of the gain was \$1800, B's \$3000, and C's \$2400, what must have been the whole capital and D's gain?

4. A, B, and C are partners, A's investment being \$9600, B's \$8100, and C's \$7500. At the end of the year they have resources amounting to \$27850, and liabilities amounting to \$3150. What is the present worth of each partner at closing?

5. Four partners, A, B, C, and D, invested equal amounts, and agreed to equally apportion the gains or losses. At the time of dissolution, the firm had resources to the amount of \$33800, and liabilities to the amount of \$51975. If the net loss was \$27460, what was the net insolvency of each partner at the time of dissolution? What was each partner's investment?

6. A and B were partners 1 year, each investing \$3500, and agreeing to equally share the gains or sustain the losses. At the close of the year their resources were: Cash, \$2650; Mdse., \$3040; accounts due them, \$3150. During the year, A drew out \$4500, and B \$5750. How much has been gained or lost? What is the solvency or insolvency of the firm? What is the present worth of each?

7. Harrison and Morton bought a section of Nebraska prairie for \$8000, Harrison paying \$5000, and Morton paying the remainder. Cleveland offered them \$8000 for one-third interest in the land; the offer being accepted, the land was surveyed and divided, each taking for his exclusive use one-third of it. How should Harrison and Morton divide the \$8000 received from Cleveland?

8. Seaman and Sullivan entered into partnership with a joint capital of \$35500, of which Seaman invested \$22000. During the existence of the partnership, each withdrew \$1500, and it was agreed that no interest account should be kept, and that Seaman should receive $\frac{2}{3}$ of the gains, and sustain the same share of the losses, if any; while Sullivan should receive $\frac{1}{3}$ of the gains, and sustain that share of the losses, if any. At the time of the dissolution, the resources and liabilities were as follows:

<i>Resources.</i>		<i>Liabilities.</i>	
Cash	\$ 2050	Notes outstanding.....	\$21500
Accounts receivable	15850	Accounts outstanding	\$16500
Real estate	8100	Insurance and interest due	2000

Find the net loss of the firm, and each partner's net insolvency at closing.

OPERATION AND EXPLANATION.

Total liabilities.....	\$40000.00	Seaman's investment	\$22000
Total resources.....	26000.00	Seaman's withdrawal	1500
Net insolvency	\$14000.00	Seaman's net investment.....	\$20500
Seaman's $\frac{5}{8}$ of net loss.....	\$29062.50	Whole investment.....	\$35500
Sullivan's $\frac{3}{8}$ of net loss	17437.50	Seaman's investment	22000
Total loss.....	\$46500.00	Sullivan's investment.....	\$13500
<i>Proof.</i>		Sullivan's withdrawal	1500
Seaman's net insolvency....	\$8562.50	Sullivan's net investment	\$12000
Sullivan's net insolvency ...	5437.50	Seaman's net investment.....	\$20500
Net insolvency of firm.....	\$14000.00	Sullivan's net investment	12000
		Firm's net investment.....	\$32500
		Firm's insolvency	14000
		Firm's net loss.....	\$46500

Seaman's $\frac{5}{8}$ of loss, \$29062.50, less his net investment, \$20500 = \$8562.50, Seaman's net insolvency.

Sullivan's $\frac{3}{8}$ of loss, \$17437.50, less his net investment, \$12000 = \$5437.50, Sullivan's net insolvency.

841. To Divide the Gain or Loss, according to the Amount of Capital Invested, and Time it is Employed.

EXAMPLE.—A, B, and C are partners in business; A invested \$3000 for four years, B invested \$5000 for three years, and C invested \$4500 for two years. How should a gain of \$15000 be divided?

OPERATION AND EXPLANATION.

A's investment of \$3000 for 4 yr. = an investment of $\$3000 \times 4$, or \$12000, for 1 yr.

B's investment of \$5000 for 3 yr. = an investment of $\$5000 \times 3$, or \$15000, for 1 yr.

C's investment of \$4500 for 2 yr. = an investment of $\$4500 \times 2$, or \$9000, for 1 yr.

A's investment for 1 yr. = \$12000

B's investment for 1 yr. = \$15000

C's investment for 1 yr. = 9000

Total investment for 1 yr. = \$36000

Proportion:

\$36000 : 12000 :: 15000 : A's gain.

3 5000

\$5000 = A's part of gain.

\$36000 : 15000 :: 15000 : B's gain.

12 5 1250

5 \times 1250 = \$6250 = B's gain.

\$36000 : 15000 :: 9000 : C's gain.

12 5 750

5 \times 750 = \$3750 = C's gain.

REMARK.—Should withdrawals of capital be made at different times, or additional investments be made, follow the steps taken above: *i. e.*, by subtracting from the whole investment for 1 year (or 1 month) the whole withdrawal for 1 year (or 1 month).

EXAMPLES FOR PRACTICE.

842. 1. Three persons traded together and gained \$900; A had invested in the business \$1000, for 6 months; B had invested \$750, for 10 months; and C had invested \$1200, for 5 months. How should the gain be divided?

2. A, B, and C were partners; A had \$800 in the business for 1 year, B had \$1000 in for 9 months, and C had \$2000 in for 8 months. How should a gain of \$2150 be divided?

3. Martin and Eaton were partners one year, Martin investing at first \$5000, and Eaton \$3000; after six months Martin drew out \$3000, and Eaton invested \$1500; they gained \$3600. What was the gain of each, and the present worth of each, at the time of the dissolution of the partnership?

4. A, B, and C hired a pasture for 6 months for \$95.10; A put in 75 sheep, and 2 months later took out 40; B put in 60 sheep, and at the end of 3 months put in 45 more; C put in 200, and after 4 months took them out. What part of the rent should each pay?

5. A, B, and C were partners, with a joint capital of \$18600; A's capital was invested for 6 months, B's for 10 months, and C's for 1 year; A's part of the gain was \$1260, B's \$1500, and C's \$1200. Find how much was invested by each.

6. A and B engaged in the grocery business for 3 years, from March 1, 1885; on that date each invested \$1600; June 1, A increased his investment \$400, and B drew out \$300; Jan. 1, 1886, each withdrew \$1000; Jan. 1, 1887, each invested \$1500. How should a gain of \$7500 be divided at the time of the expiration of the partnership contract?

7. A commenced digging a ditch, and after working 6 days was joined by B, after which the two worked together 9 days, when they were joined by C. The three then worked 12 days, at the end of which time A left the job and D worked with the other two 3 days and the work was completed. If \$92 was paid for the work, how much should each receive?

8. July 1, 1885, A and B commenced business with a capital of \$7500, for which A furnished $\frac{2}{3}$ and B the remainder; May 1, 1886, B invested \$1500, and A withdrew \$600; Oct. 1, 1886, they admitted C as a partner, with an investment of \$4500; Jan. 1, 1887, each partner invested \$1000, and on Jan. 1, 1888, each partner withdrew \$500. On closing business, Oct. 1, 1888, it is found that a net loss of \$3000 has been sustained. Find each partner's proportion of the loss.

9. Olsen and Thompson dissolved a three-year's partnership Aug 1, 1888, having resources of \$16500, and liabilities of \$2150. At first Olsen invested \$2750, and Thompson \$2500; at the end of the first year Olsen drew out \$1500, and Thompson invested \$3000; six months later each invested \$1200. No interest account being kept, what has been the gain or loss, and the share of each partner, if apportioned according to average investments?

10. Simmons and Sawyer commenced business with \$25500 capital, of which Simmons invested \$13500. It was agreed that Sawyer should have \$1200 a year salary for attending to the business, and that the net gain should be divided in proportion to investments. At the close of 1 year the partnership was dissolved, the firm having resources to the amount of \$37500, and liabilities, other than for Sawyer's salary, to the amount of \$4150. If neither made withdrawals during the year, what was the interest of each partner at closing?

11. Drew, Allen, and Brackett, each invested \$15500 in a business that gave the firm a profit of \$21000 in one year. Nine months before dissolution, Drew increased his investment \$3000, and Allen and Brackett each withdrew \$3000; six months before dissolution, Allen invested \$2000, and Drew and Brackett each drew out \$2000; three months before dissolution, Brackett invested \$1000, and Drew and Allen each drew out \$1000. If no interest account was kept, and the gain be divided according to average investment, what is each partner's share?

12. A and B formed a copartnership for 3 years, A investing \$7200, and B investing \$5400. At the end of 6 months A increased his investment by \$1500, and B withdrew \$900; one year before the expiration of the partnership, each withdrew \$1000; and 6 months later each invested \$500. The net loss was \$2400. How much should be sustained by each, if sustained according to average investment; and if each be credited for interest at 6% on investments and be charged interest on withdrawals, what will be the present worth of each at closing?

13. Sept. 1, 1883, Martin and Gould engaged in partnership for 5 years, Martin investing \$13000, and the firm assuming his debts, amounting to \$2750; Gould investing \$9600, and the firm assuming his debts, to the amount of \$1650. At the end of the first year Martin withdrew \$2000, and Gould invested \$800. At the end of the second year Cole was admitted as an equal partner, he making an investment of \$6000. One year later each drew out \$1000; and six months before the partnership contract expired, each invested \$2500. Sept. 1, 1888, the partnership was dissolved, at which time it was found that a net loss of \$75 0 has been sustained. If the loss was shared in proportion to average investment, what was the loss of each partner?

MISCELLANEOUS EXAMPLES.

1. Hart, of Kansas, and Brown, of New York, form a copartnership in the grain business; Hart to make purchases, Brown to effect sales, and they agree to share equally the gains or losses. Brown sent Hart \$12,000 cash; Hart bought grain to the value of \$14,382.50, and sent Brown 40 car loads of corn, of 600 bushels each, which Brown sold at 65¢ per bushel. Hart paid traveling expenses to the amount of \$438.20, and Brown paid freight \$1249.70. At the close of the season Hart had in his possession wheat to the value of \$1128.42, and Brown had on hand 8300 bushels of oats, worth 28¢ per bushel in the New York market. They then dissolved partnership, each taking the grain in his possession at the values stated. What has been the gain or loss, and how should the partners settle?

REMARK.—By application of the principles of debit and credit, as used in book-keeping, a book-keeper may with ease and certainty close up the affairs of a partnership involving any agreed division of gains or losses, interest conditions, or those of prior or subsequent insolvency.

OPERATION.

Dr.	HART.	Cr.	Dr.	BROWN.	Cr.	Dr.	GRAIN.	Cr.
\$12000.00	\$14382.50		\$15600.00	\$12000.00		\$14382.50	\$15600.00	
1128.42	438.20		2324.00	1249.70		438.20	1128.42	
	1491.01			1491.01		1249.70	2324.00	
\$13128.42				\$14740.71		\$16070.40		
3183.29				3183.29		1491.01		
\$16311.71	\$16311.71					1491.01		
			\$17924.00	\$17924.09		\$19052.42	\$19052.42	

\$17924.00, Brown's debit.

\$19052.42; sales of grain.

14740.71, Brown's credit.

16070.40, purchases of grain.

\$3183.29, excess received by Brown, or
the amount due from Brown to Hart.

2) 2982.02, net gain of firm.

1491.01, net gain of each.

EXPLANATION.—Credit Brown for the \$12000 cash sent by him to Hart, and debit Hart for the same amount. Credit Hart for the \$14382.50 paid by him for grain, and debit Grain for the same amount. Credit Grain for \$15600, the price received by Brown for the 40 car loads of corn, and debit Brown for the same amount. Credit Hart for the \$438.20 expenses paid by him, and debit Grain for the same amount, as an element of its cost. Credit Brown for the \$1,249.70 freight paid, and debit Grain for the same amount as an added element of its cost. Now under the dissolution agreement, debit Hart for \$1128.42, the inventory value of the grain taken by him, and credit Grain for the same amount, as having virtually been sold to Hart. Debit Brown for \$2324, the inventory value of the oats taken by him, and credit Grain for that amount, as having virtually been sold to Brown. Having now disposed of all the grain, the difference between its cost, *Dr.*, and the returns from its sales, *Cr.*, will show the gain or loss. Foot the debits, and find the total cost to have been \$16070.40; foot the credits, and find the total receipts from sales to have been \$19052.42, showing a net gain of the difference, or \$2982.02, one-half of which, or \$1491.01, should go to the credit of each partner. Debit Grain for Hart's one-half of the gain, \$1491.01, and credit Hart for the same amount, to which he is entitled by the partnership agreement; and for like reasons, debit Grain for \$1491.01, as Brown's one-half of the gain, and credit Brown for the same amount, as his one-half of the gain, and find that while Brown is entitled, as shown by his credits, to only \$14740.71, he has actually received, as shown by his debits, \$17924, or that he has received the difference \$3,183.29, more than he is entitled to receive. Also find that while Hart is entitled, as shown by his credits, to receive \$16311.71, he has actually received, as shown by his debits, only \$13128.42, or that he has received the difference, \$3183.29, less than is due him. If then, Brown pays the excess, \$3183.29, that he has received, over to Hart, the accounts of both, as well as the Grain account, will be in balance, and the obtained results will be shown as follows:

1st. Net gain, \$2982.02. 2d. Net gain of each, \$1491.01. 3d. Brown owes Hart \$3183.29.

2. Hopkins and Hawley formed a partnership Sept. 1, 1886, for two years, and agreed that the gains or losses in the business should, on settlement, be adjusted according to the average investment. Sept. 1, 1886, Hopkins invested \$6250, and Hawley invested \$4500. Three months later each invested \$1750. On Mar. 1, 1888, Hopkins drew out \$3000, and Hawley invested \$2000. How should a gain of \$9400 be divided?

3. Three boys bought a watermelon for 24¢, of which price Charles paid 9¢, John 8¢ and Walter 7¢. Ralph offered 24¢ for one-quarter of the melon, which offer was accepted and the melon divided. How should the 24¢ received from Ralph be divided among the other three boys?

4. At the time of closing business, the resources of a firm were: Cash, \$931.50; Mdse., per inventory, \$13196.25; notes and accounts due it, \$8154; interest on same, \$211.50; real estate, \$11150. The firm owed, on its notes, acceptances and bills outstanding, \$7142, and interest on the same, \$348.50; and there was an unpaid mortgage on the real estate of \$2500, with interest accrued thereon of \$88.50. If the invested capital was \$22500, what was the net solvency or net insolvency of the firm at closing, and how much has been the net gain or net loss?

5. Gray, Snyder and Dillon entered into partnership with equal investments, and agreed that, in case no withdrawals of capital were made, and no added investments made by either, they should share the gains or losses equally; but in case either party increased or diminished his investment, the gains or losses should be shared according to average investment. At the end of 6 months Gray withdrew \$2000, and Snyder \$3000, and Dillon invested \$5000. Three months later Gray invested \$1000, and Snyder and Dillon each withdrew \$1500. At the end of the year they dissolved the partnership, having as total resources, \$51000; total liabilities, \$16500. No interest account having been kept, what was the present worth of each at closing, and what was the gain of each, the whole gain being \$6900?

6. Phelps, Rogers, and Wilder enter into partnership for five years. Phelps invested \$10000; Rogers, \$20000; and Wilder, \$30000. At the end of each year Phelps withdrew \$1000; Rogers, \$1600; and Wilder, \$1800. Upon final settlement, the value of the partnership property was \$57200. How much of this sum should each receive?

7. Apr. 1, 1884, Smith and Jones commenced business as partners, Smith investing \$8000, and Jones \$6000; six months later each increased his investment \$1500; and on Jan. 1, 1885, Brown was admitted as a partner with an investment of \$2400. On Oct. 1, 1885, each partner drew out \$1500; on Apr. 1, 1886, Smith and Jones each drew out \$1000, and Brown invested \$6000. On Jan. 1, 1889, it was found that a net gain of \$37500 has been realized. What was the share of each? If by agreement Smith, at final settlement, was to be allowed \$1200 per year for keeping the books of the concern, what was the present worth of each?

8. Burke, Brace, and Baldwin became partners, each investing \$15000, and each to have one-third of the gains or sustain one-third of the losses. Burke withdrew \$2100 during the time of the partnership, Brace \$1800, and Baldwin \$2000. At close of business their resources were: Cash, \$3540; Mdse., \$14785; notes, acceptances, and accounts receivable, exclusive of partner's accounts, \$16250; real estate, \$28500. They owed on their outstanding notes \$8125, and on sundry personal accounts \$1950. Find the present worth of each partner at closing.

9. Parsons and Briggs became partners Apr. 1, 1887, under an agreement that each should be allowed 6% simple interest on all investments, and that, on final settlement, Briggs should be allowed 10% of the net gains, before other division, for superintending the business, but that otherwise the gains and losses be divided in proportion to average investment. Apr. 1, 1887, Parsons invested \$18000, and Briggs \$4000; Jan. 1, 1888, Parsons withdrew \$5000, and Briggs

invested \$3000; Aug. 1, 1888, Briggs withdrew \$1500; Dec. 1, 1888, the partners agreed upon a dissolution of the partnership, having resources and liabilities as follows:

<i>Resources.</i>		<i>Liabilities.</i>	
Cash on hand and in bank....	\$ 1101.05	Notes and acceptances.....	\$6520.00
Accounts receivable.....	16405.50	Outstanding accounts.....	21246.50
Bills receivable.....	2550.00	Rent due.....	1200.00
Int. accumulated on same...	287.41		
Mdse. per inventory.....	9716.55		

If, of the accounts receivable, only 80% prove collectible, what has been the net gain or loss? What has been the gain or loss of each partner? What is the firm's net insolvency at dissolution? What is the net insolvency of each?

10. Bradley and Maben became partners July 1, 1885, under a 3-year's contract which provided that Bradley should have \$1500 each year for superintending sales, and that Maben should have \$1000 each year for keeping the books of the concern, and that these salaries should be adjusted at the end of each year and before other apportionment of gains or losses was made. July 1, 1885 each invested \$12500. Six months later each increased his investment \$5000. July 1, 1886, Bradley drew out \$3600, and Maben drew out \$3000. Oct. 1, 1886, Bradley withdrew \$1000 and Maben invested \$2000. July 1, 1887, each drew out \$1500. At the expiration of the time of the contract the resources exceeded all liabilities \$47280. What was the gain of each, and the present worth of each?

11. Clark, Wilkin and Ames bought a section of Kansas land for \$6400, of which Clark paid \$1600, Wilkin \$2000, and Ames the remainder. Wheeler offered \$4000 for one-fourth of the land; the offer was accepted, and each of the four had set apart a quarter-section for his exclusive use. How shall the money received from Wheeler be divided?

12. A, B, and C, formed a copartnership for 2 years, investing equal sums, with the agreement that each shall receive interest at the rate of 6% on all sums invested, be charged interest at the same rate on all sums withdrawn, and the gains or losses shown on final settlement be apportioned according to average net investment. Three months after the formation of the partnership A drew out \$1200, and six months later B and C each drew out \$1000, and A invested \$6000; at the end of the first year each drew out \$500. On closing the affairs of the firm, the following statement was made: net gain, \$15000; present worth, \$75000. What was the original investment of each? What was the present worth of each at the time of dissolution? What was each partner's share of the gain?

13. A and B became partners for one year; A investing $\frac{3}{4}$ of the capital, and B $\frac{1}{4}$; the agreement being that the gains or losses shall be apportioned according to average net investment, and that each partner be allowed 6% interest per annum on all investments, and be charged interest at that rate on all sums withdrawn. At the end of the year the firm had as resources: Mdse., per inventory, \$21460; real estate, \$15000; cash, \$1950; bills receivable, \$13146.50; interest accrued on the same, \$519.25; accounts due it, \$11218.50;

store furniture, \$1320; delivery wagons and horses, \$2100. The liabilities were: mortgage on real estate, \$7000; interest on same accrued, \$210; notes outstanding \$26950; interest accrued on same, \$811.75. The firm owes Barnes, Clay & Co., of Boston, \$33560. It is found that 33½ per cent. of the accounts due the firm are uncollectible. If the firm's losses during the year have been \$12000, how much was invested by each partner? What is the present worth or net insolvency of the firm, and of each partner, at closing?

14. Clay and Hard commenced business Nov. 1, 1883, with the following resources:

Clay invested cash.....	\$10000	Hard invested Mdse., valued at..	\$13500
Store, valued at.....	12000	Cash	3000
Marble fixtures, valued at.....	1500	Good will of trade, valued at...	7500

The firm assumed an outstanding mortgage on the store of \$6000, and a note made by Hard for \$3000, and due without interest July 1, 1884. Jan. 1, 1884, each partner withdrew \$300; May 1, 1886, Clay withdrew \$2000, and Hard invested the same amount. Jan. 1, 1887, Dunn was admitted to the partnership, with a cash investment of \$4500. Nov. 1, 1887, each partner invested \$1000; and on Nov. 1, 1888, the partners agreed upon a dissolution, the following being shown from the ledger of the firm:

<i>Resources.</i>		<i>Liabilities.</i>	
Mdse., per inventory.....	\$48450.50	Notes and acceptances.....	\$3825.00
Cash.....	10918.20	Interest on notes.....	114.60
Accounts receivable.....	23416.80	Balance of mortgage unpaid..	2150.00
Real estate.....	15000.00	Taxes on store, due.....	75.40
Movable fixtures and sundries,	3114.50	Due Hard for keeping the books	5000.00

It was agreed that Hard should, at the time of dissolution, be allowed \$1000 per year for keeping the books of the concern. If no interest account was kept and the gains or losses be apportioned according to average investment, what are the net resources of the firm at closing? What has been the net gain or loss? What has been the gain or loss of each partner? What is the present worth of each at closing?

APPENDIX.

STOCKS AND BONDS.

843. **Stocks** is a term applied to shares in the capital stock of banking, insurance, railroad and other incorporated or joint stock companies.

844. A **Stock Certificate** is a written or printed instrument of a Joint Stock Company or Corporation, signed by the officers of the company, certifying that the holder of the certificate is the owner of a certain number of shares of its capital stock.

A *share* represents simply a certain component part of the capital stock, which is usually divided into shares of \$25, \$50, \$100, \$1000. The Stock Certificate represents the number of shares specified therein.

The Capital Stock of a company is the sum of all the shares issued, at their *par* value.

The Par Value of stock is the sum for which stock is issued. The Market Value is the sum for which stock can be sold.

Stocks are at *par* when they sell for the value written on their face.

Stocks are *below* par when worth less than their face value, and *above* par when worth more than their face value.

845. The **Preferred Stock** of a corporation is stock on which dividends are payable before those on the original shares or common stock.

Preferred Stock is usually issued to take up certain floating indebtedness of a corporation, and agreed dividends are declared at certain intervals out of the net earnings, and before any dividend can be declared on the common stock. Such stock is usually issued upon the reorganization of railroads and consolidated joint stock companies.

846. A **Bond** is a written or printed obligation of government, Joint Stock Company or Corporation. It is conditioned to pay a certain sum of money at a specified time and at an agreed or fixed rate of interest, payable at regular intervals.

Bonds of business corporations are usually secured by mortgage on their real estate. *Municipal Bonds* are issued by a vote of the people or their representatives, and for the payment of which a Sinking Fund is accumulated by a yearly rate per cent. levied on all the real property within the limits of the municipality.

847. **Government Bonds** are bonds issued by the general government. Their names are usually derived from the interest they bear and the time when due; as 4-Twenties, 4½-'91's, U. S. Cur. 6's.'97, etc.

848. A **Coupon Bond** is one with certificates attached showing amount, date of interest and when due. When paid the coupons are detached and canceled as vouchers.

The interest coupons on government coupon bonds are payable to the bearer, and will be cashed by any bank or banker in the United States. Coupon bonds may be converted into registered bonds of the same issue.

849. A Registered bond is one which is payable to the owner as registered in the books of the corporation or government issuing it. Registered bonds can be transferred only by assignment and registry on the books of the corporation.

The interest on registered bonds is paid by checks, which are made to the order of the registered owner and sent to him by mail. The checks for interest on registered government bonds, when properly endorsed, will be cashed by any bank or banker.

850. A Corporation is a fictitious person. It consists of several natural persons, who in the name of the corporation are authorized by law to transact business.

The instrument which defines the rights and duties of a corporation is called a *Charter*. It is issued by government, under seal.

851. Stock Quotations are the public prices or rate per share that stock sells for.

852. A Dividend is a certain profit divided among stockholders.

Dividends and assessments are declared at a certain per cent. on the capital stock. Dividends are declared yearly, semi-annually and quarterly.

853. An Assessment is a sum levied upon the stockholders of a corporation to make up its losses, etc.

854. Premium is the per cent. profit of stock over 100%, or its par value. Discount is the per cent. loss of stock below 100%, or par value.

855. To Find the Dividend on Stocks, the Capital Stock and Rate Per Cent of Dividend being given.

EXAMPLE.—The Wilson Manufacturing Co., of Trenton, N. J., has a net profit of \$17812.50 for the year, to be adjusted. Its capital stock is \$250000, divided into 2500 shares of \$100 each, and the directors have declared a dividend of $5\frac{1}{2}\%$. How much will the dividend be, and how much will the sinking fund be after the dividend is transferred? also, how much of the dividend will be due C. B. Henry, who owns 15 shares?

OPERATION.

Capital = \$250000.

Rate of dividend = $5\frac{1}{2}\%$.

Net earnings = \$17812.50.

Dividend = $.05\frac{1}{2} \times \$250000 = \13750 .

Sinking fund = $\$17812.50 - \$13750 = \$4062.50$.

C. B. Henry's dividend.

Value 15 shares = \$1500.

$5\frac{1}{2}\%$ of \$1500 = \$82.50.

EXPLANATION.—Since the dividend is a certain per cent. of the capital stock, $5\frac{1}{2}\%$ dividend represents $5\frac{1}{2}\%$ of \$250000 or \$13750. As the net earnings are \$17812.50, and the dividends are \$13750, there will remain for the sinking fund, the difference between the two amounts; and $\$17812.50 - \$13750 = \$4062.50$. Since C. B. Henry has 15 shares, the par value of which is \$1500, his dividend will be $5\frac{1}{2}\%$ of \$1500 = \$82.50.

Rule.—I. *To find the total dividend: Multiply the par value of the capital stock by the rate per cent. of dividend.*

II. *To find the dividend due a stockholder: Multiply the par value of one share by the number of shares owned, and multiply this product by the rate per cent. of dividend.*

EXAMPLES FOR PRACTICE.

856. 1. The Citizens Gas Co., of Rochester, N. Y., declares a dividend of $7\frac{1}{2}\%$ on its capital stock of \$300000. Find the total dividend and the amount due Martin Bishop, who owns 25 shares of the par value of \$50 each.

2. A bank with a capital of \$200000 declares a dividend of $4\frac{1}{2}\%$. Find the amount of dividend, and B's dividend who owns 12 shares, of the par value of \$100 each.

3. A Boston railroad company having a capital stock of \$3500000, divided into 35000 shares, declares a semi-annual dividend of $2\frac{3}{4}\%$. How much is the total dividend, and how much is due A, who holds a certificate for 16 shares?

4. The Johnson Machine Co., of Cincinnati, O., declared a quarterly dividend of $1\frac{3}{4}\%$ on its capital stock of \$150000. Find the total dividend due stockholders, and C's portion, who owns 50 shares of the par value of \$100 each.

5. A manufacturing concern has a capital stock of \$200000. Its net earnings for the year are \$28512.75. Pursuant to its charter 3% of the net earnings is set aside as a Sinking Fund to cover losses, 8% is declared for dividends, and the remainder is transferred to an account called Undivided Profits, from which to declare subsequent dividends or to meet current expenses. Find the amount carried to dividend account, sinking fund account, and undivided profits account.

857. To Find the Rate Per Cent. of Dividend, the Capital Stock and Net Earnings being given.

EXAMPLE.—An insurance company having a capital stock of \$500000, has net earnings of \$45813.50. After setting aside $1\frac{1}{2}\%$ for a sinking fund, how great an even per cent. dividend may be declared out of the remainder?

OPERATION.

Capital stock	=	\$500000.00
Net earnings	=	45813.50
Sinking fund, $1\frac{1}{2}\%$	=	687.20
Remainder,	\$45126.30	
1% of capital	=	\$5000
\$45126.30 ÷ \$5000	=	9 times,
Plus \$126.30 undivided profits.		
Rate dividend	=	9%

PROOF.

Sinking fund, $1\frac{1}{2}\%$	=	\$ 687.20
Dividend, 9%	=	45000.00
Undivided profits	=	126.30
Net earnings	=	\$45813.50

EXPLANATION.—Since the rate per cent. of dividend is a certain number of hundredths of the capital stock, the capital stock is the base. As $1\frac{1}{2}\%$ is to be set aside from the net earnings for a sinking fund, which equals \$687.20, then \$45813.50 — \$687.20 = \$45126.30, the remainder from which to declare dividends. As 1% of the capital stock = \$5000, there can be as great an even per cent. dividend declared as \$5000 is contained an even number of times in \$45126.30; and \$45126.30 ÷ \$5000 = 9 times, with a remainder of \$126.30; therefore the rate of dividend is 9% , and undivided profits = \$126.30.

Rule.—*From the net earnings deduct the amount to be set aside for the sinking fund, if any, and divide the remainder by 1% of the capital stock; the quotient will represent the rate per cent. of dividend.*

EXAMPLES FOR PRACTICE.

858. 1. If the capital is \$1500000, and the net earnings are \$52500, what rate per cent. of dividend may be declared?

2. A railroad company having a capital of \$400000 has net earnings of \$33500 to divide among its shareholders. What is the greatest even per cent. of dividend that may be declared?

3. A gas company having 2000 shares, at the par value of \$50 each, has net earnings of \$18500. What even rate per cent. of dividend may be declared?

4. On Jan. 1st, 1894, a manufacturing company issued 2000 shares of its capital stock at the par value of \$100 each. At the end of the year the net earnings were \$16850. What even rate per cent. of dividend was declared, and how much remained for undivided profits?

5. The Excelsior Bicycle Co.'s loss and gain accounts show the following for the year: Gross earnings, \$29518.75; expenses, \$9618.75. Its capital stock being \$200000, what even per cent. of dividend may be declared, and what will be the amount of undivided profits, if 1% is first set aside as a sinking fund?

859. To Find the Par Value, the Premium or Discount being given.

EXAMPLE.—Sold Mutual Gas Co. stock for \$18000, at a gain of $12\frac{1}{2}\%$. Find the par value of the stock.

OPERATION.

100 % = par value.

$12\frac{1}{2}\%$ = premium.

$112\frac{1}{2}\%$ = sale, \$18000.

1 % = \$160 unit of measure.

100 % = \$16000, par value.

EXPLANATION.—The par value is 100% and the gain, $12\frac{1}{2}\%$, is the premium, therefore, $112\frac{1}{2}\%$ = sale of \$18000. As $112\frac{1}{2}\%$ = \$18000, the unit of measure, or 1% = $\$18000 \div 112\frac{1}{2} = \160 ; and 100%, or par value, = $100 \times \$160 = \16000 .

Rule.—*Divide the sale by 100% plus the rate of premium, or minus the rate of discount; the quotient will represent 1% of the par value, which, multiplied by 100, will represent 100%, or the par value.*

When the net gain or the net loss, and the per cent. of gain or loss are given, divide the gain or loss by the rate and multiply the quotient by 100.

When dividends or assessments and the amount of the sale are given, proceed as under the rule.

EXAMPLES FOR PRACTICE.

860. Find the par value.

1. \$16640 sale, 4% premium.

2. \$24000 sale, 4% discount.

3. \$17510 sale, 3% premium.

4. \$82616 sale, 8% loss.

5. \$17940 sale, 8% loss.

6. \$750 assessment, 5% rate.

7. \$1450 dividend, 2% rate.

8. \$640 discount, 4% rate.

9. \$845 loss, $2\frac{1}{2}\%$ rate.

10. \$648 gain, 3% rate.

861. To Find the Market Value, the Premium or Discount being given.

EXAMPLE.—Find the market value of 150 shares Vermont Central Railroad stock, par value \$100 each, quoted at $93\frac{1}{2}\%$.

OPERATION.

Par value = $150 \times \$100 = \15000 .
 $100\% =$ par value.
 $93\frac{1}{2}\% =$ market value.
 $.93\frac{1}{2} \times \$15000 = \14025 market value.

EXPLANATION.—The par value of 1 share is \$100. The par value of 150 shares equals $150 \times \$100 = \15000 . 100% = the par value, and the market value = $93\frac{1}{2}\%$ of the par value, therefore $.93\frac{1}{2} \times \$15000 = \14025 , market value.

Rule.—Multiply the par value by 100% plus the rate per cent. of premium or minus the rate per cent. of discount. The product will be the market value.

862. Find the market value of the following stock of the par value of \$100 per share.

- | | |
|---|---|
| 1. 50 shares quoted at $92\frac{3}{4}$. | 6. 58 shares quoted at $62\frac{1}{2}$. |
| 2. 75 shares quoted at $31\frac{1}{2}$. | 7. 150 shares quoted at 85. |
| 3. 80 shares quoted at $75\frac{3}{8}$. | 8. 300 shares quoted at 73. |
| 4. 100 shares quoted at $43\frac{1}{2}$. | 9. 500 shares quoted at $154\frac{1}{8}$. |
| 5. 200 shares quoted at 72. | 10. 450 shares quoted at $112\frac{1}{2}$. |

863. To Find the Rate Per Cent. of Investment, the Cost and Dividend being given.

EXAMPLE.—What per cent. profit is made on stock costing 80 and paying a dividend of 10%?

OPERATION.

80% = cost.
 10% = dividend.
 1% = unit of measure.
 1% of 80% = .8%.
 $10\% \div .8\% = 12\frac{1}{2}$ or $12\frac{1}{2}\%$.

EXPLANATION.—To find the rate of investment refers to the cost, therefore, 1% of cost = 1% of 80%, or .8%. As 1% of cost equals .8% profit on the investment, 10% profit will equal as many per cent. as .8% is contained times in 10%; and $10\% \div .8\% = 12\frac{1}{2}$, or $12\frac{1}{2}\%$. Proof: $12\frac{1}{2}\%$ of 80% = $.12\frac{1}{2} \times 80\% = 10\%$.

Rule.—Divide the rate of income by 1% of the cost or market value. The quotient will represent the rate of investment.

EXAMPLES FOR PRACTICE.**864. Find the rate of investment.**

Rate of Income.	Cost.	Rate of Income.	Quotations.
1. $12\frac{1}{2}\%$.	75.	6. $16\frac{2}{3}\%$.	$83\frac{1}{3}$.
2. 10%.	90.	7. 10%.	80.
3. 33%.	110.	8. 6%.	95.
4. 20%.	120.	9. 5%.	96.
5. 40%.	130.	10. 15%.	120.

MISCELLANEOUS EXAMPLES.

865. 1. What per cent profit is made on the investment if stock paying 6% dividend is bought at 120?

2. A manufacturing company having a capital of \$100000 has net earnings of \$6500. What rate per cent dividend may be declared?

3. A natural gas company declares a semi-annual dividend of $7\frac{1}{2}\%$ on its capital of \$300000. What is the total dividend, and how much is due C, who owns 68 shares of the par value of \$50 each?

4. How much money will be required to purchase 150 shares D. R. & B. railroad stock, par value \$100 each, quoted at $76\frac{1}{4}\%$, brokerage $\frac{1}{8}\%$?

5. I paid \$4440 for Kansas City & Pacific Railroad stock at 74. How many shares at \$50 each did I buy?

6. H. M. Barton bought through a broker, at $\frac{1}{8}\%$ brokerage, the following shares of railroad stock of the par value of \$100 each: 75 shares Mobile & Ohio, at $65\frac{5}{8}$; 100 shares Milwaukee & St. Paul, at 123; 80 shares Louisville & Nashville, at $113\frac{1}{4}$; 20 shares N. Y. Central, at $123\frac{1}{4}$. Find the total cost.

7. A broker bought for a customer, at $\frac{1}{8}\%$ brokerage, 800 shares New Jersey Central Railroad stock, par value \$100, at a total cost to his customer of \$100100. Find the market quotation and brokerage.

8. A business man owns 18 shares of gas stock of the par value of \$100 per share, upon which he was paid \$99 as a dividend. If the capital stock was \$150000, what was the total dividend paid to stockholders?

9. A street railway company declared a dividend of $2\frac{1}{4}\%$ per cent. If the amount of the dividend was \$16875, what was the capital stock?

10. The Silver Lake Ice Co. declares a semi-annual dividend of $2\frac{1}{4}\%$ on its capital stock of \$100000. Find the total dividend and the amount due Horace Brown who owns 75 shares of the par value of \$100 each.

11. When U. S. 5's, 1907, are selling at $119\frac{1}{4}$, how much must be invested in them to produce a quarterly income of \$1250?

12. Brown bought bank stock paying a regular annual dividend of 8% and realized 6% on his investment. What did he pay for the stock?

13. I purchased through a broker \$5000 Rock Island 5's at $101\frac{1}{4}$. What did the bonds cost me, the broker's charges being $\frac{1}{8}\%$?

14. A father invested \$5712.50 in U. S. 4's, 1907, for his son. If the bonds were purchased at $114\frac{1}{4}$, what was the son's annual income from the bonds?

15. What is the cost of four \$1000 U. S. 4's, 1907, reg. bonds at $114\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

16. The net earnings of a railroad company is \$1336375.48, and its capital stock, \$25000000. If 6% is set aside for a sinking fund, what even per cent. of dividend can be declared, and how much will remain for undivided profits?

17. Find the total cost of 1000 shares of American Sugar Ref., at 82; 500 shares of General Electric, at $37\frac{1}{2}$; 75 shares Michigan Central, at 99; 10 shares Manhattan Elevated, $113\frac{5}{8}$. Brokerage $\frac{1}{8}\%$.

18. With the proceeds of Louisville & Nashville stock which I sold at $52\frac{3}{4}$, I purchased \$21000 in Rock Island 5's, at $100\frac{3}{8}$. What was the par value of the stock sold, brokerage $\frac{1}{8}\%$?

19. What is the total par value and the total market value of 50 shares Amer. Ex., at 114; 125 shares Adams Ex., at $146\frac{1}{2}$; 200 shares Chicago Gas, at $73\frac{1}{4}$; 150 shares N. Y. C. & St. L. 1st preferred, at 65?

20. Jones bought 100 shares of railroad stock paying a regular annual dividend of $4\frac{1}{2}\%$, at 75. Smith bought the same number of shares of bank stock paying 8% dividend, at 125. Which made the better investment, and what per cent. better?

21. I bought Canadian Pacific stock at 60, which paid regular annual dividends of 4%. What rate per cent. of income will I receive on my investment?

22. Purchased 1000 shares of Amer. Sugar Refinery at $80\frac{1}{4}$; 500 shares General Electric at $37\frac{1}{2}$; 125 shares of Manhattan Elevated at $113\frac{3}{8}$. Sold the Sugar at $81\frac{1}{4}$, the General Electric at $37\frac{1}{8}$, and the Manhattan at $113\frac{1}{2}$. What was the net gain or loss, brokerage $\frac{1}{8}\%$ each way, no interest?

23. Jones bought Ches. & Ohio bonds at 25% below par and by so doing realized 8% on his investment. He also purchased Texas Pacific bonds bearing the same rate of interest at 15% below par and received on these an annual income of \$1800. What did he pay for the Texas Pacific bonds?

24. I own 150 shares of Western Union Telegraph stock, for which I paid \$13500. If I realize 5% annually on my investment, what is the rate per cent. of dividend?

25. A capitalist invested \$5306.25 in "industrial" stock at 106, paying 9% annual dividends; \$2678.13 in U. S. cur. 6's, '97, at 107; \$5012.50 in express stock at 50, annual dividends 3%; and \$7471.88 in railroad stock at $99\frac{1}{4}$, annual dividends 4%; brokerage $\frac{1}{8}\%$. How much was invested, and what was the total annual income?

EXCHANGE.

866. Exchange is a system by which debts are paid in distant places without the transmission of money. This is done by the means of written orders called Bank Drafts, Bills of Exchange, Commercial Paper, Express Orders, Telegraphic Money Orders and Post Office Money Orders.

867. Bills of Exchange are classed as Domestic or Inland, and Foreign.

868. Domestic Bills of Exchange are those payable at some place in the same country in which they are drawn.

Domestic Bills are called Drafts whether drawn at sight or on time.

869. Foreign Bills of Exchange are those payable at some place in another country.

870. A Place of Exchange is some great money center.

The principal centers of exchange in the United States are New York, Boston, Philadelphia, Chicago, St. Louis, Baltimore, Cincinnati and San Francisco. Those of Europe are London, Paris, Antwerp, Geneva, Amsterdam, Hamburg, Frankfort, Bremen, Berlin and Vienna.

DOMESTIC EXCHANGE.

871. A Bank Draft or Domestic Bill of Exchange is an order written by one bank directing another bank to pay a specified sum of money to a third party or to his order. Following is a form of

BANK DRAFT.

ALLIANCE BANK.

No. 768.

ROCHESTER, N. Y., Nov. 18, 1894.

PAY TO THE ORDER OF

.....Williamson Publishing Co.,

\$155.86.

One Hundred Fifty-five.....⁸⁶/₁₀₀ Dollars.

To Ninth National Bank, }
New York.

C. J. BARTON,
Cashier.

Nearly all banks keep money deposited with some one bank, called a correspondent, at one or more commercial centers, against which they draw drafts to sell to their customers for remittance to creditors; these drafts pass as cash in the section of country tributary to the commercial centers where the banks are located upon which the drafts are drawn.

872. Commercial Paper consists of Sight and Time Drafts drawn by one person or firm, called the drawer, directing a second person or firm, called the drawee, to pay to a third person or firm, called the payee, or to his order, a specified sum of money. Following is a common form of

COMMERCIAL PAPER.

\$418.50.

ALBANY, N. Y., Dec. 1, 1894.

*At one day's sight**Pay to the order of M. F. Brownell & Son,**Four Hundred Eighteen.....and $\frac{50}{100}$ Dollars,
and charge to the account of**To Barker Bros., }
Newark, N. J. }*

W. B. REYNOLDS & Co.

Bank drafts are drawn against cash balances on deposit, while commercial drafts are drawn to collect a debt due.

873. United States Post Office Money Orders are drawn by the postmaster, or his clerk, at one office, directing the postmaster of another office to pay to the person named in his private letter of advice, the sum specified in the order.

Applications for money orders must be in writing, and must state the amount of each order wanted, the name and address of the person to whom the order is to be paid, and the name and address of the remitter. Application blanks may be obtained at any money order office.

The maximum amount for which a single money order may be issued at an office designated as a "Money Order Office" is \$100, and at an office designated as a "Limited Money Order Office," \$5. When a larger sum is to be sent, additional orders must be obtained. But postmasters are instructed to refuse to issue in one day to the same remitter, and in favor of the same payee, on any one post office of the fourth class, money orders amounting in the aggregate to more than \$300, as such office might not have funds sufficient for immediate payment of any large amount.

The payee who desires a money order to be paid to another person must fill in and sign the form of transfer which appears on the face of the order. More than one transfer is prohibited by law.

If a money order is lost, a certificate should be obtained from both the paying and issuing postmasters that it has not been paid, and will not be paid, and the Department at Washington will issue another on application. If a money order is not collected within one year from date, it is invalid, and can be paid only by the Department at Washington on application through the issuing or paying postmaster.

U. S. MONEY ORDER RATES.

DOMESTIC.	INTERNATIONAL.
For \$2.50 or less 3 cents.	For \$10.00 or less 10 cents.
Over \$ 2.50 to \$ 5.00 5 cents.	Over \$10.00 to \$ 20.00 20 cents.
Over \$ 5.00 to \$ 10.00 8 cents.	Over \$20.00 to \$ 30.00 30 cents.
Over \$10.00 to \$ 20.00 10 cents.	Over \$30.00 to \$ 40.00 40 cents.
Over \$20.00 to \$ 30.00 12 cents.	Over \$40.00 to \$ 50.00 50 cents.
Over \$30.00 to \$ 40.00 15 cents.	Over \$50.00 to \$ 60.00 60 cents.
Over \$40.00 to \$ 50.00 18 cents.	Over \$60.00 to \$ 70.00 70 cents.
Over \$50.00 to \$ 60.00 20 cents.	Over \$70.00 to \$ 80.00 80 cents.
Over \$60.00 to \$ 75.00 25 cents.	Over \$80.00 to \$ 90.00 90 cents.
Over \$75.00 to \$100.00 30 cents.	Over \$90 to \$100 1 dollar.

The above and following tables of rates are given to aid the student in working the examples relating to money orders in this subject. These rates, while current at the time of the writing of this chapter, are subject to change, and hence should not be regarded as authentic by the business man, until verified.

874. An Express Money Order is an order drawn by the agent of an express company at the office where the order is bought, directing another agent of the company, at some place designated, to pay to the person named therein a certain sum of money.

Express orders are transferable by indorsement the same as checks.

EXPRESS MONEY ORDER RATES.

Payable in U. S., Canada or Europe.

Not over \$5.00-----	5 cents.	Over \$ 40.00 to \$ 50.00-----	18 cents.
Over \$ 5.00 to \$10.00-----	8 cents.	Over \$ 50.00 to \$ 60.00-----	20 cents.
Over \$10.00 to \$20.00-----	10 cents.	Over \$ 60.00 to \$ 75.00-----	25 cents.
Over \$20.00 to \$30.00-----	12 cents.	Over \$ 75.00 to \$100.00-----	30 cents.
Over \$30.00 to \$40.00-----	15 cents.	Over \$100.00 at above rates.	

875. Telegraphic Money Orders represent a system of exchange by which the remitter sends a message to the payee, directing him to call at a certain telegraph office for the sum named therein. The telegraph agent at the sending office instructs the agent at the receiving office, to pay to the person named in the message, the sum specified, upon his personal application and proper identification.

Express companies also contract to transmit orders for funds by telegraph, but instead of requiring the payee to call for the funds in person, they deliver them to him at his residence or place of business.

TELEGRAPHIC TRANSFER RATES.

For not more than \$50, 50¢; \$50 to \$100, 1%; \$100 to \$200, \$1.25; over \$200 to \$300, \$1.50; over \$300 to \$400, \$1.75; over \$400 to \$500, \$2.00; over \$500, special rates.

The above rates are in addition to cost of telegraphic service, which is based upon distance and the number of words contained in the message.

876. To Find the Cost of Domestic Exchange.

EXAMPLE.—A hardware merchant of Toledo, Ohio, owes Morgan & Co., of Taunton, Mass., an account of \$750. He buys of his banker a draft on the Commercial Bank of New York, at a cost of 10¢ per hundred. How much did the draft cost?

OPERATION.	
Draft, face =	\$750.00
Exchange = $7\frac{1}{2} \times 10\text{¢} =$	<u>.75</u>
Cost of draft, =	\$750.75

EXPLANATION.—The cost of the draft will be its face, \$750, plus the cost of exchange, 75¢ or \$750.75. Credit will be given by Morgan & Co., however, for only \$750, for should they deposit this draft with their banker, or transfer it to any one on account, the draft would be worth only its face, \$750.

Rule.—*Add to the face of the draft, the charge for drawing same; the sum will represent the cost of the draft.*

Formerly domestic exchange was at a premium or discount in the city where purchased, according as the balance of trade between that city and the one on which the draft was drawn,

was in favor of or against the former city. If the drawer city owed the drawee city, exchange on the latter would be at a premium in the former. If the balance of trade was in favor of the drawer city, the conditions of exchange would be reversed in the two places.

For several years past, however, domestic exchange has been practically at par throughout this country. Bankers usually make a charge, called "exchange," for the trouble of keeping funds at commercial centers and drawing drafts against these funds, and also one, called "collection," for collecting drafts deposited with them payable at banks outside of the section in which the banks receiving them are located, or where they have no regular correspondents. Some bank drafts, as those drawn on New York, are current at par almost everywhere in this country.

Some banks make no charge for domestic exchange to regular depositors.

Instead of making remittances by exchange, merchants often send their personal checks in payment of bills. These, having to be returned for collection, command a small charge, as 15¢ or 20¢ on each check, and not a *per cent.* on the amount.

877. To Find the Cost of Exchange by Express Money Orders, U. S. Money Orders and Telegraphic Money Orders.

EXAMPLE.—W. J. Boone & Co., of Bandon, Oregon, have bills to pay as follows: T. W. Brooks, Dayton, O., \$650; E. L. Greyson & Sons, Rosedale, Oregon, \$46.53; Barnes & Snyder, Bolton, Mo., \$48.50; and their traveling salesman, W. H. Post, is wanting \$100 for expenses at Denver, Colorado. They pay the amounts by remitting as follows: T. W. Brooks and E. L. Greyson & Sons, express money orders; Barnes & Snyder, P. O. money order, and W. H. Post by telegraphic transfer in a 10-word message, the message costing 50¢. Find the total cost of the remittances.

OPERATION.

Express orders:

Dayton remittance, \$650.00
Exchange, 1.98 = \$651.98

Rosedale remittance, \$ 46.53
Exchange, 18 = \$ 46.71

U. S. money order:

Bolton remittance, \$ 48.50
Fee, .18 = \$ 48.68

Telegraphic transfer:

Agent's remittance, \$100.00
commission, 1%, 1.00
cost of message, .50 = \$101.50
Total, \$848.87

EXPLANATION.—By the table of rates under Art. 874, the cost of \$650 in express order is \$1.98 (30¢ on each \$100 and 18¢ on remaining \$50) and the total cost of the remittance to Dayton is therefore \$651.98, and to Rosedale, \$46.71. The fee on U. S. money orders being 18¢ for sums over \$40 and less than \$50, the order on Bolton costs \$48.68. The rate for sending money by telegraph being 1% on sums of not more than \$100, and the cost of the service being 50¢, the total cost of the remittance to the agent at Denver is \$101.50. The total cost of exchange for the bills paid is \$848.87.

Rule.—I. *To find the cost of a remittance by express order or P. O. money order: Add to the sum remitted the exchange as determined by the table of rates.*

II. *To find the cost of telegraphic transfer of money: Add to the sum transferred the commission, as determined by the table, and the cost of sending the message.*

878. To Find the Proceeds of Commercial Paper Drawn at Sight.

EXAMPLE.—Barnum & Co., of St. Paul, drew a sight draft for \$1400 on Martin & Cole, 415 High St., Boston, on account of an invoice of hides shipped them, valued at \$3000, as per B. of L. attached to draft. They sold the draft at a bank at $\frac{1}{2}\%$ discount. What were the proceeds?

OPERATION.	
Face of draft =	\$1400
Dis. = $\frac{1}{2}\%$ of \$1400 =	<u>7</u>
Proceeds =	\$1393

EXPLANATION.—As the draft is at sight, no time is considered, and as the B. of L., valued at \$3000 is attached to the draft, that, together with the indorsement of Barnum & Co., is a sufficient guarantee to the bank for the paper. The discount is $\frac{1}{2}\%$ of \$1400 or \$7, which subtracted from the face of the draft leaves \$1393, the proceeds.

Rule.—1. *Multiply the face of the draft by the rate per cent. of discount, the product will be the discount. Subtract the discount from the face of the draft, the result will be the proceeds.* Or,

2. *Multiply the face of the draft by 100%, minus the rate of discount.*

Commercial paper is generally bought and sold at a discount, the rate depending somewhat upon the cost of transmitting the money from the place where the paper is purchased to the place on which it is drawn, and the condition of the money market in the former place.

879. To Find the Proceeds of Commercial Paper Drawn on Time.

EXAMPLE.—A commission merchant of Charleston, S. C., bought a 90-day commercial draft for \$800 on a Boston firm at $\frac{1}{2}\%$ discount. If money be worth 6%, what did the draft cost him?

OPERATION.	
Bank dis. on \$1.00 for 93 ds. =	\$.0155
Commercial dis. on \$1.00 =	<u>\$.005</u>
Total dis. on \$1.00 =	\$.0205
Proceeds of \$1.00 = \$1.00 —	\$.0205 = \$.9795
\$800 draft costs $800 \times$	\$.9795 = \$783.60

EXPLANATION.—As the bank discount on \$1.00 for 93 days is \$.0155 and the commercial discount on \$1.00 at $\frac{1}{2}\%$ is \$.005, the total discount on \$1.00 is \$.0205, and the proceeds are \$1.00 — \$.0205 = \$.9795. Since each \$1.00 of the draft cost \$.9795, the face of the draft will cost $800 \times$ \$.9795 = \$783.60.

Rule.—*Find the cost of \$1.00 by deducting the bank discount for the time and the commercial discount at the given rate. Multiply the cost of \$1.00 by the face of the draft, the product will represent the cost of the draft.*

EXAMPLES FOR PRACTICE.

880. 1. A grocer at Kingsboro, Pa., had a debt of \$38.17, due in New York, which he paid by U. S. money order. Find the cost of the order.

2. The proceeds of a 15-day draft, which was sold at $\frac{3}{4}\%$ discount, are \$1943.44. What was the face of the draft, interest 6%?

3. Find the cost of a bank draft for \$2580 on Third National Bank of New York, at the rate of 10 cents exchange for each \$100.

4. What is the face of a time draft which can be bought for \$313.14, if payable 24 days after sight, discount $\frac{3}{4}\%$, interest 5%?

5. Gates & Son, of Memphis, drew a sight draft on Perrin & Boon, of Portland, Me., for \$8750.85, which they sold at the Cotton Exchange Bank at $\frac{5}{8}\%$ discount. How much were the proceeds?

6. I drew a 60-day draft on one of my customers and sold it to a broker at $\frac{3}{4}\%$ discount, receiving \$1354.18 as proceeds. What was the face of the draft, interest 6%?

7. Jno. W. Williams, of Middlesex, Mass., remitted Janis Bros. & Co., of Milwaukee, \$1750 by draft on New York, exchange 15 cents per each \$100; Martin & Co., of Allentown, Pa., by American Express money order, \$89.75; and Theodore Emens, \$28.50, by P. O. money order. Find the total cost of exchange.

8. An Atlanta broker bought, at $\frac{3}{4}\%$ discount, a 90-day (after date) draft on a Nashville merchant, 4 days after the draft was drawn, and gave his check for \$660.17 in payment. What was the face of the draft, interest 7%?

9. L. C. Thompson, of St. Louis, bought of Glenn & Garson a 60-day draft for \$3800 on Leroy & Co., of Boston, at $\frac{3}{4}\%$ discount and less 6% interest for the time to run. He sent the draft to a Boston broker who sold it at $\frac{1}{2}\%$ discount and less 6% interest for 54 days. If the broker's commission was $\frac{1}{2}\%$, what amount did he remit Thompson and what did Thompson make by the transaction?

10. A wholesale grocer owed for an invoice of \$5425.40, purchased in New York, subject to a discount of 6% if paid within 10 days. Within the required time he discounted the bill and remitted for balance as follows: A sight draft which he bought of E. M. Brooks on Gunn & Baker for \$4000, at $\frac{1}{2}\%$ discount, and a bank draft for remainder, the exchange being 10 cents for each \$100. How much was required to settle the bill, and how much was gained by discounting it?

FOREIGN EXCHANGE.

§81. Foreign Exchange is the name given to drafts or bills of exchange drawn in one country and payable in another.

Foreign bills of exchange are usually drawn in the moneys of account of the countries in which they are payable. Thus, drafts on England, Ireland, and Scotland are drawn in pounds, shillings, and pence; on France, Switzerland, and Belgium, in francs; on Germany, in marks; on Holland, in guilders.

Foreign bills of exchange are usually drawn in duplicate or triplicate of the same tenor and date, one of which being paid the others are void. Formerly it was the practice to send the different bills of a set of foreign exchange by different routes or vessels, to guard against loss or accident. The present practice, however, especially between Europe and America, where the mail service is both rapid and sure, is to send only the original bill and retain the others of the set.

Foreign bills of exchange are sometimes used as a means of collecting debts due in foreign countries. The method employed is similar to that used in collecting debts by means of domestic bills. That is, the drawer leaves the bill with his local banker, who forwards it to a correspondent in the place where the drawee resides, by whom it is collected, the drawee paying the equivalent of the face of the bill in local current funds. The correspondent then remits the proceeds of the bill in funds current in the place where the draft was drawn, and when received by the local banker he pays or credits the drawer the face of the bill, less the charges for collecting the same.

SET OF EXCHANGE.

1	Exchange for £100.	NEW YORK, Nov. 2, 1894.
	Thirty days after sight of this First of Exchange.....	
	(Second and Third of the same tenor and date unpaid),	
	Pay to the order of Messrs. E. P. Reed & Co.....	
	One Hundred Pounds Sterling.....	
	Value received, and charge the same to account of	
	To UNION BANK, London.	
	No. 1305.	CHRYSTIE & JANNEY.

2	Exchange for £100.	NEW YORK, Nov. 2, 1894.
	Thirty days after Sight of this Second of Exchange.....	
	(First and Third of the same tenor and date unpaid),	
	Pay to the order of Messrs. E. P. Reed & Co.....	
	One Hundred Pounds Sterling.....	
	Value received, and charge the same to account of	
	To UNION BANK, London.	
	No. 1305.	CHRYSTIE & JANNEY.

3	Exchange for £100.	NEW YORK, Nov. 2, 1894.
	Thirty days after sight of this Third of Exchange.....	
	(First and Second of the same tenor and date unpaid),	
	Pay to the order of Messrs. E. P. Reed & Co.....	
	One Hundred Pounds Sterling.....	
	Value received, and charge the same to account of	
	To UNION BANK, London.	
	No. 1305.	CHRYSTIE & JANNEY.

882. The **Par of Exchange** is the established value or equivalent of the standard unit of money of one country, expressed in the standard unit of money of another country. It is of two kinds, *intrinsic* and *commercial*.

The *intrinsic par value* refers to bullion value. Thus, the pound sterling of Great Britain contains 113 grains of pure gold, and the dollar of the United States contains 23.22 grains pure gold. Since 113 grains are 4.8665 times 23.22 grains, the dollar is worth 100 cents, and the pound is worth 486.65 cents.

The *commercial par value* refers to the value of the coin or currency of one country compared with that of another country, as determined by its market value, or by the requirements of trade or commerce.

The following quotations of the foreign moneys of account are used as the basis for comparisons in the United States: Great Britain, £1 = \$4.8665; France, 1 Franc = \$.193; German Empire, 1 Mark = \$.238; Spain, 1 Peseta = \$.193; Italy, 1 Lira = \$.193; Mexico, 1 Dollar = \$.79; Brazil, 1 Milreis = \$.546; Cuba, 1 Peso = \$.932; Hayti, 1 Gourde = \$.965.

Computations in the following examples will depend upon the quotations given in each.

883. The Commercial Rate of Exchange is the market value in one country of the drafts on another.

Quotations of foreign exchange are given by means of equivalents, no reference being made to the par value.

The commercial par of exchange cannot be greater or less than the intrinsic par to a point beyond the transportation charges and insurance of shipping coin or bullion from one country to the other. When the cost of exchange is greater than the intrinsic value of the coin represented by its face, to a point beyond the cost of transmitting such coin to that country, gold can be exported at a profit; when less than its intrinsic value, beyond the cost of shipment and insurance, gold can be imported at a profit.

American exchange on Great Britain (sterling exchange) is quoted by giving the exchange value of £1 in dollars and cents; on France, Belgium, and Switzerland by giving the exchange value of \$1 in francs and centimes; on Holland by giving the exchange value of one guilder in cents; on Germany by giving the exchange value of four reichmarks in cents.

884. Documentary Exchange is a bill drawn by a shipper upon his consignee against merchandise shipped, accompanied by the letter of hypothecation, the bill of lading endorsed to order of payee, and the insurance certificates covering the property against which the bill is drawn.

885. Cable Transfer.— Within a few years the practice has arisen of transferring money to foreign countries by telegraph, or, as it is termed, “cable transfer.”

By cable transfer a merchant who desires to ship wheat to London can complete the transaction in a few hours. He can ship the wheat, telegraph the fact to the consignee at London, obtain particulars concerning the conditions of the market, and, if he thinks best, have the wheat sold at once, “to arrive,” and to remit the proceeds through a London banker. A bill does not appear at all in the transaction. The amount of business done in this manner has materially reduced the volume of bills in some places. In the eastern trade with London, in which competition is exceedingly keen and the margin of profit consequently small, the telegraphic transfer system has been in use for several years. The amount of cable transfer between this country and European countries is constantly increasing.

886. A Letter of Credit is a circular letter issued by a banking house to a person who desires to travel abroad. The letter is usually addressed to the foreign correspondents of the bank issuing it, requesting them to furnish the traveler such funds as he may require up to the aggregate amount specified in the letter.

When the traveler desires funds he goes to the correspondent in the city which he is then visiting and draws a draft for the amount on the correspondent mentioned in the body of the letter of credit. The draft is signed in the presence of the local correspondent, who carefully compares the signature with the one on the letter, and if found to agree, the draft is cashed, and the amount inscribed on the back of the letter. The last draft drawn is attached to the letter itself.

The difference between a bill of exchange and a letter of credit is that the former is payable at a certain designated place, at a specified time and in one amount, while the latter is payable at several places, at different times and in variable amounts.

887. Travelers Cheques are a substitute for letters of credit and bills of exchange. They are similar in form to bank bills. They are issued for fixed printed amounts, with the equivalent of each denomination in the money of the principal European countries, and are payable to order, after being signed and countersigned by the purchaser or holder.

Travelers Cheques are cashed, without discount or commission, by an extended list of banks and bankers, and are received in settlement of hotel bills by the principal hotels in Europe.

Following is the form of Travelers Check issued by the American Express Company:

TRAVELERS CHEQUE.

THE AMERICAN EXPRESS COMPANY.

Established 1841.

CAPITAL, \$19,000,000.

PRINCIPAL OFFICE, 65 BROADWAY, N. Y.
London Bankers:

National Provincial Bank of England.

Paris Bankers: Credit Lyonnais.

No CHEQUE issued until its full value has been paid in. Payment is assured however long checks may remain outstanding.

CHEQUES are issued for \$10, \$20, \$50 and \$100 each or foreign equivalents.

This CHEQUE will be cashed by any of the 8000 agencies of the Company in United States or Canada, and in Europe at the Company's agencies, 35 Mark St., near Cheapside, London, E. C.; 13 Water St., Liverpool; 4 Rue Scribe, Paris; 117 Langenstrasse, Bremen.

By Credit Lyonnais, Paris, Havre, Lyons, Nice, Brussels, Madrid, Geneva, Constantinople, Smyrna, Alexandria, Cairo and St. Petersburg. Munster & Leinster Bank, Cork and Dublin; Northern Banking Co. and Ulster Bank, Belfast. National Bank of Scotland, Royal Bank of Scotland, and British Linen Co. Bank of Edinburgh and Glasgow; Dresdener Bank, Berlin and Dresden; Bank für Handel & Industrie, Frankfurt, A. M.; Bayerische Vereins Bank, Munich; Anglo-Austrian Bank, Vienna; Maguay, Hooker & Co., Rome, Florence, Leghorn and Pisa; Hope & Co., Amsterdam; Landmandsbanken, Copenhagen; T. J. Hefftye & Sons, Christiania; Kredit Arbejdsbank, Stockholm, and other Banks, Bankers, Hotels, etc., as per list.

In Austria, Russia, Spain, Portugal, etc., Cheques are paid at the current rate of exchange.

When Countersigned Below with
this Signature

THE
AMERICAN EXPRESS COMPANY
TRAVELERS CHEQUE.

39368

London, July 4, 1894.
M. F. Perry

The American Express Company

Will pay to the Order of *Travelers Hotel* \$20⁰⁰

IN UNITED STATES AND CANADA.	ENGLAND, IREL., SCOTL.	FRANCE, BELG., SWITZ.	GERMANY.	ITALY.	NORWAY, SWED., DEN.	HOLLAND.
<i>Twenty</i> Dollars	£ s d 4 1 7	Francs Cent. 102 50	Marks Pfgs. 82 50	Lira Cent. 102 50	Kravas Ore. 74 07	Florins Cent. 49 02

COUNTERSIGNED: (SEE SIGNATURE ABOVE.)

Chas. G. Clark,
M. F. Perry.
TREAS.

To provide the necessary security in case cheques are lost and a simple means of identification, the intended user must at once place his or her signature in the upper left hand corner of each check under heading "When countersigned below with this signature," leaving the other spaces blank until the necessity arises for making use of same. This will prevent the use of the cheque by any other than the person whose signature is so written.

When the occasion arrives to obtain funds at a bank, or to pay hotel or other bills or accounts, the user fills out the check to the order of the party to whom payable, and places his or her signature in the lower space thereon under heading "Countersigned" (see signature above), thereby completing the issuance and insuring the identification of the rightful owner, as the two signatures must agree. No identification is required.

888. To Find the Cost of a Foreign Bill of Exchange, the Face of the Bill and the Rate of Exchange being given.

EXAMPLE.—Find the cost of a draft on London for £380, 10s. 6d. sterling, exchange being quoted at \$4.86 $\frac{1}{2}$.

OPERATION.

Face draft = £380.525.

Monetary unit = £1.

Market quotation of £1 = \$4.86 $\frac{1}{2}$.

$380.525 \times \$4.86\frac{1}{2} = \$1851.729.$

Cost of draft = \$1851.73.

EXPLANATION.—Changing 10s. 6d. to the decimal of a pound = $\frac{10}{20} + \frac{6}{240} = \frac{1}{2} + \frac{1}{40} = \frac{21}{40} = .525$, and the face of the draft in sterling exchange is worth £380.525. The value of £1 is \$4.86 $\frac{1}{2}$, therefore, £380.525 is worth $380.525 \times \$4.86\frac{1}{2} = \$1851.73.$

Rule.—*Change the given denominations to units and the decimal of a unit of the money of account of the country upon which the exchange is drawn. Multiply the value of 1 monetary unit, or market quotation, by the number of monetary units, the product will represent the cost in United States money.*

889. To Find the Face of a Foreign Bill of Exchange, the Amount and Rate being given.

EXAMPLE.—The cost of a bill of exchange on London, bought at 4.86 $\frac{1}{2}$, was \$3654.47. What was the face of the bill?

OPERATION.

4.865) 3654.47

£751 + £.175 rem.

20

3s. + .5s. rem.

12

6d.

EXPLANATION.—Since £1 costs \$4.865, \$3654.47 will buy as many £ as 4.865 is contained in 3654.47, or £751.175. Reducing the decimal of a pound to shillings and pence, we have 3s. 12d.

Rule.—*Divide the cost or market value by the quotation, if both express similar monetary units; or multiply the quotation by the cost or market value, if they express different monetary units.*

If brokerage is included, divide the gross cost in purchasing a draft by 100% plus the rate of brokerage; or the net proceeds in selling a draft by 100% minus the rate of brokerage. The quotient will be the net cost or market value of the draft.

EXAMPLES FOR PRACTICE.

890. 1. I bought a bill of exchange on Paris and paid \$2156. What was the face of the bill, the quotation being 5.17 $\frac{3}{4}$?

Since \$1 will buy 5.17 $\frac{1}{2}$ francs, \$2156 will buy $2156 \times 5.17\frac{1}{2}$ francs, or 11157.30 francs. .30 francs = 30 centimes.

2. An importer owes a bill in London of £400, 8s. 10d. How much will it cost him to settle the bill in London exchange, quoted at \$4.86 $\frac{1}{2}$?

3. I bought a bill of exchange on London at 4.87 $\frac{1}{4}$, and paid for the same \$2487.57. What was the face of the bill?

4. A New York importer who owed a Dresden manufacturer 21320 reichsmarks, bought a bill of exchange on Berlin at $95\frac{3}{4}$, and paid for the same by check. What was the face of the check?

5. What is the face of a bill of exchange on London which can be bought for \$5807.25 if quoted at 4.85, brokerage $\frac{1}{8}\%$?

6. An exporter sold through a broker a bill of exchange on Hamburg, at $95\frac{3}{4}$, and received \$5953.49 as net proceeds. What was the face of the bill, brokerage $\frac{1}{8}\%$?

7. Hibbard & Co., of Brooklyn, purchased a bill of exchange on London at 3 days' sight for £342, 12s. 6d., at $4.86\frac{1}{2}$. How much did the bill cost?

8. What will be the face of a bill of exchange on Paris that can be bought for \$1284.40, exchange being quoted at $5.17\frac{1}{2}$?

9. An importer purchased a sixty-day bill of exchange on Bremen at $95\frac{3}{4}$ for \$446.20. What was the face of the bill?

10. A New York diamond merchant purchased a bill of exchange on Amsterdam at 3 days' sight for 63892 guilders, at $40\frac{1}{2}$. What did the bill cost?

11. I purchased a bill of exchange on Paris for 33250 francs and paid \$6412.72. What was the course of exchange?

12. The Armour Packing Company received \$3149.25 for a bill of exchange on Hamburg. What was the face of the bill, exchange being $94\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

13. An importer purchased a bill of exchange on Amsterdam for 3575 guilders and paid \$1443.41 for it. What was the course of exchange?

14. A Manchester manufacturer drew a bill of exchange at 3 days' sight for £450, 10s. 8d. on a Rochester, N. Y., merchant. The draft was presented to the payee by a local bank, and paid by check. What was the face of the check, exchange being $4.85\frac{1}{4}$, collection $\frac{1}{4}\%$?

15. Langdon & Perry, of New York, owed on foreign invoices as follows: T. C. Shepherd Sons, London, £1800, 8s.; J. L. Von Buesche, Berlin, 1600 marks; Perrie, Buzzell & Co., Paris, 4016 francs; F. Gonzalez, Mexico, 816 dollars. They bought at their bank: Exchange on London, at $4.86\frac{1}{8}$; on Berlin, $96\frac{1}{4}$; on Paris, $5.19\frac{1}{2}$; on Mexico, $79\frac{1}{2}$, and issued one check to cover the total purchase. What was the amount of the check?

16. When N. Y., N. H., & H. stock is quoted in New York at $181\frac{1}{2}$, and sterling exchange at $4.86\frac{1}{2}$, what should be the London quotation of the stock?

NOTE.—American securities are quoted in London at an assumed valuation of \$5 to the pound sterling, instead of the actual value of \$4.8665. Hence, to obtain the equivalent London quotation, multiply the American quotation by 5 and divide the product by the rate of exchange.

17. If N. Y. C. R. R. stock is quoted in London at $103\frac{1}{2}$, what is the equivalent New York quotation, sterling exchange being quoted in New York at $4.87\frac{1}{2}$? Multiply the London quotation by rate of exchange, and divide the product by 5.

18. When sterling exchange is quoted in New York at $4.86\frac{1}{4}$, what is the equivalent London quotation of Can. Pacif. R. stock, listed in New York at $64\frac{3}{8}$?

19. Pennsylvania R. R. stock is quoted in London at 54. What is the equivalent New York quotation if the rate of exchange is $4.86\frac{1}{4}$?

20. When Manhattan Elevated stock is quoted in New York at $113\frac{1}{2}$, what is the equivalent London quotation, the course of exchange being $4.86\frac{1}{2}$?

ANSWERS.

Page 12.

Art. 64.

1. 45.
2. 306.
3. 217.
4. 1647.
5. 979.
6. 262.
7. 853.
8. 599.
9. 1053.
10. 1610.

Art. 65.

1. 3342.
2. 22512.
3. 26052.
4. 161840.
5. 223732.
6. 2967515.
7. 813496.
8. 21423493.
9. 24543879.
10. 8179519.

Art. 66.

1. 133.36.
2. 530.80.
3. 553.61.
4. 629.23.
5. 421.34.
6. \$36.91.
7. 948.69.
8. 91.30.
9. 314.61.
10. 296.19.
11. 488.35.
12. 260.54.
13. 473.43.

Art. 67.

1. \$3102.

Page 13.

2. 5530 pounds.
3. 33200 feet.
4. 5114836333.

5. 6457434373.

6. 515.
7. 599100.
8. £919760700.
9. \$519949564.88.
10. £87197090.
11. 168 in.
12. \$178586.

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13. 513281.
14. 50291783.
15. 3501409.

Page 15.

16. \$3361127356.

Page 18.

Art. 80.

1. 613.
2. 1609.
3. 2022.
4. 13890.
5. 50000.
6. 64365.
7. 151223.
8. 57006.
9. 1407503.
10. 213305.
11. 449.
12. 30889825.
13. 790000.
14. 500.

Page 19.

15. 3175 bushels.
16. 139886 feet.
17. 1594.
18. 5 and 38576 rem.
19. 929496.
20. 1984 dollars.
21. 12960 acres.
22. \$53440.
23. \$6250.
24. \$140.
25. 708 miles.

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Art. 91.

1. 126.
2. 124.
3. 54.
4. 300.
5. 204.
6. 450.
7. 182.
8. 87.
9. 114.
10. 475.
11. 408.
12. 4088.
13. 750.
14. 680.
15. 1248.
16. 693.
17. 1197.
18. 832.
19. 3330.
20. 712.
21. 1440.
22. 572.
23. 585.
24. 3015.
25. 1300.
26. 6987.
27. 11184.
28. 817.
29. 2553.
30. 4554.
31. 7735.
32. 1540.
33. 2250.
34. 3450.
35. 1298.
36. 13590.
37. 8550.
38. 5250.
39. 4500.
40. 8679.

Art. 93.

1. 1608.
2. 2535.
3. 12012.

4. 1866.
5. 7245.
6. 9624.
7. 2650.
8. 23188.
9. 7665.
10. 3516.
11. 23413.
12. 9576.
13. 12976.
14. 14427.
15. 42084.
16. 1743.
17. 4152.
18. 12342.
19. 8333.
20. 35610.
21. 7872.
22. 27120.
23. 6454.
24. 53130.

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Art. 94.

1. 19512.
2. 496736.
3. 7188.
4. 28210.
5. 559790.
6. 6410556.
7. 17180824.
8. 229291455.
9. 6605212120.
10. 89336820048.
11. 1486262400360.
12. 2651761850220.
13. 463437665439.
14. 10768229616048.
15. 321453090615.

Page 24.

Art. 99.

1. 615.
2. 357.
3. 2664.
4. 41652.
5. 90855.

6. 8352192.
7. 7809840.
8. 7809840.
9. 7809840.
10. 7809840.
11. 7809840.
12. 7809840.
13. 5184.
14. 5184.
15. 5184.

Page 25.**Art. 100.**

1. 10017000.
2. 18941400.
3. 106326.
4. 257322000.
5. 41325000.
6. 252000.
7. 864450.
8. 46232352.
9. 145152.
10. 109515.
11. 1305 cents.
12. 990 dollars.
13. 3030 dollars.
14. 2700 dollars.
15. 57708.
16. 238800800.
17. 2285 dollars.
18. 15378 dollars.
19. \$645 gained.
20. 3720 dollars.

Page 26.**Art. 101.**

1. \$385560000.
2. \$189739175.
3. \$123224.
4. \$15147.50.
5. \$116816.
6. 26784 ft.
7. 6717 ϕ .
8. \$200.
9. 1689 ϕ .
10. 3361 ϕ .
11. \$844.
12. 92700 pairs.

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13. 358302 ϕ .
14. 2915 lb.
15. 7170 ϕ .

16. \$8158 gained.
17. \$675.
18. \$2230.
19. \$1899600.
20. \$1649 gain.

Page 29.**Art. 112.**

1. 8, 4, 2.
2. 10, 5, 4, 2.
3. 14, 7, 28, 8, 4.
4. 18, 30, 15, 6, 10.
5. 5, 3, 9, 15.
6. 9, 2, 3, 18, 4.
7. 12, 36, 6, 3.
8. 12, 21, 42, 7, 4.
9. 20, 4, 50, 25, 10.
10. 4, 12, 6, 2, 3.
11. 25, 5.
12. 12, 4, 16, 8, 24.
13. 8, 16, 2, 32, 4.
14. 6, 40, 15, 24, 10.
15. 20, 5, 8, 4, 10.
16. 12, 18, 24, 36, 48, 6.
17. 5, 25, 35.
18. 16, 12, 3, 8, 6.
19. 36, 54, 12, 18, 9, 4.
20. 40, 20, 10, 25, 50.

Page 30.**Art. 117.**

1. 823.
2. 315.
3. 281.
4. 529.
5. 3945 $\frac{3}{4}$.
6. 7644 $\frac{1}{2}$.
7. 1721 $\frac{1}{2}$.
8. 13023 $\frac{5}{11}$.
9. 62341 $\frac{1}{8}$.
10. 417230 $\frac{1}{11}$.
11. 12870.
12. 10880 $\frac{3}{16}$.
13. 2675 $\frac{3}{8}$.
14. 4637.
15. 475.
16. 18461 $\frac{7}{16}$.
17. 1361080 $\frac{1}{16}$.
18. 56026 $\frac{1}{17}$.
19. 11137 $\frac{2}{13}$.

20. 111930 $\frac{4}{10}$.
21. 706369 $\frac{4}{12}$.

Page 31.**Art. 119.**

- | | |
|---------|---------|
| 1. 3. | 7. 217. |
| 2. 20. | 8. 217. |
| 3. 11. | 9. 217. |
| 4. 217. | 10. 45. |
| 5. 217. | 11. 45. |
| 6. 217. | 12. 45. |

Page 32.**Art. 121.**

1. 442 $\frac{1}{2}$.
2. 192 $\frac{1}{2}$.
3. 264 $\frac{1}{2}$.
4. 34 $\frac{10}{105}$.
5. 179 $\frac{31}{125}$.
6. 371 $\frac{55}{144}$.
7. 371 $\frac{55}{144}$.
8. 371 $\frac{55}{144}$.
9. 371 $\frac{55}{144}$.
10. 371 $\frac{55}{144}$.
11. 371 $\frac{55}{144}$.
12. 5173 $\frac{3}{4}$.
13. 831 $\frac{887}{4000}$.
14. 110057 $\frac{1}{2}$.
15. 796 $\frac{7}{24}$.

Page 33.**Art. 124.**

1. 1232 $\frac{2}{10}$.
2. 3264 $\frac{3}{1000}$.
3. 5184 $\frac{3}{1000}$.
4. 60 $\frac{404}{16000}$.
5. 1384 $\frac{51}{1000}$.
6. 4622 $\frac{550}{10000}$.
7. 314 $\frac{75}{1000}$.
8. 66424 $\frac{6}{110000}$.
9. 422 $\frac{281}{1200000}$.
10. 15 $\frac{5274}{110000}$.
11. 21579011 $\frac{1}{1000000}$.
12. 582521467 $\frac{1}{1000000}$.

Art. 125.

1. \$103055.
2. 16000 acres.
3. 621 acres.
4. \$3.
5. 4 $\frac{84}{144}$ miles, and \$1381 $\frac{23}{144}$.
6. 36666667.
7. 15353637-6 rem.

8. 13.
9. 6.
10. 715 acres.

Page 34.

11. 211 $\frac{15}{36}$.

Page 35.**Art. 127.**

1. 36.
2. 15.
3. 5.
4. 12.
5. 501 $\frac{15}{8}$.
6. 3796 $\frac{7}{25}$.
7. 481 $\frac{38}{100}$.
8. 123 $\frac{55}{100}$.
9. 315 $\frac{23}{100}$.
10. 28231 $\frac{1}{11}$.
11. 746 $\frac{50}{156}$.
12. 117 $\frac{48}{251}$.
13. 310 $\frac{204}{3144}$.
14. 61286 $\frac{857}{1535}$.
15. 2650951149 $\frac{1}{1888}$.
16. 396042289.
17. 11471721 $\frac{1}{509}$.
18. 7837 $\frac{3503}{11181}$.
19. 247 $\frac{5557}{30006}$.
20. 1000 $\frac{72109}{1000000}$.
21. 180531 $\frac{617}{20313}$.
22. 29438576 $\frac{1}{999125}$.
23. 18555 $\frac{345}{10000}$.
24. 50679701283 $\frac{3}{1778}$.
25. 160212125 $\frac{6}{156}$.
26. 10000 $\frac{50344}{100000000}$.
27. 49986 $\frac{4553}{22553}$.
28. 300557 $\frac{13278}{870084}$.
29. 800044 $\frac{80}{12345}$.
30. 250673 $\frac{21823}{1000000}$.

Art. 128.

1. 2305111 $\frac{1}{1000000}$.
2. \$7605 $\frac{2061}{2511}$.
3. 4025.
4. 54141700.
5. \$15416 $\frac{329}{7230}$.
6. 18 $\frac{4}{10}$ lb.
7. \$641025 $\frac{74}{117}$ and \$534181 $\frac{13}{140}$.
8. 14 $\frac{230}{100}$ miles.
9. 348409 $\frac{40}{100}$ lb. copper, and 104 070 $\frac{40}{100}$ lb. tin.
10. 173 $\frac{50}{80}$ miles.
11. 1720 bbl.

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12. $164\frac{247}{385}$ years.
 13. $\$2906070732\frac{2}{17}$
 and $\$14378213\frac{1}{8}$.
 14. $257\frac{130}{132}$ men.
 15. $4\frac{161}{111}\frac{230}{1132}$.
 16. $4\frac{5213}{12240}$ miles.
 17. $\$115009119\frac{37}{12}$ and
 $\$544888262\frac{1}{12}$.
 18. $34\frac{4}{11}$.
 19. $5\frac{77240}{1182417}$.
 20. $86781\frac{9883}{22223}$.
 21. $6\frac{88876371}{120083504}$.
 $3\frac{20238698}{86322898}$.
 22. $4\frac{64572411}{119444598}$.

Page 37.

Art. 131.

1. 46.
 2. $82\frac{2}{3}$.
 3. $69\frac{2}{3}$.
 4. $48\frac{2}{3}$.
 5. 47 miles.
 6. $\$137641\frac{1}{11}$.
 7. 76ϕ .
 Art. 133.
 1. 64.
 2. 29.
 3. 885.
 4. 296.
 5. 19.
 6. 742.
 7. 8751.
 8. 8906.
 9. 71237.
 10. 17959.

Page 39.

Art. 139.

1. 3, 3, and 3.
 2. 3, 3, and 13.
 3. 3, 5, and 11.
 4. 3, and 31.
 5. 2, 2, 2, 3, 3, 3,
 and 11.
 6. 2, 3, 5, 5, and 7.
 7. 2, 2, 2, 2, 3, and 3.
 8. 5, 5, 5, 5, 5, and 5.
 9. 2, 2, 2, 2, 2, 3,
 3, 3, and 13.
 10. 2, 3, 5, 7, 13, 17,
 and 19.

Page 40.

Art. 146.

1. 11. 7. 14.
 2. 12. 8. 50.
 3. 16. 9. 151.
 4. 18. 10. 63.
 5. 52. 11. 70.
 6. 45. 12. 25.

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Art. 148.

1. 4. 7. 7.
 2. 24. 8. 23.
 3. 2. 9. 21.
 4. 17. 10. 131.
 5. 51. 11. 1.
 6. 4. 12. 25.

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Art. 155.

1. 480.
 2. 450.
 3. 1872.
 4. 840.
 5. 840.
 6. 9504.
 7. 7920.
 8. 840.
 9. 2520.

Page 44.

Art. 158.

1. $64\frac{2}{3}$.
 2. $\frac{29}{17}$.
 3. $\frac{181}{220}$.
 4. 120.
 5. $97\frac{1}{2}$.
 6. 90.
 7. $\frac{2064}{137149}$.
 8. $33\frac{1}{15}$.
 9. 1080.
 10. $86\frac{2}{3}$.
 11. 45 bu.
 12. 5 bbl.
 13. 33 bales and 528
 yards.
 14. 720 yd.
 15. $71\frac{1}{2}$.
 16. $5\frac{1}{4}$ pieces.
 17. 44.
 18. 14.
 19. $41\frac{7}{16}$ bbl.
 20. $2\frac{2}{3}$ sections.

Page 46.

Art. 172.

1. $\frac{20}{4}$. 6. $\frac{150}{10}$.
 2. $\frac{23}{9}$. 7. $\frac{70}{5}$.
 3. $\frac{12}{13}$. 8. $\frac{297}{11}$.
 4. $\frac{24}{8}$. 9. $\frac{9543}{23}$.
 5. $\frac{26}{12}$. 10. $\frac{64125}{67}$.

Art. 174.

1. $\frac{7}{2}$. 6. $\frac{215}{4}$.
 2. $\frac{23}{5}$. 7. $\frac{863}{85}$.
 3. $\frac{24}{5}$. 8. $\frac{1701}{10}$.
 4. $\frac{206}{7}$. 9. $\frac{20801}{20}$.
 5. $\frac{193}{12}$. 10. $\frac{14527}{15}$.

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Art. 176.

1. $4\frac{1}{2}$. 6. $2\frac{5}{11}$.
 2. $6\frac{5}{8}$. 7. $23\frac{21}{25}$.
 3. $2\frac{7}{8}$. 8. $1\frac{8}{39}$.
 4. $8\frac{1}{11}$. 9. $33\frac{113}{155}$.
 5. $8\frac{1}{11}$. 10. $14\frac{489}{775}$.

Art. 178.

1. $\frac{3}{8}$. 6. $\frac{25}{8}$.
 2. $\frac{3}{8}$. 7. $\frac{85}{72}$.
 3. $\frac{5}{12}$. 8. $\frac{11}{12}$.
 4. $\frac{17}{12}$. 9. $\frac{5}{6}$.
 5. $\frac{13}{12}$. 10. $\frac{7}{6}$.

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Art. 180.

1. $\frac{10}{12}$. 6. $\frac{23}{8}$.
 2. $\frac{15}{12}$. 7. $\frac{10}{116}$.
 3. $\frac{12}{12}$. 8. $\frac{34}{128}$.
 4. $\frac{23}{12}$. 9. $\frac{10}{19}$.
 5. $\frac{8}{12}$. 10. $\frac{7}{147}$.

Art. 182.

1. $\frac{810}{1080}$, $\frac{180}{1080}$, $\frac{600}{1080}$,
 and $\frac{432}{1080}$.
 2. $\frac{12110}{24570}$, $\frac{1388}{24570}$,
 $\frac{12656}{24570}$, $\frac{10530}{24570}$,
 $\frac{12285}{24570}$, and
 $\frac{24120}{24570}$.
 3. $\frac{8840}{22080}$, $\frac{13800}{22080}$,
 $\frac{5520}{22080}$, $\frac{44160}{22080}$,
 $\frac{23052}{22080}$, $\frac{78080}{22080}$,
 and $\frac{13248}{22080}$.
 4. $\frac{246}{33}$, $\frac{160}{33}$, $\frac{4}{33}$, $\frac{23}{33}$,
 $\frac{16}{33}$, $\frac{72}{33}$, and $\frac{24}{33}$.
 5. $\frac{13}{20}$, $\frac{115}{20}$, $\frac{14}{20}$, $\frac{765}{20}$,
 and $\frac{240}{20}$.

6. $\frac{237}{189}$, $\frac{266}{189}$, $\frac{2628}{189}$,
 $\frac{1184}{189}$, and $\frac{2205}{189}$.
 7. $\frac{2375}{2000}$, $\frac{5000}{2000}$, $\frac{7500}{2000}$,
 $\frac{8400}{2000}$, $\frac{5400}{2000}$, and
 $\frac{2250}{2000}$.
 8. $\frac{1800}{720}$, $\frac{5580}{720}$, $\frac{8370}{720}$,
 $\frac{16945}{720}$, $\frac{288}{720}$, $\frac{640}{720}$,
 $\frac{3600}{720}$, and $\frac{720}{720}$.
 9. $\frac{1442000}{945000}$, $\frac{708780}{945000}$,
 $\frac{6881500}{945000}$, and
 $\frac{945000}{945000}$.
 $\frac{7560000}{945000}$, $\frac{1575000}{945000}$,
 $\frac{13635000}{945000}$, and
 $\frac{732000}{945000}$.
 10. $\frac{54000}{74250}$, $\frac{58400}{74250}$,
 $\frac{47025}{74250}$, $\frac{35145}{74250}$,
 $\frac{8800}{74250}$, $\frac{24000}{74250}$,
 and $\frac{1455000}{74250}$.

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Art. 184.

1. $\frac{48}{120}$, $\frac{100}{120}$, $\frac{15}{120}$,
 and $\frac{10}{120}$.
 2. $\frac{56}{504}$, $\frac{288}{504}$, $\frac{315}{504}$,
 $\frac{252}{504}$, and $\frac{84}{504}$.
 3. $\frac{425}{1980}$, $\frac{360}{1980}$, $\frac{18240}{1980}$,
 $\frac{880}{1980}$, $\frac{4850}{1980}$, and
 $\frac{1825}{1980}$.
 4. $\frac{1080}{1560}$, $\frac{1248}{1560}$, $\frac{1300}{1560}$,
 $\frac{1365}{1560}$, $\frac{3120}{1560}$, $\frac{8820}{1560}$,
 and $\frac{1560}{1560}$.
 5. $\frac{12}{60}$, $\frac{10}{60}$, $\frac{6}{60}$, $\frac{5}{60}$, $\frac{20}{60}$,
 $\frac{240}{60}$, and $\frac{800}{60}$.
 6. $\frac{24}{90}$, $\frac{40}{90}$, $\frac{990}{90}$, $\frac{990}{90}$,
 $\frac{165}{90}$, $\frac{810}{90}$, and
 $\frac{90}{90}$.
 7. $\frac{4230}{180}$, $\frac{2592}{180}$, $\frac{1422}{180}$,
 $\frac{1035}{180}$, and $\frac{140}{180}$.
 8. $\frac{1428}{84}$, $\frac{204}{84}$, $\frac{1212}{84}$,
 $\frac{735}{84}$, $\frac{72}{84}$, and
 $\frac{420}{84}$.
 9. $\frac{1260}{2520}$, $\frac{1880}{2520}$, $\frac{1820}{2520}$,
 $\frac{2016}{2520}$, $\frac{2100}{2520}$,
 $\frac{2160}{2520}$, $\frac{2205}{2520}$,
 $\frac{2240}{2520}$, and $\frac{2250}{2520}$.
 10. $\frac{54}{180}$, $\frac{86}{180}$, $\frac{153}{180}$,
 $\frac{492}{180}$, $\frac{1440}{180}$, $\frac{16}{180}$,
 $\frac{180}{180}$, $\frac{2280}{180}$, and
 $\frac{48}{180}$.
 11. $\frac{1800}{1980}$, $\frac{1788}{1980}$, $\frac{1760}{1980}$,
 $\frac{27780}{1980}$, $\frac{1980}{1980}$,
 $\frac{1710}{1980}$, $\frac{72080}{1980}$,
 $\frac{2355}{1980}$, $\frac{10692}{1980}$,
 and $\frac{21780}{1980}$.

12. $\frac{855}{900}, \frac{1620}{900}, \frac{3060}{900},$
 $\frac{840}{900}, \frac{2700}{900}, \frac{2500}{900},$
 $\frac{3708}{900}, \frac{1800}{900}, \frac{820}{900},$

Art. 186.

1. 3. 6. $3\frac{1}{2}$.
 2. $\frac{1}{5}$. 7. $2\frac{1}{2}$.
 3. 4. 8. 4.
 4. $3\frac{1}{2}$. 9. $2\frac{1}{2}$.
 5. $4\frac{1}{7}$. 10. $2\frac{1}{2}$.

Page 50.**Art. 188.**

1. $4\frac{2}{3}$.
 2. $4\frac{7}{30}$.
 3. $3\frac{123}{250}$.
 4. $4\frac{1}{84}$.
 5. $3\frac{1}{2}$.
 6. $4\frac{13}{30}$.
 7. $4\frac{23}{84}$.
 8. $4\frac{171}{200}$.
 9. $3\frac{793}{250}$.
 10. $4\frac{97}{120}$.

Art. 190.

1. $21\frac{989}{2520}$.
 2. $16\frac{109}{250}$.
 3. $21\frac{139}{180}$.
 4. $58\frac{1}{15}$.
 5. $105\frac{59}{70}$.
 6. $329\frac{9}{16}$.
 7. $180\frac{7}{70}$.
 8. $715\frac{11}{60}$.
 9. $709\frac{59}{140}$.
 10. $108\frac{3}{4}$.

Page 51.**Art. 191.**

1. $2\frac{1}{15}$. 7. $4\frac{1}{3}$.
 2. $3\frac{1}{2}$. 8. $11\frac{1}{4}$.
 3. $2\frac{173}{210}$. 9. $4\frac{1}{2}$.
 4. $3\frac{101}{120}$. 10. $4\frac{2}{15}$.
 5. $2\frac{1}{2}$. 11. $3\frac{2}{3}$.
 6. $2\frac{8}{9}$. 12. $5\frac{1}{20}$.

Art. 192.

1. $1051\frac{459}{3080}$.
 2. $244\frac{9}{10}$.
 3. $6054\frac{432}{445}$.
 4. $1055\frac{17}{180}$.
 5. $421\frac{1}{2}$.
 6. $412\frac{1}{10}$.
 7. $100\frac{11}{32}$ acres.
 8. $1104\frac{251}{364}$.

9. $2644\frac{1}{16}$ lb.
 10. $5693\frac{1}{8}$ bu. and
 $\$3341\frac{9}{40}$.

Page 52.**Art. 194.**

1. $\frac{3}{5}$. 9. $\frac{3}{7}$.
 2. $\frac{1}{3}$. 10. $2\frac{1}{11}$.
 3. $1\frac{2}{11}$. 11. $\frac{1}{15}$.
 4. $\frac{2}{3}$. 12. $\frac{1}{19}$.
 5. $\frac{3}{8}$. 13. $5\frac{13}{18}$.
 6. $\frac{2}{3}$. 14. $1\frac{1}{2}$.
 7. $1\frac{1}{2}$. 15. $\frac{4}{35}$.
 8. $\frac{1}{3}$.

Art. 195.

1. $\frac{7}{15}$. 9. $\frac{1}{3}$.
 2. $\frac{39}{211}$. 10. $\frac{19}{15}$.
 3. $\frac{1}{2}$. 11. $2\frac{1}{58}$.
 4. $\frac{193}{405}$. 12. $\frac{9}{16}$.
 5. $\frac{3}{10}$. 13. 4.
 6. $\frac{1}{15}$. 14. $\frac{1}{5}$.
 7. $\frac{1}{4}$. 15. $\frac{31}{144}$.
 8. $\frac{8}{5}$. 16. $2\frac{1}{10}$.

Art. 197.

1. $\frac{1}{6}$. 7. $1\frac{3}{24}$.
 2. $\frac{1}{6}$. 8. $\frac{1}{30}$.
 3. $\frac{1}{2}$. 9. $\frac{5}{24}$.
 4. $\frac{7}{18}$. 10. $\frac{3}{4}$.
 5. $\frac{7}{23}$. 11. $\frac{8}{21}$.
 6. $\frac{2}{23}$. 12. $\frac{3}{26}$.

Art. 198.

1. $\frac{1}{36}$. 9. $3\frac{3}{11}$.
 2. $\frac{9}{40}$. 10. $6\frac{2}{5}$.
 3. $\frac{1}{2}$. 11. $4\frac{1}{10}$.
 4. $\frac{7}{12}$. 12. $5\frac{7}{10}$.
 5. $\frac{7}{15}$. 13. $3\frac{1}{2}$.
 6. $\frac{1}{4}$. 14. $2\frac{10}{117}$.
 7. $2\frac{1}{12}$. 15. $\frac{1}{20}$.
 8. $\frac{11}{60}$. 16. $\frac{1}{24}$.

Page 53.**Art. 200.**

1. 4. 9. $9\frac{2}{3}$.
 2. $2\frac{1}{2}$. 10. $12\frac{1}{2}$.
 3. 9. 11. $9\frac{1}{2}$.
 4. $17\frac{1}{2}$. 12. $14\frac{13}{40}$.
 5. $2\frac{1}{2}$. 13. $5\frac{3}{8}$.
 6. $3\frac{3}{10}$. 14. $8\frac{1}{15}$.
 7. $7\frac{7}{18}$. 15. $99\frac{1}{30}$.
 8. $8\frac{1}{4}$. 16. $170\frac{1}{2}$.

Art. 201.

1. $2\frac{1}{8}$.

2. $12\frac{7}{10}$.
 3. $30\frac{1}{4}$.
 4. 20.
 5. $12\frac{1}{40}$.
 6. $36\frac{3}{4}$.
 7. $63\frac{63}{1800}$.
 8. $8\frac{2}{3}$.
 9. $150\frac{9}{10}$.
 10. $39\frac{1}{11}$.
 11. $198\frac{302}{421}$.
 12. $5\frac{53343}{1408}$.
 13. $59\frac{1}{2}$.
 14. $87\frac{5}{6}$.
 15. $969\frac{7}{8}$.
 16. $487\frac{1}{4}$.

Art. 202.

1. $\frac{3}{10}$.
 2. $\frac{91}{170}$.
 3. $1\frac{2}{5}$.
 4. $1\frac{1}{15}$.
 5. $6\frac{9}{15}$.
 6. $1\frac{1}{12}$.
 7. $12\frac{21}{220}$.
 8. $9\frac{7}{35}$.
 9. $5\frac{1}{2}$.
 10. $67\frac{403}{770}$.
 11. $4\frac{211}{800}$.
 12. $15\frac{2}{5}$.
 13. $1\frac{5}{12}$.
 14. $3\frac{3}{7}$.
 15. $3\frac{2}{3}$.
 16. $1\frac{1}{10}$.
 17. $6\frac{451}{1456}$.
 18. $64\frac{43}{143}$.
 19. $593\frac{3}{10}$.
 20. $49\frac{49}{120}$.
 21. $61\frac{1}{210}$.
 22. $2\frac{61}{120}$.
 23. $4\frac{1}{15}$ acres.
 24. $6\frac{2}{10}$ dollars.

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25. $\$235\frac{11}{20}$, gain.
 26. $660\frac{1321}{1980}$.
 27. $114\frac{201}{204}$.
 28. $6\frac{51}{80}$.
 29. $\$34\frac{1}{2}$.
 30. $\$12690\frac{17}{10}$.
 31. $\$373\frac{1}{2}$.

Page 55.**Art. 204.**

1. $1\frac{1}{2}$. 9. $4\frac{1}{2}$.

2. $\frac{8}{9}$. 10. $1\frac{3}{4}$.
 3. $2\frac{1}{2}$. 11. 2.
 4. $1\frac{1}{4}$. 12. 9.
 5. $5\frac{1}{11}$. 13. $3\frac{1}{2}$.
 6. 2. 14. 14.
 7. $3\frac{3}{8}$. 15. 18.
 8. 14.

Art. 205.

1. 25. 9. 344.
 2. $7\frac{5}{13}$. 10. 600.
 3. 44. 11. $9\frac{1}{2}$.
 4. 30. 12. $297\frac{1}{2}$.
 5. 124. 13. 265.
 6. $26\frac{5}{8}$. 14. 679.
 7. $462\frac{3}{8}$. 15. 7.
 8. $94\frac{1}{2}$. 16. 68.

Art. 207.

1. $3\frac{3}{4}$. 9. $3\frac{3}{8}$.
 2. $6\frac{1}{8}$. 10. $6\frac{1}{4}$.
 3. 10. 11. 12.
 4. $5\frac{1}{2}$. 12. $16\frac{1}{2}$.
 5. 35. 13. 10.
 6. 2. 14. 16.
 7. 15. 15. 36.
 8. 6.

Art. 208.

1. $46\frac{2}{7}$. 9. 63.
 2. 49. 10. 427.
 3. $33\frac{8}{9}$. 11. 45.
 4. 77. 12. 84.
 5. $49\frac{1}{5}$. 13. $65\frac{1}{12}$.
 6. $21\frac{1}{2}$. 14. $168\frac{9}{16}$.
 7. 152. 15. 6972.
 8. $3\frac{3}{4}$. 16. 448.

Page 56.**Art. 210.**

1. $\frac{4}{35}$. 9. $\frac{2}{5}$.
 2. $\frac{1}{4}$. 10. $\frac{5}{8}$.
 3. $\frac{5}{36}$. 11. $1\frac{1}{2}$.
 4. $\frac{35}{48}$. 12. $\frac{57}{180}$.
 5. $\frac{7}{12}$. 13. $1\frac{1}{4}$.
 6. $\frac{5}{12}$. 14. $\frac{150}{209}$.
 7. $2\frac{1}{16}$. 15. $\frac{1}{10}$.
 8. $\frac{1}{96}$. 16. $\frac{1}{4}$.

Art. 211.

1. $\$4\frac{1}{2}$. 4. $\frac{7}{8}$.
 2. $\frac{1}{2}$ A. 5. $\frac{6}{35}$.
 3. $\$1\frac{1}{2}$. 6. $\frac{1}{96}$.

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7. $11\frac{3}{4}$, and $8\frac{1}{4}$.

8. \$90, and \$60.
 9. \$1.
 10. \$1.
 11. \$1.
 12. \$1.
 13. \$1.
 14. $\frac{2}{3}$ gal.

Art. 212.

1. $\frac{11788}{44000}$.
 2. $617\frac{1}{27}$.
 3. $8058\frac{1}{27}$.
 4. $262790\frac{1578}{1000}$.
 5. $1340713\frac{1}{2}$.
 6. $10667478\frac{1}{125}$.
 7. $1\frac{8}{27}$.
 8. $80134846\frac{311}{10}$.
 9. $19786149735\frac{8}{7}$.
 10. $42702\frac{918}{11}$.
 11. $37\frac{1457}{4400}$.
 12. $\frac{7}{78}$.
 13. $116085\frac{5}{7}$.
 14. $\frac{7}{78}$.
 15. $\$87\frac{2}{3}$.
 16. $56\frac{2}{3}$ acres.
 17. $\$135\frac{1}{2}$.
 18. $159\frac{1}{2}$ bbl.
 19. $\$6\frac{9}{10}$.
 20. $\$35\frac{41}{40}$ gain.

Page 58.

Art. 214.

1. $\frac{2}{7}$. 9. $\frac{1}{2}$.
 2. $\frac{1}{18}$. 10. $\frac{2}{25}$.
 3. $\frac{1}{11}$. 11. $\frac{1}{11}$.
 4. $\frac{2}{21}$. 12. $\frac{1}{25}$.
 5. $\frac{1}{18}$. 13. $\frac{2}{25}$.
 6. $\frac{1}{21}$. 14. $\frac{2}{15}$ yd.
 7. $\frac{1}{11}$. 15. $\frac{7}{25}$.
 8. $\frac{1}{18}$.

Art. 215.

1. $\frac{8}{105}$. 9. $7\frac{88}{105}$.
 2. $\frac{29}{204}$. 10. $24\frac{111}{110}$.
 3. $\frac{103}{2180}$. 11. $3\frac{1}{2}$.
 4. $\frac{8}{441}$. 12. $4\frac{1}{2}$.
 5. $\frac{1}{11}$. 13. $34\frac{1}{11}$.
 6. $4\frac{1}{2}$. 14. $20\frac{5}{10}$.
 7. $3\frac{5}{7}$. 15. $14\frac{1}{10}$.
 8. $99\frac{8}{10}$. 16. $53\frac{1}{10}$.

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Art. 217.

1. $39\frac{1}{2}$. 11. $12\frac{1}{2}$.

2. $19\frac{1}{2}$. 12. $3\frac{1}{2}$.
 3. $21\frac{1}{2}$. 13. 24.
 4. $51\frac{1}{2}$. 14. $67\frac{1}{2}$.
 5. $888\frac{1}{2}$. 15. 81.
 6. 20. 16. 40.
 7. $22\frac{3}{4}$. 17. 300.
 8. $67\frac{8}{25}$. 18. $14\frac{2}{5}$.
 9. $53\frac{1}{11}$. 19. 36.
 10. $3\frac{2}{11}$. 20. 21.

Art. 218.

1. \$72.
 2. 5 shares.
 3. 40 families.
 4. 24 bu.
 5. 5 da.
 6. $\$11\frac{1}{10}$.
 7. $\frac{1}{2}$ A.
 8. $3089\frac{1}{3}$, or 3090 sacks.

Page 60.

Art. 220.

1. $\frac{8}{9}$. 9. $12\frac{2}{3}$.
 2. $2\frac{1}{2}$. 10. $\frac{8}{9}$.
 3. $1\frac{1}{2}$. 11. $\frac{8}{9}$.
 4. $\frac{4}{5}$. 12. $1\frac{1}{5}$.
 5. $1\frac{1}{10}$. 13. $1\frac{1}{10}$.
 6. $\frac{15}{16}$. 14. $\frac{8}{11}$.
 7. $\frac{2}{3}$. 15. $1\frac{2}{3}$.
 8. $\frac{2}{5}$. 16. $1\frac{2}{5}$.

Art. 221.

1. $1\frac{9}{25}$.
 2. $1\frac{1}{27}$.
 3. $\frac{7}{17}$.
 4. $\frac{4}{100}$.
 5. $1\frac{12}{100}$.
 6. $\frac{2}{100}$.
 7. $1\frac{18}{100}$.
 8. $\frac{1}{108}$.
 9. $\frac{8}{1000}$.
 10. $35\frac{5}{8}$.

11. $\frac{1}{18}$.
 12. $\frac{1}{18}$.
 13. $88\frac{1}{2}$.
 14. $1\frac{5}{11}$.
 15. $1\frac{1}{18}$.
 16. $2\frac{1}{2}$.
 17. 18 da.
 18. 13.
 19. $12\frac{1}{2}$ da.
 20. $1223\frac{1}{100}$ bu.
 21. $834\frac{1}{2}$ cords.
 22. $1841\frac{1}{2}$ bu.

23. $279\frac{1}{2}$ miles.

Page 61.

Art. 223.

1. $2\frac{2}{15}$.
 2. $7\frac{88}{100}$.
 3. $7\frac{11}{10}$.
 4. $62\frac{1}{4}$.
 5. \$14000.
 6. $\$6772\frac{1}{4}$.
 7. 3 bu.
 8. 2 lb.
 9. \$135000.
 10. $323\frac{21}{220}$, sulphur,
 $215\frac{47}{110}$ salt, and
 $1615\frac{1}{4}$ char.
 11. \$52500.
 12. \$80 watch, and
 \$35 chain.

Page 62.

13. \$5 $\frac{1}{2}$.
 14. 15 days.
 15. \$19000.
 16. \$67837 $\frac{1}{2}$.
 17. 51 + ϕ .
 18. \$2.
 19. \$248.70 $\frac{3}{4}$.
 20. $3\frac{1}{100}$ miles.
 21. \$5.
 22. \$13200.
 23. 336 trees.
 24. 100 lb.
 25. \$67 $\frac{1}{2}$.
 26. \$6, \$15 and \$16.
 27. 62 $\frac{1}{2}$ years.
 28. S \$1190, H \$476
 and R \$544.
 29. $90\frac{1}{2}$ years.

Page 63.

30. \$35 and \$40.
 31. 105.
 32. 6 $\frac{1}{2}$ bu.
 33. $22\frac{1}{2}$ da.
 34. J \$47 $\frac{1}{2}$, C \$33 $\frac{1}{2}$.
 35. 40 ft.
 36. 94 $\frac{1}{2}$ ft.
 37. Cow \$30, colt
 \$94.
 38. 2 P. M.
 39. 36 ft.
 40. 405..
 41. H \$216, C \$324.

42. $30\frac{1}{2}$ years.
 43. 6 $\frac{3}{4}$ da.
 44. $5\frac{10}{107}$ da.
 45. $42\frac{3}{4}$ da.
 46. $67\frac{1}{2}$ da.
 47. S \$180, B \$150.
 48. 150 ft.

Page 64.

49. $1\frac{1}{8}$.
 50. 176 rd.
 51. $14\frac{2}{5}$ min.
 52. Ben 7 ϕ , John 1 ϕ .
 53. 28 bu. and 80 bu.
 54. $5\frac{1}{11}$ min. past 1
 o'clock.
 55. $27\frac{1}{11}$ min. of 7
 o'clock.
 56. $10\frac{1}{11}$ min. of 10
 o'clock.
 57. $5\frac{1}{11}$ min. of 11
 o'clock.
 58. 60.
 59. 52 ϕ loss.
 60. \$1500, \$3000,
 \$4500, \$6000.
 61. $62\frac{2}{3}$ yards.
 62. A \$19.75, and B
 \$15.80.
 63. 15 hr.
 64. 4 min.
 65. 55 yr.
 66. A \$11 $\frac{8}{105}$, B
 \$14 $\frac{4}{105}$ and C
 \$10 $\frac{2}{105}$.

Page 65.

67. B $42\frac{3}{4}$, S $25\frac{1}{2}$; B
 \$16 $\frac{1}{2}$, S \$23 $\frac{1}{2}$.
 68. $11\frac{1}{11}$ da;
 C \$33 $\frac{1}{11}$,
 H \$25 $\frac{1}{11}$,
 T \$22 $\frac{1}{11}$,
 L \$19 $\frac{1}{11}$.
 69. $16\frac{1}{2}$ da.; A, \$44
 $\frac{20}{11}$; and B, \$30
 $\frac{13}{11}$.
 70. \$7350. A, \$2650;
 B, \$2700; C,
 \$2000.
 71. H, 216; M, $129\frac{1}{2}$;
 and B, $64\frac{1}{2}$.
 72. 81 d.
 73. $49\frac{1}{4}$.

74. $43\frac{1}{2}$.
 75. H, $\$33\frac{1}{2}$; M, $\$55\frac{5}{8}$,
 and B, $\$111\frac{1}{2}$.
 76. A, $135\frac{3}{8}$ da.; B,
 $169\frac{1}{2}$ da.; C,
 $188\frac{1}{8}$ da.; D,
 $67\frac{1}{2}$ da.
 77. A and B, $75\frac{1}{2}$ da.;
 A and C, $78\frac{3}{8}$
 da.; A and D,
 $45\frac{1}{2}$ da.
 78. A, B, and C,
 $53\frac{1}{2}$ da.; A, C,
 and D, $36\frac{1}{4}$
 da.; B, C, and
 D, $38\frac{3}{4}$ da.
 79. $26\frac{15}{11}$ da.
 80. A, $77\frac{8237}{2745}$;
 B, $58\frac{8123}{12769}$;
 C, $47\frac{12757}{12769}$;
 D, $115\frac{8065}{12769}$.

Page 69.**Art. 238.**

1. .026.
 2. .27.
 3. .0016.
 4. .04.
 5. .00022.
 6. 5.7.
 7. 83.0504.
 8. 710.00243.
 9. 500.05.
 10. 45.046.
 11. 1001.0100.
 12. 1890.090.
 13. 850.05.
 14. 1000.10.

Art. 239.

1. 11.107.
 2. 15.0014.
 3. .000726.
 4. 1106.0012.
 5. 1600.16.
 6. 10000000.000010.
 7. 300.65.

Page 70.

8. 25400.11.
 9. 21.0015015.
 10. .0000018018.
 11. .500.
 12. .00005.

13. .000900.
 14. .00000009.
 15. 54054054.005405-
 0054.
 16. 103.587.
 17. 640.64.
 18. 26.04002.
 19. 9019.029039.
 20. 7.7.
 21. 870.01.
 22. 479027004.00000-
 99004.
 23. 700000000000000-
 .0000000000007.
 24. 1100.0011.
 25. .000003001.
 26. .001003.
 27. .0100011.
 28. .00000605.
 29. 1890.0000000189-
 0.

Art. 240.

1. .25.
 2. .0106.
 3. .00256.
 4. 9.01.
 5. 1.476.
 6. .193024.
 7. .00504.
 8. 2146.9003.
 9. 56973.805.
 10. .1934675.
 11. 33254.81.
 12. .00001876.
 13. 10.007.
 14. .00097.
 15. 15.0015.
 16. .035700097.
 17. 219760.0801.
 18. .046700004.
 19. .00068001.
 20. 1101.10011

Page 72.**Art. 244.**

1. $\frac{3}{10}$.
 2. $\frac{63}{100}$.
 3. $\frac{21}{200}$.
 4. $\frac{22}{250}$.
 5. $\frac{27}{50}$.
 6. $\frac{561}{2500}$.
 7. $\frac{229}{2000}$.
 8. $\frac{19}{100}$.

9. $\frac{5313}{12500}$.
 10. $\frac{7207}{25000}$.
 11. $\frac{2001}{25000}$.
 12. $\frac{12021}{25000}$.
 13. $\frac{7163}{62500}$.
 14. $\frac{4922}{15625}$.
 15. $\frac{100000007}{50000000}$.
 16. $\frac{6142727}{625000000}$.
 17. $5\frac{4}{25}$.
 18. $13\frac{41}{200}$.
 19. $117\frac{301}{200}$.
 20. $3005\frac{629}{5000}$.
 21. $1600\frac{1}{250}$.
 22. 1000000.
 23. $1234500\frac{1}{100000000}$.
 24. $6540000\frac{2469}{20000000}$.
 25. $188900\frac{2667}{100000000}$.
 26. 59
 $\frac{59}{31250000}$.

Art. 246.

1. .0625.
 2. .65.
 3. .275.
 4. .09375.
 5. .1375.
 6. .52.
 7. .0525.
 8. .46875.
 9. .024.
 10. .0024.
 11. .9375.
 12. .015625.
 13. .0015625.
 14. .8875.
 15. .00024.
 16. .96875.
 17. .984375.
 18. .028.
 19. .308.
 20. .95.
 21. .94.
 22. .226.
 23. .034375.
 24. .76.
 25. .015625.

Page 73.**Art. 249.**

1. $\frac{3}{5}$.
 2. $\frac{7}{5}$.
 3. $\frac{23}{55}$.

4. $\frac{105}{25}$ or $\frac{21}{5}$.
 5. $\frac{613}{500}$.
 6. $\frac{2011}{9999}$.

Art. 251.

1. 129.341.
 2. 848.1816.
 3. 1652.461772.
 4. 12638.517762.
 5. 2602.55141194.
 6. 8688.0148502.
 7. 24018.46093544.

Page 74.

8. 13444.61870921.
 9. 1004219.317454.
 10. 57597.358230005.
 11. 7003122.0011890-
 9997.
 12. 1627.325 A.
 13. 1017.84375 A.
 14. 1798.9425 bu.
 15. 395.8125 yd.
 16. 376.
 17. 926°.

Art. 253.

1. .811357+.
 2. 2.23985+.
 3. 1.7912
 4. 1.907703945.
 5. 1384.4959234662
 2.
 6. 5569.3518126587.
 42.

Page 75.**Art. 255.**

1. .412.
 2. .52977.
 3. .6863.
 4. 5.5264.
 5. 1.545648.
 6. 54.2294.
 7. 754.6005.
 8. 10000.0999.
 9. .3148.
 10. 213.889625.
 11. .810.
 12. 135.25746.

Page 76.**Art. 257.**

1. .546.

2. .01968.
3. 1.26875.
4. 39.9024.
5. 23469.986904.
6. 4625520.705.
7. 1.
8. 9.
9. .625000.
10. .87600.
11. 7231.98325125.
12. 49.
13. .1.
14. 275400116.25610-
02754.
15. \$20217.72.
16. \$536.88.
17. \$937.04.
18. \$336.33.

Page 77.

Art. 260.

1. .25.
2. 305.
3. 250.
4. .5.
5. .05.
6. 50.
7. 500.
8. 4000.
9. 2000.
10. .002.
11. .000025.
12. 183100.
13. 5.5875.
14. 5252000.
15. 50.
16. .00007.
17. .001.
18. 4000.
19. 1000000000.
20. .000001.
21. 25.
22. 4000000000.
23. .15.
24. 60000000.
25. 1.
10.
100.
100.
1000.
.1.
1.
10.
100.
.01.

26. 1.
.01.
.0001.
.00001.
.001.
10000000.
100000.
.00000001.
.0001.
1000000000.
27. .02.
200.
.02.
.0002.
.000000002.
200000.
200000000.
.00000002.
.0000000000002.
100000000000.

Page 78.

28. 6400646464647.0-
4064640000064.
29. 250252502527.75-
05000000025.
30. 40000400000.044-
800440044.
31. 400044440440.00-
0000440004.
32. 30030330000.003-
60306.
33. 150001650151.80-
16500000015.

Page 80.

Art. 266.

1. 1991.1198244.
2. .61032010.
3. .1625.
4. $\frac{.001}{.000000}$.
5. .0038462.
6. 2116.99454.
7. 187.2996.
8. \$7.766.
9. 41.299781.
10. 7.029956572371-
000.
11. 999999999.99999-
9998.
12. .90.
13. 572501.2525.
14. \$1100.869.

15. 12711.875¢.
16. 10.625.
17. 1011.6.
18. 6.875 da.
19. \$274.58.
20. \$24.9331+.

Page 84.

Art. 287.

1. 600¢.
2. 11100¢.
3. \$2.41.
4. \$10.44.
5. \$214.68.
6. \$18.
7. \$510.
8. \$98.76.
9. 10098¢.
10. 375¢.
11. 2653¢.
12. 15732¢.

Art. 289.

1. \$88.84.

Page 85.

2. \$144.89.
3. \$4337.77.
4. \$221.53.
5. \$378.07.
6. \$377.29.
7. 384.⁵¹.
8. 1088⁶⁰.
9. 4073⁷⁶.
10. 3065⁶⁵.
11. 896⁸⁶.
12. 2094.15.

Art. 291.

1. \$8179.88.

Page 86.

2. \$4859.76.
 3. \$5669.60.
 4. \$29.52.
 5. \$839.26.
 6. \$33.73.
- Art. 293.
1. \$2284.35.
 2. \$1315.63.
 3. \$7.13.
 4. 950 bu.
 5. \$344.73 gain.
 6. \$91.10.

Page 88.

Art. 294.

7. 26 and 39.
8. 16 $\frac{1}{11}$ min. after
3 o'clock.
9. Midnight.
10. B. 12, A. 26,
C. 78.
11. B. 84¢ and O.
45¢.
12. 1900.

Page 93.

Art. 304.

1. \$143.
2. \$384.
3. \$65.875.
4. \$113.75.
5. \$291.83 $\frac{1}{4}$.
6. \$133.31 $\frac{1}{4}$.
7. \$247.60.
8. \$63.05.
9. \$1.21 $\frac{1}{2}$.
10. \$3.70.
11. \$67.71 $\frac{1}{4}$.
12. \$10.50.
13. \$841.15.
14. \$1156.97.

Page 94.

Art. 306.

1. \$75.
2. \$358.75.
3. \$22.13.
4. \$53.50.
5. \$1125.
6. \$612.
7. \$3281.25.
8. \$382.
9. \$24.06.
10. \$45.56.
11. \$34.
12. \$1567.50.
13. \$187.50.
14. \$281.25.
15. \$125.25
16. \$370.
17. \$2750.
18. \$46.25
19. \$50.25
20. \$86.25.
21. \$156.25.

Art. 307.

1. \$2507.96.

Page 95.

2. \$3324.46.
3. \$4321.26.
4. \$5282.20.
5. \$8096.48.
6. \$3364.72.

Page 96.**Art. 309.**

1. 23 lb.
2. 4627.5 yd.
3. 6 lb.
4. 763.2 yd.
5. 876.5 doz.
6. 371 lb.
7. 81.5 yd.
8. 115.2 acres.
9. 689 yd.
10. 123 lb.
11. \$40.96.

Page 97.**Art. 311.**

1. \$21.91.
2. \$92.68.
3. \$70.07.
4. \$28.35.
5. \$148.28.
6. \$31.68.
7. \$25.31.
8. \$89.10.
9. \$63.63.
10. \$35.65.

Art. 313.

1. \$24.75.
2. \$14.66.
3. \$158.76.
4. \$123.18.
5. \$53.
6. \$38.75.
7. \$19.80.
8. \$100.32.
9. \$140.81.
10. \$34.78.
11. \$68296.35.
12. \$2819.31.

Page 98.**Art. 315.**

1. \$4.02.

2. \$4.71.
3. \$78.70.
4. \$13.84.
5. \$2014.46.
6. \$2016.46.
7. \$941.63.
8. \$21.06.
9. \$2.77.
10. \$19.51.
11. \$296.24.
12. \$15.77.
13. \$56.72.
14. \$66162.39.

Page 99.**Art. 317.**

1. \$28.56.
2. \$28.08.
3. \$52.83.
4. \$39.29.
5. \$24.66.
6. \$27.15.
7. \$55.67.
8. \$58.33.
9. \$174.78.
10. \$132.36.
11. \$121.68.
12. \$242.79.
13. \$333.31.
14. \$109.57.
15. \$198.91.
16. \$28.16.
17. \$11.57.
18. \$12.06.
19. \$60.89.
20. \$30.14.
21. \$15.62.
22. \$1941.92.

Page 101.**Art. 326.**

1. \$9.03.

Page 102.

2. \$49954.08.
3. \$191.47.

Page 103.

4. \$1812.31.

Art. 327.

1. \$46731.58.

Page 104.

2. \$967.31.

3. \$66.21.

Page 105.**Art. 328.**

1. \$943.54.
2. \$57269.94.

Page 106.**Art. 329.**

1. \$115.68.
2. \$560.50.
3. \$1528.75.
4. \$190.33.
5. \$272.35.
6. \$429.14.

Page 107.

7. \$524.03.
8. \$1718.01.
9. \$168.68.
10. \$322.29.

Page 112.**Art. 355.**

1. 450 min. 51 sec.
2. 23 hr. 5 min. 29 sec.
3. 7 da. 1 hr. 30 min. 51 sec.
4. 24 yr. 1 mo. 5 da. 6 min.
5. 6332½ hr.
6. Jan. 14, 1889.
7. 3405 da.
8. 1169 da.
9. 11 mo. 9 da.
10. 1 yr. 10 mo. 19 da. 19½ hr.

Page 113.

11. 1440 min.
12. 939613.7 sec.

Art. 358.

1. 35° 54'.
2. 24° 16' 46".
3. 3° S. 1° 49' 41".
4. 1296000.
5. 4907'.
6. 205737.
7. 136° 2'.
8. 21600'.
9. 8° 39'.

Page 115.**Art. 363.**

1. 2 hr. 30 min. 24 sec.
2. 41 min. 40 sec.
3. 57 min. 44 sec. after 3 a. m.
4. 1 hr. 52 min. 8 sec.
5. 16 min. past 8 p. m.

Art. 365.

1. 47° 59'.
2. 35° 13' E.
3. 74° 58'.

Page 116.

4. 36° 52' 20" N.
5. 77° 1'.

Art. 366.

1. 180°.
2. 180°.
3. 4 5 16 p. m.
4. 6 33 40 a. m. 6 min. of 1 p. m. 6 a. m. 10 9 20 p. m. 1 21 21½ p. m. 3 min. 48 sec. past 7 a. m.

Page 117.**Art. 371.**

1. £256 4 s.
2. 248 s. 3 d.
3. £54 6 s. 10 d.
4. £195 4 s. 4 d. ½ far.

Art. 373.

1. 6480 d.
2. 956 far.
3. 38853 d.
4. 39450 far.
5. 13206 far.

Page 118.**Art. 375.**

1. \$350.26.
2. \$525.80.
3. \$63544.40.

4. \$15.03.
5. \$54.29.

Art. 377.

1. £38 3 d. 2.4 far.
2. £63 7 s. 10 d.
3. £513 14 s. 3 d.
3 far.
4. £751 14 s. 3 d.
5. £22621 2 s. 8 d.
1 far.

Page 124.

Art. 393.

1. 7653 pwt.
2. 155948 gr.
3. 4 lb. 11 gr.
4. 5 lb. 3 oz. 2 pwt.
9 gr.
5. 432 gr.
6. $\frac{4}{8}$ pwt.
7. $\frac{3}{8}$ oz.
8. $\frac{11}{16}$ lb.
9. 1 oz. 13 pwt. 18 gr.
10. 12 pwt. 12 gr.
11. $\frac{27}{128}$ lb.
12. $\frac{3}{8}$ lb.
13. 7 oz. 14 pwt.
4.8 gr.
14. 18 pwt. 2.4 gr.
15. .297916 + lb.
16. .875 oz.
17. 472 lb. 1 oz. 12 pwt. 8 gr.

Page 125.

18. 211 lb. 11 oz. 19 pwt. 21 gr.
19. 2 pwt. 20.4 gr.
20. .0067 +.
21. 8 lb. 2 oz. 13 pwt. 6 gr.
22. 18 lb. 9 oz. 14 pwt. 2 gr.
23. \$8032.50.
24. \$1924.39.
25. 5 oz. 2 pwt. 17 gr.
26. 1 oz. 13 pwt. 8 gr.
27. \$11655.

28. 73 lb. 2 oz. 8 pwt. 19 gr.
29. \$154.22.
30. \$360.67 gain.

Page 127.

Art. 395.

1. 34669 lb.
2. 15 T. 12 cwt. 75 lb.
3. 12 cwt. 50 lb.
4. 56 lb. 4 oz.
5. 7 cwt. 68 lb. 6.4 oz.
6. 12 cwt 50 lb.
7. $\frac{7883}{8000}$ T.
8. $\frac{2407}{1000}$ cwt.
9. .24125 cwt.
10. .99996875 T.

Page 128.

11. 30 T. 1 cwt. 94 lb. 11 oz.
12. \$72.81.
13. 2 T. 5 cwt. 84 lb.

Art. 397.

1. 17 lb. 9 oz. 5 dr. 1 sc.
2. 5896 dr.
3. $\frac{379}{1152}$ lb.
4. 11 oz. 3 dr. 2 sc. .8 gr.
5. 63 sc.
6. 6 lb. 9 oz. 6 dr. 11.5 gr.
7. 1 lb. 2 oz. 4 dr. 1 sc. 4 gr.
8. 7 lb. 3 oz. 4 dr. 12 gr.
9. 6 lb. 2 oz. 6 dr. 1 sc. 8 gr.
10. 15 lb. 10 oz. 5 dr. 2 sc. 8 gr.
11. 11 oz. 2 sc. $4\frac{1}{2}$ gr.

Page 129.

Art. 398.

1. \$1477.27.
2. \$2255.25.
3. 6 lb. 10 oz. 15 gr.
4. 18 lb. 6.43 oz.

5. \$254.49.
6. \$5.04.

Art. 400.

1. 928 pt.
2. 599 pt.
3. 186 bu. 3 qt. 2 pt.
4. 7 qt. $1\frac{3}{4}$ pt.

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5. 5 bu. 1 pk. 1 qt. 1 pt.
6. 83 bu. 1 pk. 3 qt. $1\frac{1}{2}$ pt.
7. \$184.12.

Art. 402.

1. 5932 gi.
2. 31 bbl. 7 gal. 1 pt. 3 gi.
3. 651.168 gi.
4. 6 gal. 2 qt. 1.16 gi.
5. \$71.75.
6. 7 gal. 2 qt. 1 pt. $3\frac{3}{8}$ gi.
7. 13 gal. 1 pt. 1 gi.
8. \$19.16.
9. \$72.58, gain.

Page 131.

Art. 403.

1. 1579 $\frac{4}{7}$ pt.
2. 134.5 + pt. gain.
3. \$.52 gain.
4. \$5.25, gain.
5. \$7.99 less.

Page 132.

Art. 409.

1. 127002 in.
2. 39 mi. 155 rd. 4 yd. 3 in.
3. $\frac{1}{128728}$.
4. 213 rd. 1 yd. 2 ft. 6 in.
5. $\frac{87}{96}$ rd.
6. 173 rd. 2 yd. 1 ft. 312 in.
7. .892 +.
8. 123 mi. 162 rd. 3 yd. 1 ft. 4 in.
9. 1593 mi. 312 rd. 2 yd. 1 ft. 8 in.

10. 484 mi. 53 rd. 1 yd. 2 ft. 6 in.

Page 134.

Art. 424.

1. 35676648 sq. in.
2. 134498442 $\frac{3}{8}$ sq. ft.
3. 112 A. 40 sq. rd. 261 sq. ft. 51.84 sq. in.

Page 135.

4. 110 sq. rd.
5. $\frac{44800}{14400}$ sq. mi.
6. .9382 + A.
7. 101 sq. rd. 2 sq. yd. 21.6 sq. in.
8. 4 A. 83 sq. rd. 6 sq. yd. 64 $\frac{1}{2}$ sq. in.
9. 31 $\frac{1}{2}$ squares.
10. 63 yd.
11. 90 ft.
12. 230 ft.
13. 25 $\frac{3}{8}$ A.
14. \$107156.25.
15. \$3277.97.
16. 357 $\frac{1}{2}$ rd.
17. 414 $\frac{1}{2}$ ft.
18. 130 $\frac{1}{2}$ A.
19. Not any.
20. 2 sq. rd.
21. 28 $\frac{1}{2}$.
22. 4 $\frac{3}{8}$ A.
23. 58 $\frac{3}{8}$ rd.
24. 12.
25. 26 A. 11 sq. rd. 4 sq. yd. 5 sq. ft. 36 sq. in.
26. \$60.06.
27. \$10.56.
28. 9 A. 110 sq. rd. 12 sq. yd. 3 sq. ft. 54 sq. in.
29. 38 A. 59 sq. rd. 12 sq. yd. 5 sq. ft. 112 sq. in.
30. 640 rd.
31. 320 rd.
32. 60 yd.
33. \$40.30.
34. \$58594.44

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35. \$1884.04.
 36. \$15.63.
 37. 43 rolls.
 38. $128\frac{1}{2}$ yd., 130 yd., and \$327.25.
 39. $4861\frac{1}{2}$.
 40. 12 sq. ft.
 41. \$9.46.
 42. 28512.
 43. 52.177 + ft. and 104.354 + ft.
 44. 147840.
 45. \$74.36.
 46. \$204.73.

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Art. 430.

1. 1.
 2. 9.
 3. 16.
 4. 25.
 5. 81.
 6. 100.
 7. 9801.
 8. 65536.

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Art. 439.

1. 14.
 2. 15.
 3. 12.
 4. 24.
 5. 35.
 6. 75.
 7. 206.
 8. 11.2.
 9. 7.09.
 10. 21.954.
 11. 5.07.

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12. 10.3156.
 13. $\frac{2}{3}$.
 14. $\frac{5}{8}$.
 15. .968 +.
 16. .85 +.
 17. 5510.8 +.
 18. 68548.66 +.

Art. 446.

1. 10.
 2. 49.777.

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3. 108.25 +.
 4. 60.81 +.
 5. 56.796; 112.592.
 6. 208.71 + ft.
 7. 1866.76 + ft.
 8. 660 ft.
 9. 124.03 - ft.
 10. 77.88 + ft.
 11. 72.56 + ft.
 12. 452 rd., 8 ft., 10.92 + in.
 13. 226.42 + rd.
 14. 720 rd.
 15. 208.80 rd.

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Art. 448.

1. 211618.48 in.
 2. 1 mi. 68 ch. 1 rd. 16 l.
 3. 3 rd. 181.59.4 in.
 4. 7624 l.
 5. 7 ch. 14 l. 2.27 $\frac{2}{7}$ in.
 6. 8927 $\frac{3}{8}$ ft.
 7. 1924 $\frac{1}{2}$ rd.
 8. 16 ch. 91 l. 5.22 in.
 9. 2738 steps, 11 $\frac{3}{8}$ in. rem.

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Art. 459.

1. 10 cu. yd. 1533 cu. in.
 2. 6178581.
 3. 8 cu. yd. 972 cu. in.
 4. 7200 cu. ft.
 5. 508 $\frac{1}{2}$ cu. yd.
 6. 247 $\frac{3}{11}$ pch.
 7. $37\frac{5}{11}$ cu. yd.
 8. .2615 + cu. yd.
 9. $\frac{55}{132}$ cu. ft.
 10. 14 cu. ft., 302.4 cu. in.
 11. \$2320.76.
 12. \$2622.38.
 13. 642081.6 bricks.
 14. 47 $\frac{1}{2}$ cd.
 15. 39 $\frac{1}{4}$ cd.
 16. 26 ft. 9 $\frac{1}{8}$ in.
 17. \$227.11.
 18. 54468 $\frac{1}{2}$ lb.

19. 4531 $\frac{1}{2}$ lb.
 20. 46656.
 21. 10 cu. yd. 20 cu. ft. 1339 $\frac{1}{2}$ cu. in.
 22. 659.773 + pch.; 254436 bricks; \$2060.93

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23. 198 cu. ft.
 24. 27.52 + bu.

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Art. 474.

1. 12.
 2. 25.
 3. 48.
 4. 404.
 5. 12.898 +.
 6. 49.21 +.
 7. 36.1 +.
 8. $\frac{4}{5}$.
 9. $\frac{1}{2}$.
 10. $\frac{48}{101}$.
 11. .8 +.
 12. .92 +.
 13. 2.93 +.
 14. 15.177 +.
 15. .743 +.
 16. .97 +.

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10 ft. 9 + in.

17. 47 ft. 5 + in.
 19. 5 ft. 4 + in. wide; 10 ft. 9 + in. high, and 37 ft. 7 + in. long.
 20. 13 ft. 10 + in.
 21. 7 ft. 7 + in. deep, and 15 ft. 2 + in. square.

Page 153.

Art. 492.

1. 78 sq. ft.
 2. 85.45 + sq. yd.
 3. 32 $\frac{1}{2}$ A.
 4. 360 A.
 5. 259.182 ft.
 6. 472.68 + ft.
 7. 12.27 + A.
 8. 88.6 + ft.
 9. 29.7 + in square.

10. \$37.70.
 11. 26649.9 gal.
 12. 45 sq. ft.

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13. 26 $\frac{3}{8}$ sq. yd.
 14. 201.0624 sq. in.
 15. 33.5104 cu. ft.
 16. 1262.3 +.
 17. 4561.6 mi.
 18. 117.6264 gal.
 19. 22.3074 gal.

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Art. 502.

1. £359, 12 s. 1.8 + far.
 2. Jan. 3, 1870.
 3. 420 sq. ft.
 4. 2.295 + A.
 5. 122.18 + pch.
 6. 17280 shingle
 7. 27 $\frac{1}{11}$ rd.
 8. \$65.66 +.
 9. \$14.04.
 10. 117.77 + A.
 11. 220.85 qt.
 12. 17 da. 13 hr. 12 min.
 13. 143823.788 fr.
 14. 1424 $\frac{1}{2}$ fr.
 15. 364764 $\frac{2}{3}$ bbl.
 16. 58 min., 39.2 + sec. after 10 o'clock A. M.
 17. \$16.56.
 18. 222.63 +.
 19. 24715.7 +.
 20. 15 yr. 80 da. 5 hr. 33 min. 20 sec.
 21. \$11.20.

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22. 65 $\frac{1}{2}$ yd.
 23. 28 T. 8.9 + cwt.
 24. 42° 30'.
 25. 229.2784 lb.
 26. 14 min. 20 sec. after 1 P. M. Aug. 19, 1909.
 27. 10716.38 + fr.
 28. \$145.10.
 29. 6289.308 + mar.
 30. 72 $\frac{3}{4}$ ft.

31. 20 A. 4 sq. rd.
31 sq. ft.
32. 7 Mm. 6 Km. 7
Hm. 2 Dm. 6
M. 8 dm. .438
cm.
33. Coat \$7.54, and
vest \$3.28.
34. 78 sq. rd., 106
sq. ft., 25.56
sq. in.
35. \$25.03 gain.
36. \$414.95.
37. 302.379 sq. ft.
38. \$4026.10.
39. \$446511.63.
40. 617 A., 120 sq. rd.
41. \$.31.
42. \$845.25.
43. 25.19 + ft.

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44. \$6189.51.
45. 161 $\frac{1}{2}$ ft.
46. \$188.67.
47. \$6431.23.
48. 64 rd. 9.6 + ft.
49. \$14.39.
50. 2.886 +.
51. 5 da.
52. 24 lb. 10 oz. 7
pwt., 2.95 gr.
53. 14 ft. 4 + in.
54. 24.33 da.
55. 2 mi. 108 rd. 6
ft.

Page 162.**Art. 523.**

1. 5.
2. 72.
3. 120.
4. 100.
5. 300.
6. $\frac{3}{10}$.
7. 360.
8. 36.
9. 340.

Art. 524.

1. 150 A.
2. 872 sheep.
3. \$28653.75.
4. 198.24 lb.
5. 2 lb. 10 pwt.

6. \$621.
7. \$86.79 gain.
8. \$14375 and \$8125.
9. \$32.50.
10. 725 $\frac{1}{2}$ bu.

Page 163.

11. \$17500.
12. \$682.50.
13. \$266666.66 $\frac{2}{3}$.
14. \$22720.50.
15. 38 $\frac{1}{2}$ A.
16. \$14.04.
17. \$36453.10.
18. \$42.50 gain.

Art. 526.

1. 14100.
2. 21500.
3. 10860.
4. 657.
5. 20000.
6. \$8480.

Page 164.

7. \$4000.
8. 250 A.
9. \$108.

Art. 527.

1. 400 bales.
2. \$740.88.
3. \$1475.
4. \$693.33 $\frac{1}{3}$.
5. \$40300.
6. \$538294.12.
7. \$90000.
8. 101040.
9. 70000.
10. \$180000.

Page 165.**Art. 529.**

1. 20%.
2. 25%.
3. 50%.
4. 12 $\frac{1}{2}$ %.
5. 33 $\frac{1}{3}$ %.
6. 12 $\frac{1}{2}$ %.
7. 25%.
8. 2000%.
9. 3000%.

Art. 530.

1. 25%.
2. 3 $\frac{1}{2}$ %.

3. 33 $\frac{1}{3}$ %.
4. 20%.
5. 62 $\frac{1}{2}$ %.
6. 53 $\frac{1}{3}$ %.
7. 57 $\frac{1}{4}$ %.
8. 362 $\frac{6}{37}$ %.
9. 33 $\frac{1}{3}$ %.
10. 56 $\frac{1}{4}$ %.
11. 25%.

Page 166.**Art. 532.**

1. 1.10, amt. per.
cent.
2. 1.75, amt. per.
cent.
3. 2.10, amt. per.
cent.
4. 1.16 $\frac{2}{3}$, amt. per.
cent.
5. 1.87 $\frac{1}{2}$, amt. per.
cent.

Art. 533.

1. 1.05 per cent.
2. 1.09 $\frac{1}{11}$ %.
3. 1.40%.

Art. 535.

1. .85, difference
per cent.
2. .62 $\frac{1}{2}$, difference
per cent.
3. .99 $\frac{1}{2}$, difference
per cent.
4. .96 $\frac{2}{3}$, difference
per cent.
5. .30, difference
per cent.

Art. 536.

1. 71 $\frac{1}{2}$ %.
2. 68 $\frac{1}{2}$ %.
3. .60, difference
per cent.

Page 167.**Art. 538.**

1. 1650.
2. 1695.
3. 462.
4. 1180.
5. 277.2.
6. 2580.
7. 840.

8. 637.
9. 450.

Art. 539.

1. \$14512.50.
2. \$456.
3. \$11200.
4. 816.
5. 3537.
6. \$886.50.

Art. 541.

1. 12.
2. 150.
3. 945.

Page 168.

4. 612.
5. 1200.
6. 500.
7. 5.
8. 567 ft.

Art. 542.

1. \$2843.75.
2. 581 $\frac{1}{2}$ A.
3. \$2053.13.
4. \$632.50.

Art. 544.

1. 600.
2. 400.
3. 300.

Page 169.

4. 100.
5. \$4000.

Art. 545.

1. \$1000.
2. \$4.51.
3. \$7950.
4. \$480.
5. \$106.
6. \$3200.
7. \$30000.
8. \$10000.
9. 500 pupils.
10. Invest. in farm,
\$280.

Page 170.**Art. 548.**

1. 64 $\frac{2}{3}$ $\frac{1}{4}$ %.
2. 68 lb.

3. 80%.
4. \$23333333.33 $\frac{1}{3}$.
5. 23 $\frac{1}{2}$, 42 $\frac{2}{3}$, and 33 $\frac{1}{3}$.
6. 77 yr.
7. \$108.
8. H.\$200, M.\$170, C. \$15.
9. \$1081.25.
10. 25 lb. warp, and 71 $\frac{1}{4}$ lb. rags.
11. \$160, and \$224.
12. \$113.78.
13. 1 $\frac{1}{8}$ % gain.

Page 171.

14. \$4387.50.
15. \$1045.45+.
16. 216%+.
17. 27 $\frac{2}{3}$ %, 21 $\frac{2}{3}$ %, and 18 $\frac{2}{11}$ %.
18. 100 head.
19. 9 $\frac{1}{11}$ %.
20. 25%.
21. \$6344.40.
22. 18 $\frac{3}{4}$ %.
23. \$1000 loss.
24. 400.
25. 3000.
26. 77 $\frac{11}{13}$ %.
27. 10 yd.
28. \$4856.25.
29. \$60250.

Page 172.

30. Grazing, 504 A.; grain, 420 A.; timber, 936 A.
31. \$192 C.
32. A \$93840, and B \$69360.
33. \$22400.
34. Not any.
35. 7500, 9750, 6825, and 9555.
36. Clover, 450; timothy, 450; orchardgrass, 150 and 50 red top.
37. \$81.20, \$101.50, \$182.70.
38. 16 $\frac{2}{3}$ %.

39. \$22629.31.
40. \$1750, \$3062.50, \$6125, and \$8575.
41. 25600 T.
42. Wife, \$21750; D., \$10600; Y. S., \$12500; and E.S., \$13750.

Page 173.**Art. 557.**

1. \$9.
2. \$48.
3. \$750.

Page 174.

4. \$50.
5. \$32.
6. \$320.
7. \$225.
8. \$2100.
9. \$700.

Art. 558.

1. \$592.50 gain.
2. \$3997.50 loss.
3. \$677.25 loss.
4. \$9.47 gain.
5. \$184.92 gain.
6. \$6 gain.
7. \$10.16 loss.
8. \$22.97.
9. \$133.59.

Page 175.**Art. 560.**

1. \$100.
2. \$3500.
3. \$10000.
4. \$4400.
5. \$40.
6. \$300.
7. \$900.
8. \$1050.
9. \$1.

Art. 561.

1. \$7.50.
2. \$500 and \$625.
3. \$600.
4. \$700.
5. \$7085.71.
6. 200 A.

7. \$2750.
8. \$240.
9. \$5000.

Page 176.**Art. 563.**

1. 10%.
2. 5%.
3. 12 $\frac{1}{2}$ %.
4. 20%.
5. 50%.
6. 33 $\frac{1}{3}$ %.
7. 150%.
8. 9 $\frac{1}{2}$ %.
9. 20 $\frac{5}{8}$ %.

Art. 564.

1. 150%.
2. 25%.
3. Oats. 12 $\frac{1}{2}$ %.
4. 33 $\frac{1}{3}$ %.
5. 9% gain.
6. 77 $\frac{7}{8}$ % gain.

Page 177.

7. 20% profit.
8. 13 $\frac{2}{3}$ % gain.
9. 11 $\frac{1}{5}$ %.
10. 66 $\frac{2}{3}$ %.
11. 6 $\frac{1}{4}$ loss.
12. 50% gain.
13. 18%.
14. 1 $\frac{1}{4}$ %.
15. \$23163.12.

Art. 566.

1. \$100.
2. \$40.
3. \$15.
4. \$75.
5. \$1000.
6. \$25600.

Art. 567.

1. \$365.71.
2. \$442.50.
3. \$7500.

Page 178.

4. \$7501.
5. 1080 lb.
6. \$4000.

Art. 569.

1. \$153.60.
2. \$500.

3. 48 yr.
4. 75% nitre, 12 $\frac{1}{2}$ % sulphur, and 12 $\frac{1}{2}$ % charcoal.

Page 179.

5. 44.
6. \$11.25.
7. \$1260 and \$840.
8. \$2865.
9. \$2890.80.
10. 37 $\frac{1}{4}$ %.
11. \$800.
12. \$2.
13. \$3111.11.
14. 33 $\frac{1}{3}$ %.
15. 2200 bbl.
16. 46 $\frac{2}{3}$ %.
17. \$31 $\frac{1}{2}$.
18. No gain or loss.
19. 40%.
20. \$5.
21. 33 $\frac{1}{3}$ %.
22. \$74.25.
23. \$1972.50.
24. 100%.

Page 180.

25. \$9350.
26. 27¢.
27. \$200 and \$250.
28. 16 $\frac{2}{3}$ loss.
29. \$2700.
30. \$288.
31. \$10000.
32. \$4312.50 gain.
33. \$110.
34. 3 $\frac{1}{4}$ % loss.
35. \$16.80.
36. 94 $\frac{1}{2}$ %.
37. \$155.
38. \$4.
39. 64¢.
40. \$5.25 loss.
41. \$59320.

Page 181.

42. \$2.62 $\frac{1}{2}$, 6 $\frac{1}{4}$ %, and \$175 loss.
43. 25%.
44. 15¢.
45. \$420.
46. 18 $\frac{1}{3}$ % loss.
47. 83 $\frac{1}{3}$ %.

48. \$3.
 49. $68\frac{1}{2}\%$.
 50. \$5000 cost, and
 \$12441.60.
 51. \$450.
 52. \$6050.85.
 53. \$160.
 54. \$125.
 55. $1\frac{1}{10}\%$ loss.
 56. \$53.60.

Page 182.

57. 78.596 yd. \$19.40
 gain.
 58. $91\frac{1}{2}$ gal.
 59. \$1100 onions, and
 \$750 potatoes.
 60. 9¢.
 61. \$681.82 pear,
 and \$1071.43
 apple.
 62. \$2273.40 gain,
 and $121\frac{1}{11}\%$.
 63. \$454.59.
 64. \$4000.
 65. 100500 corn, and
 75375 wheat.
 66. C, \$27.18 $\frac{2}{3}$; H,
 \$326.25; and
 S, \$1.81+.

Page 184.**Art. 575.**

1. \$24.48.
 2. \$51.30.
 3. \$660.
 4. \$5670.
 5. \$19.13 gain.
 6. 200 yd.
 7. \$1515.64.
 8. A, \$72.50.

Page 185.**Art. 577.**

1. \$30.
 2. \$30.
 3. \$12571.43.
 4. \$9.
 5. \$60.
 6. 25%.
 7. $2\frac{1}{2}\%$.

Page 186.

8. \$1000.

Art. 579.

1. 19%.
 2. 39.4375%.
 3. $51\frac{14}{15}\%$.
 4. $66\frac{1}{2}\%$.
 5. $7\frac{1}{2}\%$.
 6. 37.791%.
 7. $1\frac{7}{10}\%$.

Page 188.**Art. 588.**

1. \$211.
 2. \$483.96.

Page 189.**Art. 590.**

1. \$36.09.
 2. \$368.48.

Page 190.**Art. 592.**

1. \$64.50.

Page 192.**Art. 606.**

1. \$180.
 2. \$75.
 3. \$250.
 4. \$275.
 5. \$277.38.
 6. \$375.90.

Page 193.**Art. 608.**

1. \$5000.
 2. \$24500.
 3. \$282.
 4. \$8672.
 5. 9000 bu.
 6. 50 bales.

Art. 610.

1. \$12140.
 2. 30000 lb.
 3. 145 doz.

Page 194.

4. \$400 80.
 5. 1500 A., and
 \$202.50.
 6. 1750 lb.

Art. 611.

1. \$67.50.
 2. 225 bbl.

3. $4\frac{1}{2}\%$.
 4. \$10635.53.
 5. $33\frac{1}{2}\%$.
 6. 2000 lb. and
 \$16.20 com.
 7. \$1818.60.
 8. \$1184.60.
 9. \$1097.40.
 10. $4\frac{1}{2}\%$.

Page 195.

11. \$1888.
 12. 194122.3 lb. and
 \$2773.17.
 13. 1890 bbl. and
 \$1.11.
 14. \$648.
 15. \$1508.57 com.
 and \$16091.43
 16. $5\frac{1}{2}\%$.
 17. 10666 $\frac{2}{3}$ yd.
 18. \$7021.21.
 19. 45.278%.
 20. \$30859.74.
 21. 253 bbl. and
 \$9.68.

Page 196.

22. \$31403.75.
 23. 26250 corn, and
 19200 barley.
 24. 5%.
 25. \$5044.29.
 26. Remitted \$566 $\frac{2}{3}$,
 com. \$113 $\frac{1}{3}$,
 and rate 16 $\frac{2}{3}\%$.
 27. Loss of \$280.26.
 28. \$26023.50.
 29. 21071.52 bu.
 30. Barley, 14333 $\frac{1}{2}$
 bu.; hops, 560-
 67.13 lb.; and
 com. \$508.40.

Page 199.**Art. 634.**

1. \$22.80.
 2. \$300.38.
 3. \$8000.
 4. \$72.
 5. \$234.
 6. 40 gal.
 7. \$144.

Page 200.**Art. 637.**

1. \$5645.50.
 2. \$1744.80.
 3. \$1178.80.
 4. \$2188.60.
 5. \$10656.40.

Art. 638.

1. \$904.70
 2. \$686.35.
 3. \$900.90.
 4. \$3565.68.
 5. \$757.20.
 6. \$10531.70.
 7. \$1970.90.
 8. \$3812.50.

Page 201.**Art. 649.**

1. \$91.20,

Page 202.

2. \$386.25.

Art. 651.

1. $\frac{5}{8}\%$, \$62.50.
 2. \$3550.
 3. \$537.50.
 4. $4\frac{1}{2}$ mills, \$70.
 5. \$35.35.

Page 203.

6. $3\frac{1}{2}$ mills, \$888-
 83.13, S. T.
 \$27553.77.
 7. $2\frac{1}{2}$ mills; $2\frac{1}{2}$
 mills.
 8. $4\frac{1}{2}$ mills; \$110.63
 9. \$54.05.
 10. \$462.33.
 11. \$.005 $\frac{2}{3}$, and \$85-
 17.
 12. 1.25669 ¢.

Page 206.**Art. 677.**

1. \$5000.
 2. 470.
 3. \$1600, \$2400,
 \$2000.
 4. \$1576.50.
 5. \$523.75.
 6. \$4450.

Page 207.

7. \$27411.17.
8. \$11454.55.
9. \$14791.67.
10. \$16853.56.
11. $\frac{1}{4}\%$.
12. \$1967.96.
13. \$60.
14. \$6093.40.
15. \$3200.
16. \$18242.66.
17. H, \$26522.73;
M, \$43099.43;
A, \$23207.39;
Phoenix, \$265-
22.73; and Prov-
ident, \$26522.73.
18. \$5000.
19. \$47500, and \$39-
375.

Page 208.

20. G., \$630; H.,
\$150; and M.,
\$337.50.
\$142.50 gain.
 $7\frac{1}{8}\%$.

Page 211.**Art. 703.**

1. \$258.30.
2. \$47.67.
3. \$75.60.
4. \$364.50.
5. \$457.10.

Page 212.

6. \$131.39.
7. \$675.13.
8. \$570.
9. \$322.58.
10. \$523.80.
11. \$102.10.
12. \$223.12.
13. \$2800.53.
14. \$74.43.
15. \$300.28.
16. \$220.26.
17. \$1662.50.
18. \$132.86.
19. \$1798.30.
20. \$717.27.
21. \$791.78.

22. \$2161.54.
23. \$4981.67.
24. \$98.90.
25. \$780.
26. \$100.83.
27. \$96.75.
28. \$16975.

Page 213.

29. \$74.94.
30. \$2707.18.
31. \$4310.74.
32. \$2040.15.
33. \$2766.55.
34. \$1969.62.
35. \$3519.75.
36. \$2837.92.

Page 214.**Art. 705.**

1. \$343.75.
2. \$4099.71. ✓
3. \$337.41.
4. \$857.01.
5. \$27500.
6. \$826.23.

Page 215.**Art. 707.**

1. \$445.94.
2. \$10344.83.
3. \$600.
4. \$1000.
5. \$291.85.
6. \$739.13.
7. \$1954.63.

Art. 709.

1. $7\frac{1}{2}\%$.
2. 7% .
3. $6\frac{3}{4}\%$.
4. 6% .
5. 5% .
6. 6% .
7. 10% .

Page 216.**Art. 711.**

1. 2 yr. 5 mo. 24 da.
2. 3 yr. 10 mo. 12
da.
3. April 25, 1881.
4. 11 mo.
5. Sep. 22, 1889.
6. 12 yr. 6 mo.

Page 218.**Art. 715.**

1. \$4.38.
2. \$5.25.
3. \$3.71.
4. \$17.44.
5. \$9.24.
6. \$1.13.
7. \$5.83.
8. \$3.33.
9. \$1.76.
10. \$5.70.
11. \$2.34.
12. \$8.02.
13. \$6.32.
14. \$3.00.
15. \$24.58.
16. \$1.15.
17. \$4.37.
18. \$11.65.
19. \$1.34.
20. \$10.96.

Page 219.

21. \$6.61.
22. \$19.71.
23. \$25.80.
24. \$10.60.
25. \$25.38.
26. \$2.39.
27. \$4.86.
28. \$8.22.
29. \$6.18.
30. \$2.82.
31. \$7.79.
32. \$1.79.
33. \$8.78.
34. \$14.30.
35. \$1.41.
36. \$1.47.
37. \$4.42.
38. \$12.87.
39. \$10.50.
40. \$7.70.
41. \$12.
42. \$35.
43. \$1.29.
44. \$70.
45. \$5.50.
46. \$16.53.
47. \$11.66.
48. \$5.70.
49. \$29.17.

50. \$.53.

Art. 716.

1. \$79.20.
2. \$29.96.
3. \$80.84.
4. \$104.45.

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5. \$73.41.

Art. 718.

1. \$5.18.
2. \$11.53.
3. \$3.62.
4. \$2.80.
5. \$5.19.
6. \$16.81.
7. \$3.95.
8. \$5.10.
9. \$16.95.
10. \$4.50.

Art. 719.

1. \$191.26.
2. \$90.88.

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3. \$45.89.

Page 222.**Art. 729.**

1. \$448.70.
2. \$1546.70.
3. \$366.60.
4. \$2422.30.
5. \$12726.80.

Page 223.**Art. 734.**

1. \$372.96.
2. \$96.45.
3. \$459.34.
4. \$417.84.
5. \$1198.09.
6. \$2319.22.

Page 226.**Art. 737.**

1. \$2766.75.
2. \$1020.30.
3. $4\frac{1}{2}\%$.
4. 8% .

Art. 738.

1. \$889.58.
2. \$1773.73.

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3. \$1618.32.
4. \$6386.77.
5. \$879.71.

Art. 739.

1. 4 yr. 2 mo.
2. 6.716%.
3. 13.23%.
4. \$920.08.
5. \$20.72.
6. \$8681.12.
7. $1\frac{1}{2}\%$.
8. 3 yr. 7 mo. 24 da.
9. \$10505.94.
10. $7\frac{1}{2}$ yr.
11. \$6856.52.
12. $6\frac{1}{2}\%$.
13. \$4644.61.

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14. May 18, '89.
15. Gain \$875.
16. \$2728.82.
17. Grace, \$7678.96;
Mabel, \$7031.85;
Flora, \$4333.02.
18. \$299.20.
19. $4\frac{1}{8}\%$.
20. \$1372.81.
21. \$4128.37.
22. 725 M.
23. \$2660.
24. 50 years.
25. Herbert, \$5938.66;
Theodore, \$4847.72.

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26. \$536.95, better to invest in land.
27. \$33884.38.
28. \$1508.75.
29. \$12006.80.
30. Chas., \$5364.99;
John, \$4590.03;
Walter, \$3895.34.

Page 230.**Art. 744.**

1. \$540.
2. \$99.75, and \$38.75.
3. \$1654.61.

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4. Interest, \$12.29.
5. \$.55, better to pay cash.
6. No difference.
7. Loss \$11.82.
8. \$1523.15.
9. \$7.48.
10. 4% loss.
11. \$7481.30.
12. Cash offer, \$8.37.
13. $21\frac{3}{4}\%$ profit.
14. Gain \$1202.
15. \$44.85, and $8\frac{1}{8}\%$.
16. \$683.22.
17. \$4000.
18. \$340.13.
19. \$29.79.

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20. \$37.82.
21. \$1756.27.
22. \$272.58.
23. \$9736.94.
24. \$10855.79.

Page 236.**Art. 778.**

1. Bk. Dis., \$9.38;
Pro., \$740.62.
2. Bk., Dis., \$1.67;
Pro., \$284.83.
3. Bk. Dis., \$22.08;
Pro., \$1302.92.

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4. Bk. Dis., \$4.47.
5. Proc., \$988.37.
6. Gain, \$11.35.
7. Bk. Dis., \$13.92;
Pro., \$1251.50.
8. Proc., \$1749.47.

9. Jan. 28, 1889;
Term of Dis.,
27 days; Pro.,
\$284.71.
10. Feb. 29, 1888;
19 da., \$799.09.
11. Aug. 6, 1888; 66
days; \$664.99.
12. Mar. 3, 1899; 179
days; \$2382.89.
13. Dr., \$125.29.

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14. \$1900.41 to their credit.
15. \$2865.30.

Art. 780.

1. \$320.
2. \$1250.
3. \$923.87.
4. \$2461.96.
5. \$2150.
6. \$691.12.
7. \$175.08.

Page 241.**Art. 790.**

1. \$199.27.
2. \$523.68.
3. \$1100.85.
4. \$4.07.

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5. \$100.53.
6. \$1550.07.
7. \$1.73.
8. \$1890.50.

Page 242.**Art. 792.**

1. \$202.98.
2. \$640.89.
3. \$39.18.
4. \$563.53.

Page 247.**Art. 801.**

1. Oct. 16, 1888.
2. Oct. 13, 1888.
3. Sep. 7, 1888.
4. Mar. 20, 1889.

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5. Nov. 15, 1888.
6. Dec. 29, 1887.
7. Jan. 14, 1889.
8. Jan. 11, 1888.

Page 249.**Art. 803.**

1. Nov. 12, 1888.
2. Dec. 21, 1888.
3. Jan. 10, 1889.
4. Feb. 12, 1889.
5. June 13, 1888.
6. May 9, 1889.
7. May 14, 1888.
8. Mar. 7, 1889.

Page 255.**Art. 806.**

1. Nov. 28, 1886.
2. Feb. 11, 1887,
\$300.
3. \$313.18.
4. \$601.73.
5. Oct. 14, 1888.

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6. \$100. Nov. 25,
1890.
7. Sep. 21, 1889.
8. Dec. 23, 1886.
9. Sep. 4, 1887.
10. Jan. 24, 1888,
\$348.88.

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11. July 24, 1887,
\$421.
12. \$300, Dec. 27,
1887, \$300.42.
13. Feb. 16, 1889.
14. \$100, May 17, '89,
\$98.93.

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15. \$400, Jan. 4, '88,
\$435.23.
16. Dec. 7, 1888.

Page 261.**Art. 820.**

1. 4.
2. 28.
3. 7.
4. 81.
5. \$288.

6. \$4.30+.
7. $733\frac{1}{2}$ ft.
8. 478 bu.
9. \$2812.50.
10. 2 yr. 9 mo. $22\frac{1}{2}$ days.

Page 262.**Art. 823.**

1. $26\frac{2}{3}$ A.
2. 350 rd.
3. \$567.
4. \$1290.
5. \$384.75.
6. $124\frac{1}{2}$ yd.
7. $7\frac{1}{2}\%$
8. 93 da.

Page 265.**Art. 841.**

1. A., \$26250; and B., \$15750.
2. \$6589.41 gain; Hadley, \$2758.36; and Hunt, \$3831.05.
3. Whole capital, \$58800; D's gain, \$1200.
4. A, \$9409.52; B, \$7939.29; and C, \$7351.19.
5. N. Insol., \$4543.75. N. Invest., \$2321.25.
6. Gained, \$12090, A's P. W., \$504.5; B's P. W., \$37.95; Solv., \$8840.
7. Harrison, \$7000; Morton, \$1000.

Page 267.**Art. 842.**

1. A, \$276.92; B, \$346.15; C, \$276.93.
2. A, \$596.53; B, \$559.25; C, \$994.22.

3. Martin's gain, \$1737.93; Eaton's gain, \$1862.07; Martin's P. W., \$3737.93; Eaton's P. W., \$6362.07.

4. A, \$17.40; B, \$29.70; and C, \$48.

5. A's investment, \$8491.30; B's investment, \$6065.22; C's investment, \$4043.48.

6. A, \$4548.39; B, \$2951.61.

7. A, \$36; B, \$32; C, \$20; and D, \$4.

8. A, \$731.57; B, \$1483.93; and C, \$784.50.

9. Net gain, \$5200; Olsen's share of net gain, \$1640.27, Thompson's share of net gain, \$3559.73.

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10. Simmon's, \$170.20.59, and Sawyer, \$16329.41.

11. Drews' gain, \$8058.14; Allen's gain, \$6837.21; Brackett's gain, \$6104.65.

12. B, \$838.10; and A, \$1561.90.

13. Martin, \$3126.53; Gould, \$3104.46; and Cole, \$1269.01.

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2. Hopkins's gain, \$4873.27; Haw-

ley's gain, \$4526.73.

3. Charles, 12¢; John, 8¢; Walter, 4¢.

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4. Net resources, \$23564.25; net solvency, \$23564.25; net gain, \$1064.25.

5. Investm't of each, \$9866 $\frac{2}{3}$; Gray's gain, \$2161.69; Snyder's gain, \$1894.93; Dillon's gain, \$2843.33; Gray's P. W., \$11028.36; Snyder's P. W., \$7261.60; Dillon's P. W., \$16210.04.

6. Phelps, \$8000; Rogers, \$18300; Wilder, \$30900.

7. Smith, \$16170.43; Jones, \$11990.32; Brown, \$9339.25.

- Smith's P. W., \$28870.43; Jones's P. W., \$16990.32; Brown's P. W., \$16239.25.

8. Burke, \$17533.33; Brace, \$17833.33; Baldwin, \$17633.33.

9. Loss, \$22747.09; Briggs' loss, \$5907.62; Parson's loss, \$16839.47; net insolvency, \$2187.09; Briggs' P. W., \$127.38; Parson's insolv., \$2314.47.

Page 271.

10. Bradley's gain, \$7517.61; Maben's gain, \$8362.39; Bradley's P. W., \$23417.61; Maben's P. W., \$26362.39.

11. Wilkins' share, \$1000; Ames' share, \$3000.

12. \$17445.24; A's P. W. \$30165.84; B's P. W. \$22417.09; C's P. W. \$22417.08. A's share of gain, \$6033.17; B's share of gain, \$4483.42; C's share of gain, \$4483.41.

13. A's capital, \$3865.80; B's capital, \$2577.20; insolvency, \$5557; A's insolvency, \$3334.20; B's insolvency, \$2222.80.

Page 272.

14. Net resources, \$89735; net gain, \$44335; Clay's share of gain, \$17932.89; Hard's share of gain, \$24380.43; Dunn's share of gain, \$2021.68. Clay's P. W. at closing, \$34132.89; Hard's P. W. at closing, \$53080.43; Dunn's P. W. at closing, \$7521.68.

Page 275.**Art. 856.**

1. \$22500. Martin Bishop's dividend, \$93.75.
2. \$9000. B's dividend, \$54.
3. \$96250. A's dividend, \$44.
4. \$2625. C's dividend, \$87.50.
5. Sinking fund, \$855.38. Dividend, \$16000. Undivided profits, \$11657.37.

Page 276.**Art. 858.**

1. $3\frac{1}{2}\%$.
2. 8%.
3. 18%.
4. Dividend, 8%. Undivided profits, \$850.
5. Dividend, 9%. Undivided profits, \$1701.

Art. 860.

1. \$16000.
2. \$25000.
3. \$17000.
4. \$89800.
5. \$19500.
6. \$15000.
7. \$72500.

8. \$16000.
9. \$33800.
10. \$21600.

Page 277.**Art. 862.**

1. \$4637.50.
2. \$2362.50.
3. \$6030.
4. \$4350.
5. \$14400.
6. \$3625.
7. \$12750.
8. \$21900.
9. \$77062.50.
10. \$50625.

Art. 864.

1. $16\frac{2}{3}\%$.
2. $11\frac{1}{2}\%$.
3. 30%.
4. $16\frac{2}{3}\%$.
5. $30\frac{1}{2}\%$.
6. 20%.
7. $12\frac{1}{2}\%$.
8. $6\frac{1}{5}\%$.
9. $5\frac{5}{8}\%$.
10. $12\frac{1}{2}\%$.

Page 278.**Art. 865.**

1. 5%.
2. $6\frac{1}{2}\%$.
3. Dividend, \$22500. C's dividend, \$255.
4. \$11456.25.
5. 120 shares.

6. \$28781.25.
7. Brokerage, \$100.125%.
8. \$8250.
9. \$750000.
10. Dividend, \$2250. Horace Brown's dividend, \$168.75.
11. \$119250.
12. 133 $\frac{1}{2}$.
13. \$5068.75.
14. \$200.
15. \$4575.
16. 5%. Undivided profits, \$6192.95.
17. \$109509.38.
18. \$40400.
19. Market value, \$48412 50. Par value, \$52500.
20. Smith, $\frac{2}{3}\%$.
21. $6\frac{2}{3}\%$.
22. Net gain, \$515.61.
23. \$25500.
24. $4\frac{1}{2}\%$.
25. Total investment, \$20468.76. Total income, \$1200.

Page 284.**Art. 880.**

1. \$38.32.
2. \$1964.06.
3. \$2582.58.
4. \$315.51.

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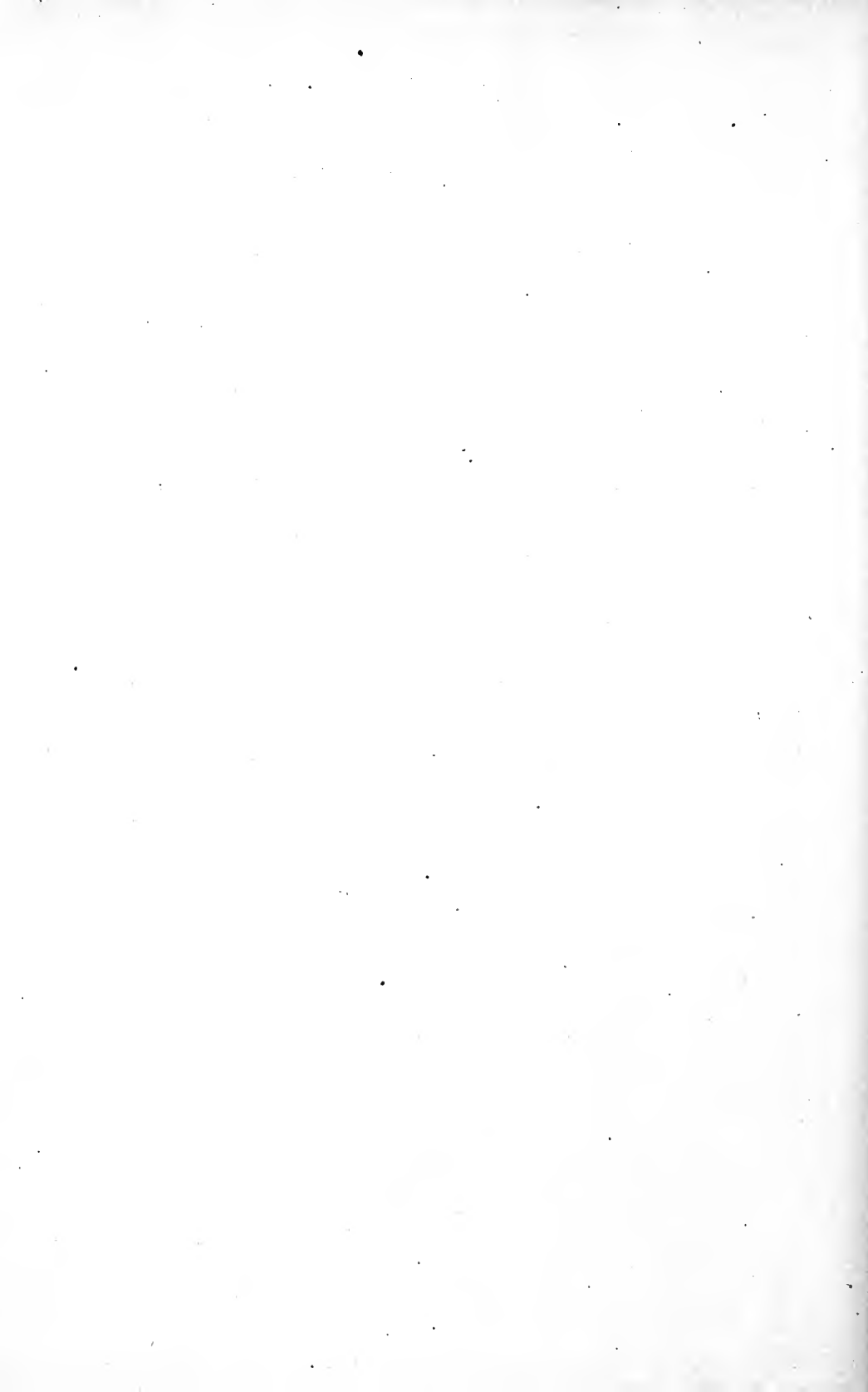
5. \$8696.16.
6. \$1379.
7. \$1871.30.
8. \$676.96.
9. \$3756.30. \$24.70, gain.
10. \$5080.98. \$344.42, gain.

Page 289.**Art. 890.**

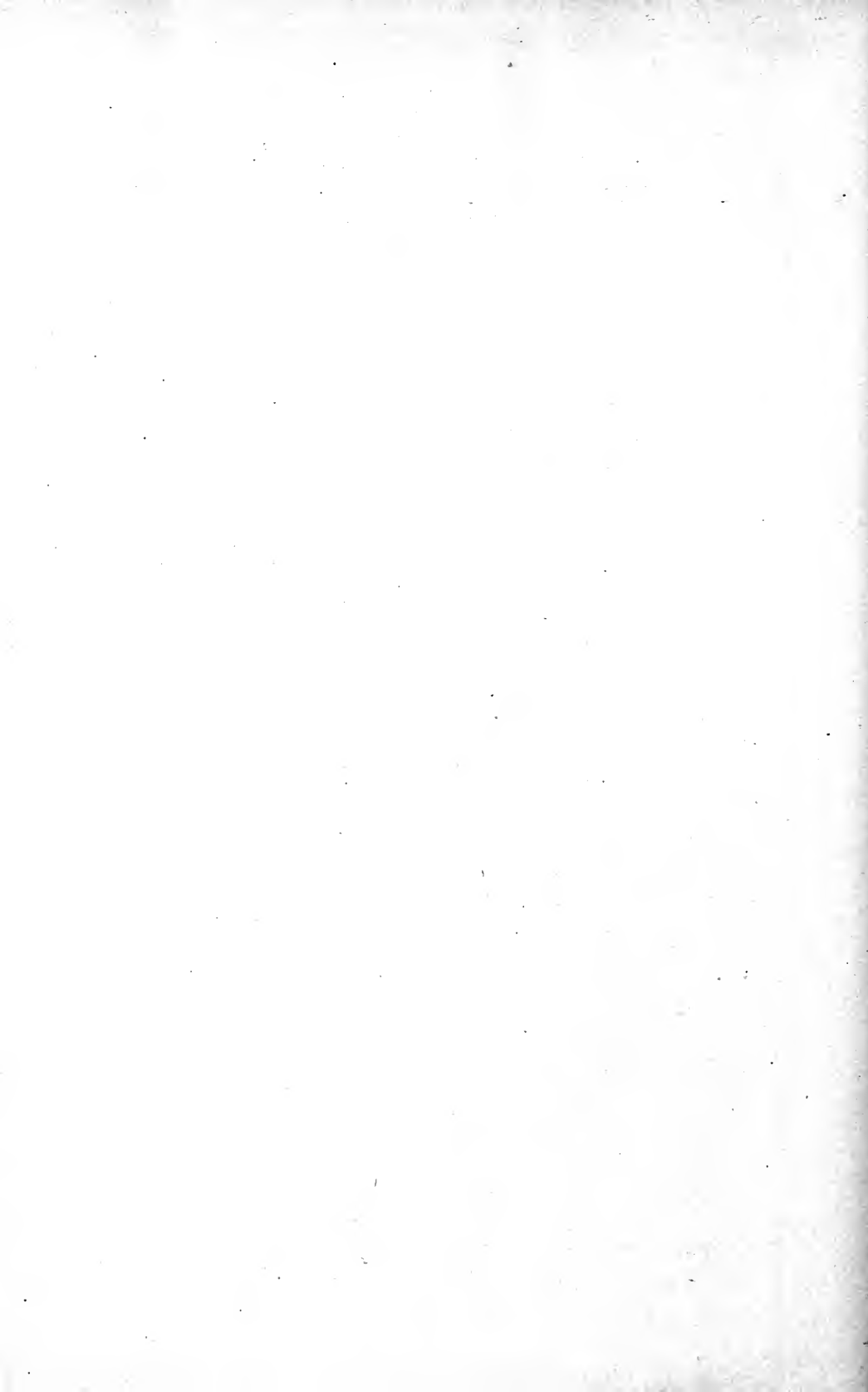
1. 11157 francs, 30 centimes.
2. \$1948.65.
3. £510, 10s. 8d.

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4. \$5103.48.
5. £1197, 1s. 3d.
6. 25000 marks.
7. \$1666.87.
8. 6646 francs, 77 centimes.
9. 1864 marks.
10. \$25876.26.
11. 5.18 $\frac{1}{2}$.
12. 13400 marks.
13. 40 $\frac{3}{4}$.
14. \$2191.68.
15. \$10558.96.
16. 186 $\frac{1}{2}$.
17. 100 $\frac{3}{4}$.
18. 66 $\frac{1}{2}$.
19. 52 $\frac{1}{2}$.
20. 116 $\frac{3}{4}$.







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